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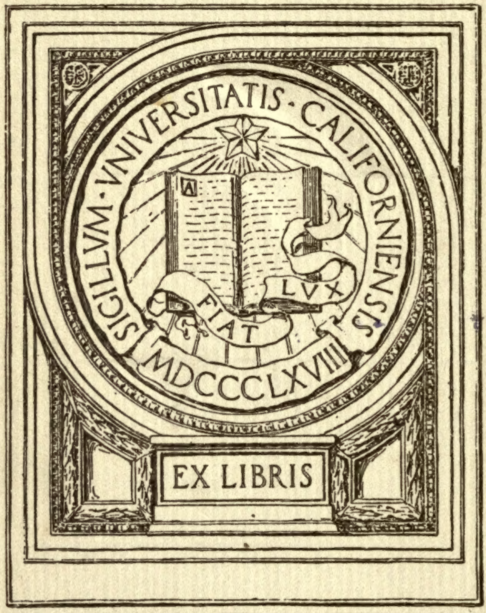

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A SHORT HISTORY OF SCIENCE

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BY

W. T. SEDGWICK AND H. W. TYLER

Professor of Biology *Professor of Mathematics*
at the
Massachusetts Institute of Technology
Cambridge

The history of science should be
the leading thread in the history of civilization.
— SARTON.

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PREFACE

This book is the outgrowth of a lecture course given by the authors for several years* to undergraduate classes of the Massachusetts Institute of Technology, the chief aims of the course being to furnish a broad general perspective of the evolution of science, to broaden and deepen the range of the students' interests and to encourage the practice of discriminating scientific reading.

There are of course excellent treatises on the history of particular sciences, but these are as a rule addressed to specialists, and concern themselves but little with the important relations of the sciences one to another or to the general progress of civilization. The present work aims to furnish the student and the general reader with a concise account of the origin of that scientific knowledge and that scientific method which, especially within the last century, have come to have so important a share in shaping the conditions and directing the activities of human life. The specialist in any branch of science is finding it more and more difficult to keep himself informed, even to the indispensable minimum extent, as to current progress in his own field, — and hence his frequent neglect of all other branches than his own.

It may reasonably be expected that some attention to the history of science on the part of students will give them a better understanding of the broad tendencies which have determined the general course of scientific progress, will enlarge their appreciation of the work of successive generations, and tend to guard them against falling into those ancient pitfalls which have bordered the paths of progress. In the words of Mach: —

> There is no grander nor more intellectually elevating spectacle than that of the utterances of the fundamental investigators in their gigantic power.

---

* By the senior author since 1889.

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> Possessed as yet of no methods — for these were first created by their labors and are only rendered comprehensible to us by their performances — they grapple with and subjugate the object of their inquiry and imprint upon it the forms of conceptual thought. Those who know the entire course of the development of science will . . . judge more freely and more correctly the significance of any present scientific movement than those who, limited in their views to the age in which their own lives have been spent, contemplate merely the trend of intellectual events at the present moment.

At a time when the forces of science are being diverted from the promotion and conservation of civilization to its destruction, and when attempts are being made to turn the waters now flowing in the stream of science back into ancient and so-called classical channels, it will be well for the general reader no less than the student of science to review its history, and to judge for himself concerning its proper place in contemporary life and education. Many volumes would be required to depict the lives of the workers, — often marked by self-denial and sometimes by persecution, — to trace the full significance of their achievements, or to portray the spirit animating their labors; — that spirit of science to which, regarding it as a critic rather than a votary, impressive tribute has been paid by one of our modern seers: —

> A greater gain to the world . . . than all the growth of scientific knowledge is the growth of the scientific spirit, with its courage and serenity, its disciplined conscience, its intellectual morality, its habitual response to any disclosure of the truth.
>
> — *F. G. Peabody.*

It has naturally been foreign to the purpose of the authors to admit matter too technical for the general student or, on the other hand, too slight in its influence on the general progress of science. The division of responsibility between them corresponds roughly to that implied by the title "mathematical" and "natural sciences", and emphasis has been laid on interrelations rather than on distinctions between the various sciences. The mathematical group from their relatively greater age and higher development afford the best examples of maturity; the natural sciences illustrate more clearly recent progress. No attempt has been made by the authors to follow an encyclopædic plan, under which all fields should receive proportional space and treatment, each by a competent representative, but some fullness of presentation has been aimed at in the particular branches with which they are themselves familiar, with briefer indication of developments along other lines.

The authors gladly acknowledge their indebtedness to many men of science interested in their undertaking, and to the special histories already referred to, on which their own work is largely based. Many brief typical quotations from the more important authorities are given as a basis for wider or more special study, but no systematic attempt has been made to examine original sources. No one can possibly be more aware than are the authors of the shortcomings of their work, and corrections of errors, from which a book of this kind cannot hope to have escaped, will be welcomed.

MASSACHUSETTS INSTITUTE OF TECHNOLOGY,
CAMBRIDGE, 1917.

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A SHORT HISTORY OF SCIENCE

CHAPTER I

EARLY CIVILIZATIONS

'The night of time far surpasseth the day' said Sir Thomas Browne; and it is the task of Archaeology to light up some parts of this long night. — *Charles Eliot Norton.*

THE ANTIQUITY AND ANCESTRY OF MAN. — It is now generally agreed that men of some sort have been living upon this earth for many thousand years. It is also, though perhaps less generally, agreed that mankind has descended from the lower animals, precisely as the men of to-day have descended from men that lived and died ages ago.

The history of science, however, is not so much concerned with the ancestry or origin of mankind as with its antiquity; for while science is a comparatively recent achievement of the human race, its roots may be traced far back in practices and processes of prehistoric and primitive times. Mankind is very old, but science so far as we know had no existence before the beginning of history, *i.e.* about 6000 years ago, and until 2500 years ago it occurred if at all only in rudimentary form. The best opinion of to-day holds that man has been on this earth at least 250,000 years, and in spite of wide variations is of one zoölogical " kind " or " species " and three principal types or " races," viz., white or Caucasian, yellow or Mongolian, and black or Ethiopian (Negroid). These great races are believed to have had a common ancestry in a more primitive race, and this in turn to have descended from the lower animals. It is furthermore held that there was probably one principal place of origin, or "cradle," of the human race from which have spread all known varieties of mankind, alive or extinct, and that this was probably in "Indo-Malaysia" in that remarkable valley which lies between the rivers Tigris and Euphrates and in its upper part is known as Mesopotamia (between the rivers).

> Mesopotamia, or the broad valley of the Tigris and Euphrates, was the cradle of civilization in the remotest antiquity. There can be little doubt that man evolved somewhere in southern Asia, possibly during the Pleiocene or Miocene times . . . . [And] as paleolithic man was certainly interglacial in Europe, we may assume that he was preglacial in Asia. . . .
>
> The earliest known civilization in the world arose north of the Persian Gulf among the Sumerians . . . . but the Babylonians of history were a mixed people, for Semitic influences according to Winckler began to flow up the Euphrates Valley from Arabia during the fourth millennium B.C. This influence was more strongly felt, however, in Akkad than in Sumer, and it was in the north that the first Semitic Empire, that of Sargon the Elder (about 2500 B.C. according to E. Meyer) had its seat. . . . The supremacy of Babylon was first established by the Dynasty of Hamurabi (about 1950 B.C., earlier according to Winckler) which was overthrown by the Hittites about 1760 B.C. Then followed the Kassite dominion, which lasted from about 1760 to 1100 B.C. . . . It was probably due to them that the horse, first introduced by the Aryans, became common in southwest Asia; it was introduced into Babylon about 1900 B.C. but was unknown in Hamurabi's reign. — *Haddon.*

ARCHÆOLOGY. — The study of antiquity, and especially of prehistoric antiquity, is known as archæology (the science of antiquities or beginnings), and is based upon finds of ruins, tools, weapons, caves, skeletons, carvings, ornaments, and similar remains or evidences of human life and action in prehistoric times. It has been well described as "unwritten history." Remains of all kinds have long been roughly but conveniently classified into three groups corresponding to three periods of development, viz.: a Stone Age, a Bronze Age, and an

Iron Age, according to the use of stone, bronze and iron implements.

PREHISTORIC MAN. — If therefore we would begin the history of science at the very beginning, we must turn far backward in imagination to a time when the human race was barely superior to the beasts that perish. Absorbed in a fierce struggle for existence, the passing generations had little history and left behind them no permanent records. In one respect nevertheless mankind stood far above the beasts; namely, in possessing the power of language, by which they could not only communicate more readily one with another, but also convey to their descendants through oral *tradition* something of whatever they might possess of accumulated knowledge. Eventually, though slowly, the generations began to leave behind them more enduring *records*, — at first crude and fragmentary, in the form of tools, cairns, and other monuments, or in drawings, paintings, or carvings, on ivory or rocks or trees, or on the walls of caverns, — which should serve to inform or instruct other men. Finally, but still slowly, and especially out of this so-called "picture-writing," grew the art of writing, which furnished a means of keeping permanent records of the past and a new and more perfect way of communication between living men and races of men. We who have ourselves witnessed some of the consequences of improvements in the arts of communication between men and nations, such as have recently been effected by steam transportation and telegraphy and telephony, can to some extent realize how much the introduction of the rudiments of the art of writing may have meant in the progress of prehistoric and primitive mankind.

THE SCIENCE OF MANKIND. ANTHROPOLOGY. — The various steps in the evolution of mankind and in the earliest development of civilization and the arts form the subject matter of one of the youngest of the sciences, anthropology, to works upon which the reader is referred who would pursue these matters further. One of the earliest and still one of the most interesting of these, Man's Place in Nature, by Huxley, is now a classic. Another, also somewhat out of date but still very valuable, entitled

"Anthropology," is of special interest because its author, E. B. Tylor, was the founder of the science and is still living (in 1916).[1]

THE CHILDHOOD OF THE RACE. — There is reason to believe that the human race, in its long and slow development, has passed through periods of essential childhood and youth, very much as the individual human being passes slowly through infancy onwards; and that, precisely as the individual begins his intellectual life in wonder, questioning, and curiosity, so the race has advanced from a condition of childish wonder, questionings, and interpretations of mankind and the external world, — sun, moon, and stars, thunder and lightning, wind, rain, and snow, — which have gradually developed into more mature and more scientific explanations. This principle of an essential parallelism between individual development and racial, named by Haeckel "the biogenetic law," will be found especially pertinent at many stages in the history of science.

PRIMITIVE INTERPRETATIONS OF NATURE. — As the child thinks he sees in almost everything some living agency, — because most of the things that happen about him are obviously connected with himself, or his parents, or his nurses, or other children, or with his pets, — so man in the childhood of the race and in its earlier development sees in the wind some hidden being or personality bending the tree, or shaking the leaves, or moaning or sighing in the forest, or roaring angrily in thunder. Only a slightly different imagination is required to see in the sun, moon, and planets supernatural beings or gods travelling across the heavens, and by association, since they seem to visit his heavens daily or monthly or at other regular intervals, to believe that they are somehow concerned with himself and his welfare or destiny. From this primitive interpretation to the modern astronomical knowledge of the immensity, the movements and the paths, the temperatures, and even the chemical composition, of those enormous lifeless masses which we call sun, moon, and stars, has been a long and laborious journey, — how long no one can tell. It is still almost always possible to find tribes or peoples somewhere on the earth living under one or more of the various conditions which the more highly developed peoples have apparently passed through, and there is no great difficulty in finding primitive tribes to-day holding such childish interpretations of nature as we have just described. This circumstance enables anthropologists, ethnologists, and historians to draw with considerable confidence the broader outlines of the probable history of the more highly developed nations, such as those of western Europe and North America, — nations in the progress of which, since the beginning of the nineteenth century, science has played a notable part.

---

[1] The latest edition of Sir John Lubbock's [Lord Avebury's] "Prehistoric Times" should also be consulted. Other easily accessible volumes are A. C. Haddon's "The Wanderings of Peoples" (Cambridge Manuals of Science and Literature) and J. L. Myres' "The Dawn of History" (Home University Library Series). The chapters on "Modern Savages" in Lord Avebury's "Prehistoric Times" are especially instructive. Most important of all is Professor H. F. Osborn's recent work, "Men of the Old Stone Age."

The first stepping-stones towards scientific knowledge are wonder and curiosity, and peoples are still to be found so low in intelligence as to be almost destitute of curiosity. As a rule, however, most human beings, no matter how primitive, have some curiosity concerning, and some sort of explanation for, the commonest events, such as day and night, life, death, sickness, health, sun, moon, stars, winds, seasons, and the like. And one of the commonest, simplest, and probably most natural, is that already referred to as the childish or personal interpretation of nature; viz., that which assumes everything to be in a sense alive and possessed of some sort of being, animation, or personality, kindred to man's own. This primitive interpretation has been called *animism.* At present, however, the term *animatism* finds more favor among certain anthropologists, apparently for the reason that the notion of mere diffuse vitality, or general "animation," is even more primitive, as observed in certain peoples of low development, than is the idea of a specific "soul" (*anima*) differentiated from the body and possessing a separate existence. For example, a tree blown by the wind may seem to a man of very low development to be merely quivering with life, and bending before some more powerful but invisible influence, diffused, hazy, unembodied, and without personality or name (*animatism*). Or it may seem to be an individual tree, bent by an invisible but powerful being like a man and perhaps having a name such as "Boreas" (the Greeks' name for the north wind). In this latter case we have the assumption of personality and, by analogy with man, of the presence and influence of a spirit or soul (*animism*).

PREVALENCE OF ANIMISM IN ANTIQUITY. — Judging by the opinions and beliefs of races which still exist in very low stages of development, prehistoric man when he pondered at all, reasoned largely in the direction of animism. He interpreted himself and his actions by his own ideas, will, feelings, and desires, and reasoned that other things were actuated likewise. If, for example, he killed an ox or a man by a blow, and later an ox or a man were killed by lightning, it was reasonable to assume that some invisible and manlike being had given the ox or man an invisible blow. The oldest records of the human race confirm this idea. The ancient Assyrians, Babylonians, and Egyptians "animated" much of what we today call inanimate, *i.e.* inorganic, nature; and Greek and Hebrew poetry are full of survivals of this view of man and nature, which on the higher levels passes into personification and anthropomorphism. The establishment of a hierarchy of the gods of Greece, such as was supposed to dwell upon Mt. Olympus, is merely a further differentiation of the same kind. "The Hellenic gods and goddesses are glorified men and women."

SOURCES OF INFORMATION CONCERNING PREHISTORIC AND ANCIENT TIMES. — These are of three kinds, tradition, monuments (including tools, implements, pottery, and other objects which have survived to the present time, more or less in their original form), and inscriptions. Of these *tradition,* because readily subject to perversion, is the least reliable and need not be further considered. It is *monuments,* such as ruins, tombs, weapons, pottery, implements, ornaments, furniture, and the like, upon which we must chiefly depend for our knowledge of prehistoric times, and the evidence which has been gradually accumulated from finds of this sort is extensive and trustworthy and correspondingly valuable. With the introduction of *inscriptions* of all sorts, including drawings, pictures, hieroglyphics, and writings of every kind, upon tablets, monuments, walls, caves, clay cylinders, papyri, parchments, and the like, from about the eighth or tenth century B.C., we enter upon the historical period. From that time forward we have more or less of the raw material from which we may reconstruct the beginnings, not only of civilization and art, but also of literature and science.

SOME ANCIENT LANDS AND PEOPLES. — From the standpoint of European history, and especially the history of science, the most important peoples of antiquity were the Babylonians, Assyrians, Egyptians, and Phœnicians. The Babylonians and Assyrians occupied the fertile valley of the Tigris and Euphrates; the Egyptians, that of the Nile; and the Phœnicians the eastern slopes of the Mediterranean basin (modern Syria). The first three peoples were chiefly agricultural; the last, chiefly seafaring, mercantile, and industrial.

BABYLONIA AND ASSYRIA. — These, lying almost side by side, may be considered together, although Babylonia furnishes the older and the more important civilization. Babylon and Nineveh were the chief cities of the two countries, the former in Mesopotamia on the Euphrates, the latter above and to the northeast, and much nearer the mountains, on the Tigris.

In that part of Asia which borders upon Africa, to the north of Arabia and the Persian Gulf, in an almost tropical region at the foot of the Armenian highlands, defended by mountains on the east and bounded by desert on the west, opens the broad valley of the Tigris and the Euphrates rivers which, flowing from the same mountains and in the same direction and maintaining for a long distance a parallel but independent course, join at last and fall together into the Persian Gulf. In the month of April these two rivers, swollen by the melted snows in the mountains of Armenia, overflow, sinking again to the level of their beds in June. The country around them therefore was very similar to the Nile valley. A large number of canals joined the Tigris to the Euphrates, and distributed the water rendered by the tropical climate necessary for agriculture.

The upper part of the country inclosed between the two rivers was properly called Mesopotamia, a term used also roughly to designate the whole. The valley of the Upper Tigris, or Upper Mesopotamia, was Assyria, and the lower part of both valleys Babylonia. . . . In these two fertile regions flourished two empires, the Chaldean-Babylonian and the Assyrian.

The Chaldeans, says a trustworthy authority, appear to have been a branch of the great Hamite race of Akkad, which inhabited Babylonia from the earliest times. With this race originated the art of writing, the building of cities, the institution of a religious system, and the cultivation of all science, and of astronomy in particular. In the primitive Akkadian tongue were preserved all the scientific treatises known to the Babylonians. It was in fact the language of science in the East, as the Latin was in Europe during the Middle Ages. When Semitic tribes established an empire in Assyria in the thirteenth century B.C., they adopted the alphabet of the Akkad, and with certain modifications applied it to their own language. . . . The mythological, astronomical, and other scientific tablets found at Nineveh, are exclusively in the Akkadian language, and are thus shown to belong to a priestly class, exactly answering to the Chaldeans of profane history and of the Book of Daniel. . . .

From about 747 B.C., the accession of Nabonassar, the line of kings at Babylon is supplied by the well-known work of Ptolemy, the geographer. . . . Babylon, according to ancient historians, was surrounded by walls over three hundred feet in height and eighty in thickness, and was divided into two parts by the river Euphrates, which flowed through it. Narrow streets led to the river, on which they opened by gates. Quays enclosed the water, and towards the centre a bridge crossed it, but the bridge was movable and was only used during the day. At night the two sides of the river were completely separated. . . . When, at the present time, we visit these formerly prosperous countries, we can scarcely believe in the universal fertility that so many witnesses have described. The carelessness of the Turkish administration has allowed the irrigation canals to be silted up, and the inundations now form unhealthy swamps in the delta of the Tigris and Euphrates. Mesopotamia was wonderfully productive in wheat and barley, the enormous returns obtained by Bablyonian farmers from their corn-lands being unexampled in modern times; but it possessed neither olives, figs, nor vines; millet and sesame, however, grew luxuriantly. Date-palms abounded, and furnished a large part of the food of the inhabitants.

The people of Assyria and Chaldea were as skilled in manual handicrafts as in the cultivation of the earth. They wove cloths of brilliant colors; they also ornamented their garments with a profusion of embroideries, and wore magnificent tiaras. Babylonian embroidery was celebrated even in the days of the Roman empire. The manufacture of carpets, one of the chief luxuries in the East, attained wonderful perfection at Babylon, as well as the manufacture of personal attire. Their furniture, by its richness and shape, differed completely from anything we find in present use amongst Orientals; the Assyrians used arm-chairs or sat on stools, and dined as we do from tables. The tables and chairs were handsomely decorated and in good taste, and it is curious to note that the same designs for ornamentation were in use then as we have now — lions' claws, animals' heads, etc.; and even at the present time the ancient models might be studied with profit and copied with advantage. They were skilful in working hard as well as soft materials. The cylinders of jasper and crystal and the bas-reliefs of Khorsabad sculptured in gypsum or in basalt equally denote their proficiency. They were acquainted with glass and with various kinds of enamel, and they knew how to bake clay for the manufacture of bricks or of porcelain vases. Moreover, the art of varnishing earthenware and of covering it with paintings by means of coloured enamel was well known at Nineveh.

The cuneiform writing — so called because it is formed by pressure of the stylus on the soft surface of the clay tablets, producing a mark like a wedge or arrow-head — is a development of hieratic, itself an improvement on the primitive hieroglyphic. The hieratic characters had been scratched with the point of the stylus on the clay that served the Mesopotamian peoples for paper. The use of the stylus in cuneiform, gave a single element, by the employment of which in various combinations, all the letters of the alphabet were formed. When the Persians conquered Mesopotamia they published their decrees, etc., in the three chief dialects of their subjects — the Persian, Median, and Assyrian. Hence the trilingual inscriptions which have supplied the key to cuneiform interpretation. The discovery of the interpretation of the famous inscription at Behistun, on the Persian frontier, in three languages, Persian, Median, and

Assyrian, enabled Sir Henry Rawlinson to find the key to the Assyrian characters. . . .

It is very difficult, in spite of the numerous texts deciphered by modern savants, to form any idea of Assyrian literature; yet the literature must have been considerable, for Layard found a complete library founded by King Asshurbanipal in two of the rooms of his palace at Nineveh. This library consisted of square tablets of baked earth, with flat or slightly convex surface, on which the cuneiform writing had been impressed while the clay was soft, before baking. The characters were very clearly and sharply defined, but many of them so minute as to be read only with the help of a magnifying glass. These tablets, which are preserved at the British Museum, contain a kind of grammatical encyclopedia of the Assyrio-Babylonian language, divided into treatises; and also fragments of laws, mythology, natural history, geography, etc. Treatises on arithmetic were also found in the library, proving that mathematical sciences were known, with catalogues of observations of the stars and planets. We have already mentioned that astronomy was greatly honored amongst the Chaldean priesthood, who had studied the course of the moon with so much precision that they were able to predict its eclipse.

Science and literature developed, in spite of a primitive writing engraved upon clay tablets; the art of sculpture was already highly refined; monuments, which without being majestic like the Egyptian were imposing in their size and splendid in their colours; rare elegance in clothing and furniture, denoting great wealth, the result of active commerce; a cruel, even ferocious character, revealed by their treatment of prisoners, and indeed by all their history; a learned caste, devoting themselves to the sciences and also to the unscientific methods of astrology; a religion elevated by the primitive idea of a supreme god, yet degraded by polytheism and often by gross debauchery; kings sufficiently intelligent to construct splendid palaces and immense cities, and yet inflated with pride and glorying in the most stupid cruelty — such is the picture opened to us by the records of Assyrian and Babylonian history. When we observe on the Assyrian bas-reliefs all the industries and all the arts, we are inclined to acknowledge that they were superior to the nations that surrounded them, and we understand how the Greeks drew inspiration from Assyrian work as well as from Egyptian. — *Verschoyle.* History of Civilization.

If, in a final summing up, the question be asked, What was the legacy which Babylonia and Assyria left to the world after an existence of more than three millenniums, the answer would be, that through the spread of dominion the culture of the Euphrates Valley made its way throughout the greater part of the ancient world, leaving its impress in military organization, in the government of people, in commercial usages, in the spread of certain popular rites such as the various forms of divination, in medical practices and in observation of the movements of heavenly bodies—albeit that medicine continued to be dependent upon the belief in demons as the source of physical ills, and astronomy remained in the service of astrology — and lastly in a certain attitude towards life which it is difficult to define in words, but of which it may be said that, while it lays an undue emphasis on might, is yet not without an appreciation of the deeper yearnings of humanity for the ultimate triumph of what is right. — *Morris Jastrow, Jr.* The Civilization of Babylonia and Assyria.

Egypt. — Another highly important ancient civilization whose beginnings are lost for us in the darkness of prehistoric times is that which flourished in the valley of the Nile.

Near the point where Africa approaches Asia lies a narrow valley, walled in by two ranges of mountains, enclosed on the farther side by two deserts, and fertilized by the periodical inundations of a mighty river. This long and narrow strip of verdure, surrounded by mountains and menaced by the desert sands, is Egypt. . . . A few years ago, the beginnings of Egyptian history, and even the source of the great river that fertilizes the land of Egypt, were hidden in mystery. The sources of the Nile have been at last discovered, and archæologists have now retraced the commencement of a history which is practically the commencement of all authentic history. To Speke and Grant in 1862, and to Baker in 1864, we owe the knowledge of the lakes Victoria Nyanza and Albert Nyanza, whence come the abundant waters, that swollen by the equatorial rains, at fixed intervals overflow and fertilize with their mud the soil that borders their bed, and refresh a land which lies beneath a sky where a rain-cloud is seldom seen. We know, too, how the abundant harvests that regularly result from the inundations of the Nile, returning ample food to moderate labour, promoted the development of the Egyptian nation;

how the Nile itself supplied to them a highway for communication, rendered doubly useful by the north winds that blow up stream more than eight months of the year, carrying the traffic up into the interior while the current carries it down; how the Arabian desert on the east and the Libyan on the west secured to them comparative immunity from invasion and opportunity for internal progress. . . .

One of the prizes of Napoleon's expedition, a black basalt stone, disinterred at Rosetta in 1798, and now in the British Museum, bore three parallel and horizontal inscriptions, all quite distinct. One was in hieroglyphics, the second in the characters called demotic or popular, the third in Greek. Although a great many scientific men exhausted their skill upon this trilingual inscription, which was a triple inscription of the same text, no one could make the Greek characters exactly apply to the hieroglyphic signs. Champollion was the first to obtain any success, as early as 1812, but further progress was largely aided by the labors of Thomas Young [a name associated in the history of science with those of Fresnel and Helmholtz]. . . .

Protected from invasion by the same deserts that isolated them, the people who came from Asia and settled in the Nile valley applied themselves to the regulation of the periodical inundations, and to the distribution of the water. They built towns on the hillocks, in order that the water should not reach them; and afterwards, with the stones that the two mountain ranges of Libya and Arabia contain in abundance, and by the means of transit afforded by the Nile, they erected monuments that have defied the course of the centuries. . . .

The paintings in the tombs also show us men at work upon all the arts and all the handicrafts. 'We see there the workers in stone and in wood, the painters of sculpture and of architecture, of furniture and carpenters' work; the quarrymen hewing blocks of stone; all the operations of the potter's art; workmen kneading the earth with their feet, or with their hands; men at work making stocks, oars and sculls; curriers, leather-dyers, and shoe-makers, spinners, cloth-weavers with various shaped looms, glass-makers; goldsmiths, jewellers, and blacksmiths.' Among the antiquities still in a state of good preservation there is much pottery, including vessels of simple earthenware and enamelled faience, enamelled and sculptured terracotta, glass, often resembling Venetian, metal work and jewellery, and linen cloth as fine as Indian muslin. — *Verschoyle.*

PHŒNICIA. — Two other civilizations of importance, the Phœnician and the Hebrew, existed in antiquity between the Mediterranean Sea and the great Arabian desert, in what are to-day called Syria and Palestine.

By the side of the Hebrew nation, which owed its grandeur to its moral and religious development, dwelt the Phœnicians, a people who owed their fame to their maritime and commercial enterprise. They occupied a narrow strip of land between Lebanon and the Mediterranean, Phœnicia proper being but 28 miles long by one to five miles broad, and the territory of the Phœnicians being, at the utmost, no more than 120 miles long by 20 wide. . . . The forests which clothed the chain of Lebanon supplied the Phœnicians with timber for their ships, and they soon made the Mediterranean a high road for their navy. Enclosed by mountains in a country that prevented their acquiring any inland empire, they became a maritime power, the first in the ancient world in order of importance as in order of time. Egyptian documents mention the Phœnician towns of Gebal, Beryta, Sidon, Sarepta, etc., as early as sixteen or seventeen centuries before the Christian era. The Phœnicians served as middlemen to the great civilizations of the Nile and the Euphrates, their vessels easily coasting along to the mouth of the Nile, and their caravans having but a short journey to reach the point where the middle Euphrates almost touches Upper Syria, whence the current would carry them down to the quays of Babylon. . . . To the westward the Phœnicians sailed beyond the Mediterranean and ventured upon the Atlantic Ocean. They coasted the western side of Africa, and early accounts record their discoveries of wonderful islands of marvellous fertility and charming climate, the 'Fortunate Isles,' — probably Madeira and the Canaries. They also sailed along the coasts of Spain and Western France and reached Northern Europe. Gades (Cadiz) was the starting point for these long and dangerous voyages, which extended as far as Great Britain, where a considerable trade in tin was carried on. . . . The Phœnicians were the great mining people of the ancient world. Gold, silver, iron, tin, lead, copper, and cinnabar were obtained from Spain, still the chief metalliferous country of Southern Europe. The details given by Diodorus concerning the Spanish mines are very circumstantial. 'The copper, gold, and silver mines are wonderfully productive,' and 'those who work the copper mines draw from the rough ore one quarter of the weight in pure metal.' . . . The Phœnicians not only brought the mineral wealth of Spain to the Eastern world, but they had also a great trade in wheat, wine, oil, fruits of all kinds, and fine wool. They provided Asia with the products of Spain and Gaul, Sicily and Africa with the products of Asia. But this maritime commerce could only be supplied by an inland trade, which served to connect the countries that were a long distance from the sea. Phœnicia found itself one of the ports of Asia, the merchandise of distant countries was brought to it, and from it was exported all the produce of the Asian continent. The caravans supplemented the fleets, and the fleets distributed the burdens of the caravans. The land trade was chiefly in three directions — to the south it followed the route to Arabia and India; to the east, that to Assyria and Babylon; to the north, that to Armenia and the Caucasus.

The Phœnicians were not only the great maritime, the great commercial, and the great mining power of antiquity, they were also one of the chief manufacturing powers. Like the Egyptians and Assyrians, they were skilful potters, and they discovered the art of making glass. 'It is said,' writes Pliny the elder, 'that some Phœnician merchants, having landed on the shores of the river Belus, were preparing their meal, and not finding suitable stones for raising their saucepans, they used lumps of *natron*, contained in their cargo, for the purpose. When the *natron* was exposed to the action of the fire, it melted into the sand lying on the banks of the river, and they saw transparent streams of some unknown liquid trickling over the ground; this was the origin of glass.' No matter how it may have originated, there is no doubt that the Phœnicians manufactured glass on a large scale, and their glass-work became celebrated all over the world. Dyeing works, however, take the first rank among Phœnician industries, and Tyrian purple was one of the chief objects of luxury among the ancients. The word 'purple' was not used only for a single colour, but for a particular kind of dye, for which animal colours obtained from the juice of certain shellfish were used. The dyeing works could not be carried on without cloths, for the Phœnicians dyed woollen materials chiefly with their famous purple. The wool came from Damascus, and the greater part of their export of woollen stuffs was doubtless of their own manufacture. Sidon was the first town that became noted for these fabrics. Homer often mentions tunics from that town, but afterwards they were manufactured all over Phœnicia, and particularly at Tyre. Among the products of Phœnician industry we must also mention the numerous ornaments and the articles whose value depends largely on their workmanship. The trade of barter which they had so long maintained with barbaric races, amongst whom these objects always find an appreciative market, had incited the Phœnicians to apply themselves to these industries. Chains of artistically worked gold were worn by Phœnician navigators in Homer's time, and Ezekiel mentions their curious work in ivory, which they procured through Assyria from India, and from Ethiopia. Accident has preserved the names of only a small number of the articles produced by the Phœnicians, but the existence of these among a rich and luxurious people implies the existence of others.

The Phœnician religion was a worship of personified forces of nature, especially of the male and female principles of reproduction. It was in a popular and simple form a worship of the sun, the moon, and the five planets, regarded as intelligent powers actively affecting human life. . . . And the Phœnician religion not only consecrated licentiousness, it also sanctioned cruelty. Living children were offered as burnt sacrifices to Baal as well as to Moloch. One can scarcely understand how human sacrifices could have been endured by an intelligent people; but this abominable ritual was in force in all the colonies, and especially at Carthage, where during the siege of the city by Agathocles, about 307 B.C., two hundred boys of the best families were offered as burnt sacrifices to the planet Saturn.

Though we have but few fragments of Phœnician antiquities and literature, we at least know their system of writing. It is now proved that the Phœnicians did not invent writing; they merely communicated letters to the Greeks. . . . The Greeks adopted the Phœnician characters with only a few modifications; the Latin races used the same letters designed more simply; they had received them at a very remote date, for the Latin tongue was a sister not a daughter of the Greek. The French, Spanish, and Italian languages are all derived from the Latin and use the same characters, while even the Teutonic languages, like English and German, have adopted this alphabet. The Phœnicians must, on this ground alone, take high rank in the history of civilization. . . .

The Phœnicians were not only the pioneers of industry, but by their commerce they brought together the peoples of the three continents of the Old World. The first carriers by sea, acting as intermediate agents between the different nations, they exchanged ideas as well as merchandise; their exploration of different countries led to the discovery of new riches; they endowed the West with the products of the East, and the East with the products of the West. They proved to the world that cities can attain a high degree of prosperity by labour, activity, and economy, and they remain examples of the highest development of purely commercial qualities. — *Verschoyle.*

THE HEBREWS. — South of Phœnicia and lying between the Arabian Desert and the Mediterranean was Palestine: —

The whole literature of the Hebrews is included in the collection of prose and poetry which we call the Bible, or, more accurately, the Old Testament. The simplicity of its narratives, the enthusiasm of its hymns, the joyful or plaintive melody of the psalms, the fiery eloquence of the prophets, place the Bible, independently of its religious and historical importance, high among the great literary monuments of antiquity. Their literature is a proof that the poetic imagination was fully developed amongst the Hebrews, and that the people were deeply thoughtful as well as passionately religious. . . . The Hebrews were not an artistic or an industrial people; but they possessed an indisputable superiority to all other nations of antiquity in their purely spiritual religion, and in their appreciation of the supreme importance of morals as the proper expression of religion. Religion was their rule of life, the maker of their laws, the pervading spirit of the whole community, as in no other nation before or since.
— *Verschoyle.*

THE EMERGENCE OF EUROPEAN CIVILIZATION. — Until very recently little was known of European events before the writings of Herodotus (484–425 B.C.). Within the last half century, however, and largely as a result of the labors of the archæologists Schliemann and Evans, the existence of a wonderfully rich and complete prehistoric civilization has been revealed on the shores and islands of and near the Greek Peninsula.

The recent discoveries in Crete have added a new horizon to European civilization. A new standpoint has been at the same time obtained for surveying not only the ancient classical world of Greece and Rome, but also the modern world in which we live. — *Sir Arthur Evans.*

Ægean Civilization in the Bronze Age. — The newer European civilization had also its prehistoric times, and the investigations and especially the excavations of the last half century have revealed such treasures as the site of Homeric Troy, the palace and tomb of Agamemnon, and the cities of Minos and others of the sea kings of Crete. It is now known that the Trojan War was fought about Hissarlik on the eastern shore of the Dardanelles; that Agamemnon's palace was at Mycenæ in Greek Argolis; and that Minos had his home and his naval base of Mediterranean sea power on the island of Crete. In Mycenæ and in Crete the arts were highly developed. Painting, sculpture, and pottery, tools, weapons, implements of various kinds, with systems of water supply and drainage, testify to the remarkable degree of civilization attained in the later Bronze Age, although this has left behind it no written records and was formerly known to us only through the poems of Homer.

Even to classical students twenty, nay, ten years ago, Crete was scarcely more than a land of legendary heroes and rationalized myths. It is true that the first reported aeronautical display was made by a youth of Cretan parentage, but in the absence of authenticated records of the time and circumstances of his flight, scholars were sceptical of his performance. And yet within less than ten short years we are faced by a revolution hardly more credible than this story; we are asked by archæologists to carry ourselves back from A.D. 1910 to 1910 B.C., and witness a highly artistic people with palaces and treasures and letters, of whose existence we had not dreamed. . . .

The theme is a fresh one, because nothing was known of the subject before 1900; it is important, because the Golden Age of Crete was the forerunner of the Golden Age of Greece, and hence of all our western culture. The connection between Minoan [Cretan] and Hellenic civilization is vital, and not one of locality alone, as is the tie between the prehistoric and the historic of America, but one of relationship. Egypt may have been foster-mother to classical Greece, but the mother, never forgotten by her child, was Crete. . . .

Members of three foreign nations have worked in friendly rivalry to learn the buried history of Crete . . . and Cretan soil may be said to have been found to teem with pre-Hellenic antiquities. The hopes of archæologists have been abundantly justified. We have followed them and arrived at the home of the first European civilization. — *Hawes.* Crete, the Forerunner of Greece.

Even at this exceedingly early stage of human progress, the various branches of industry had become fairly separated and specialized, more so, perhaps, than in the Homeric period, and a considerable variety of tools was employed in the various crafts. The carpenter was evidently a highly skilled craftsman, and the tools which have survived show the variety of work which he undertook. At Knossos a carefully hewn tomb held, along with the body of the dead artificer, specimens of the tools of his trade — a bronze saw, adze, and chisel. 'A whole carpenter's kit lay concealed in a cranny of a Gournia house left behind in the owner's hurried flight when the town was attacked and burned. He used saws long and short, heavy chisels for stone and light for wood, awls, nails, files, and axes much battered by use; and what is very important to note, they resemble in shape the tools of to-day so closely that they furnish one of the strongest links between the first great civilization of Europe and our own.' Such tools were, of course, of bronze. Probably the chief industry of the island was the manufacture and export of olive oil. The palace at Knossos has its Room of the Olive Press, and its conduit for conveying the product of the press to the place where it was to be stored for use; and probably many of the great jars now in the magazines were used for the storage of this indispensable article. — *Baikie.* Sea Kings of Crete.

THE IRON AGE. THE GREEKS OR HELLENES. — Soon after the arrival of the Iron Age, and probably not far from 1200–1000 B.C., a new people became prominent on the shores of the Ægean. These were the Greeks or, as they called themselves, Hellenes, — inhabitants of Greece or Hellas. Their precise origin is unknown, but they were undoubtedly of Indo-European stock and probably came, in part at least, from the north. It has been conjectured that their conquest of the existing inhabitants was facilitated by, if not due to, their possession of weapons of iron. Of the earlier part of this new period (1200–800 B.C.) we have only the legendary accounts of the Homeric and Hesiodic poems, which are now generally believed to be based upon, if not actually descriptive of, episodes of this age.

The Hellenes soon supplanted the Phœnicians as traders in the southern Ægean; and "if we now leave the monuments of the Egyptian temple or the Assyrian palace and turn to the pages of the Iliad and the Odyssey . . . at once we are in the open air, and in the sunshine of a natural life. The human faculties have free play in word and deed. . . From the first the Greek is resolved to confront the facts of life." — *Jebb.*

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CHAPTER II

EARLY MATHEMATICAL SCIENCE IN BABYLONIA AND EGYPT

In most sciences one generation tears down what another has built and what one has established another destroys. In Mathematics alone each generation builds a new story to the old structure. — *Hankel.*

A HISTORY of science may be based on some more or less definite logical system of definitions and classifications. As a matter of historical evolution, however, such systems and such points of view belong to relatively recent and mature periods. Science has grown without very much self-consciousness as to how it is itself defined, or any great concern as to the distinction between pure and applied science, or as to the boundaries between the different sciences. Mathematics, for example, has had its roots in the human need of exact statement as to both number and form in all sorts of affairs, and on the other hand in the analytical faculties of the human mind, which have shaped the development of the pure science and given it in course of time its deductive stamp.

The origin of a science can seldom be precisely determined, and the more ancient the science the more difficult is the attainment of such precision. The periods at which primitive men of different races began to have conscious appreciation of the phenomena of nature, of number, magnitude, and geometric form, can never be known, nor the time at which their elementary notions began to be so classified and associated as to deserve the name of science. Very early in any civilization, however, mathematics must obviously have taken its rise in simple processes of counting and adding, of time measurement in primitive astronomy, of the geometry and arithmetic involved in land measurement and in architectural design and construction. We can safely sketch certain rough outlines of the prehistoric picture, and we can to some extent verify these, on the one hand, by archæological evidence, on the other, by present-day observations of backward races — still in their prehistoric stage.

Primitive Astronomical Notions. — On the astronomical side the most obvious fact is the division of time into periods of light and darkness by the apparent revolution of the sun about the earth. With closer attention it must soon have been observed that the relative length of day and night gradually changes, and that this change is attended by a wide range of remarkable phenomena. At the time of shortest days, vegetable and animal life (in the north temperate zone) is checked by severe cold. With the gradually lengthening days, however, snow and ice sooner or later disappear, vegetation is revived, birds return from the warmer south, all nature is quickened. In the symbolism of the beautiful old myth, the sleeping princess, our earth, is aroused by the kiss of the sun-prince. The longest days and those which succeed them are a period of excessive heat and of luxuriant vegetation, followed by harvests as the days shorten, towards the completion of the great annual cycle. In time, closer observers, noting the stars, discovered that corresponding with this great periodic change are gradual variations in the starry hemisphere visible at night, that in other words the sun's place among the stars is progressively changing, that it is in fact describing a path completed in a large number of days, which after repeated counting is found to be 365. It is also found that the midday height of the sun above the southern horizon shares in the annual cycle. The determination of the number of days in the year is a matter of very gradual approximation, possible only to men who have already attained some command of numbers and the habit of preserving records extending over a long series of years. For there is no well-marked beginning of the year as of the day. An erroneous determination of the number of days becomes apparent only after a number of years, increasing with the accuracy of the original approximation. If, for example, the year is assumed to be exactly 365 days, that is, about six hours too short, the festivals and other dates will slip back about 24 days in a century, and thus lose their original correspondence with climatic conditions. A revision of the calendar will become necessary.

Still another natural period is introduced by the motion of the moon, which seems like the sun to have a daily motion about the earth, and also to describe a closed path among the stars in a period of about 29 days. Unlike the sun, however, the moon has during this period a remarkable change of apparent shape and luminosity from "new" to "full" and back again. The study of the day, the year, the month, thus naturally determined by the great heavenly bodies has led to the development of the calendar with greater and greater accuracy, the most recent rectification of the length of the year dating only (in England) from 1752. The difficulty of expressing the precise length of the month and the year in days, causing the imperfection of early calendars, has, on the other hand, reacted to the advantage of mathematical astronomy by demanding the greatest possible precision both of observation and of the computation based upon it.

The Planets. — Another celestial phenomenon, though less obvious than the foregoing, must have found wide recognition in prehistoric times. The stars vary widely in grouping and individual brilliancy, but in general their relative positions are sensibly constant. To this constancy, however, five exceptions are easily discovered in the wandering motion of the planets Mercury, Venus, Mars, Jupiter, and Saturn, which like sun and moon have their several paths among the stars but with seemingly irregular motions. Corresponding to these seven bodies there was set up by prehistoric people an arbitrary division of time into weeks of seven days, "the most ancient monument of astronomical knowledge." The correspondence with the planets is still preserved in the names of the days of the week in several modern languages.[1] The

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	FRENCH	ITALIAN	
[1] Sunday	dimanche	domenica	(Sun)
Monday	lundi	lunedi	(Moon)
Tuesday	mardi	martedi	(Mars)
Wednesday	mercredi	mercoledi	(Mercury)
Thursday	jeudi	giovedi	(Jupiter)
Friday	vendredi	venerdi	(Venus)
Saturday	samedi	sabato	(Saturn)

further division of time into hours, minutes, and seconds has followed more arbitrarily, and in connection with the development of progressively improved methods of time measurement.

Astrology and Cosmology. — Side by side with the development of elementary astronomy on its observational and mathematical sides were evolved in intimate connection with it, but sometimes in extraordinary imaginative forms, astrology and cosmology, dealing respectively with the supposed influence of the heavenly bodies on human affairs, and with the structure and organization of the world. Both these pseudo-sciences were inextricably blended, under priestly and literary influences, with a bewildering mass of superstition and mythology, legend and invention. In their earlier stages, both doubtless contributed powerfully to interest and progress in real science. Ultimately both have had to be torn away, as the scaffolding from a cathedral, in the never ending process of releasing truth from error.

Primitive Counting. — On the arithmetical side the present counting processes of primitive peoples have particular interest. The distinction between one and two similar objects, and that between two and three or more, belong to a relatively early stage of development, but tribes are known to-day in which the entire number scale is one, two, many (*i.e.* more than two). The process of counting is naturally facilitated by the use of fingers and toes as counters, their number 10 being the well-known anatomical basis for our denary or decimal number system. This may be illustrated by the following passages from E. B. Tylor's Primitive Culture: —

> Father Gilij, describing the arithmetic of the Tamanacs on the Orinoco, gives their numerals up to 4; when they come to 5, they express it by the word *amgnaitone,* which being translated means 'a whole hand'; 6 is expressed by a term which translates the proper gesture into words *itacono amgnapona tevinitpe,* 'one of the other hand,' and so on up to 9. Coming to 10, they give it in words as *amgna aceponare,* 'both hands.' To denote 11 they stretch out both the hands, and adding the foot they say *puitta-pona tevinitpe,* 'one to the foot,' and so on up to 15, which is *iptaitone,* 'a whole

foot.' Next follows 16, 'one to the other foot,' and so on to 20, *tevin itoto,* 'one Indian;' 21, *itacono itoto jamgnar bona tevinitpe,* 'one to the hands of the other Indian'; 40, *acciache itoto,* 'two Indians,' and so on for 60, 80, 100, 'three, four, five Indians,' and beyond if needful. South America is remarkably rich in such evidence of an early condition of finger-counting recorded in spoken language.

The Zulu counting on his fingers begins in general with the little finger of his left hand. When he comes to 5, this he may call *edesanta* 'finish hand;' then he goes on to the thumb of the right hand, and so the word *tatisitupa* 'taking the thumb' becomes a numeral for 6. Then the verb *komba* 'to point,' indicating the forefinger, or 'pointer,' makes the next numeral, 7. Thus, answering the question 'How much did your master give you?' a Zulu would say '*U kombile*' 'He pointed with his forefinger' *i.e.* 'He gave me seven,' and this curious way of using the numeral verb is shown in such an example as '*amahashi akombile*' 'the horses have pointed' *i.e.* 'there were seven of them.' In like manner, *kijangalobili* 'keep back two fingers,' *i.e.* 8, and *kijangalolunje* 'keep back one finger' *i.e.* 9, lead on to *kumi,* 10; at the completion of each ten the two hands with open fingers are clapped together.

The most instructive evidence I have found bearing on the formation of numerals, other than digit-numerals, among the lower races, appears in the use on both sides of the globe of what may be called numeral-names for children. In Australia a well-marked case occurs. With all the poverty of the aboriginal languages in numerals, 3 being commonly used as meaning 'several or many,' the natives in the Adelaide district have for a particular purpose gone far beyond this narrow limit, and possess what is to all intents a special numeral system, extending perhaps to 9. They give fixed names to their children in order of age, which are set down as follows by Mr. Eyre: 1, *Kertameru;* 2, *Warritya;* 3, *Kudnutya;* 4, *Monaitya;* 5, *Milaitya;* 6, *Marrutya;* 7, *Wangutya;* 8, *Ngarlaitya;* 9, *Pouarna.* These are the male names, from which the female differ in termination. They are given at birth, more distinctive appellations being soon afterwards chosen.

The mathematical advantage of 12 as a base conveniently divisible has often been pointed out, but the choice unfortunately had to be made long before its real significance could possibly be apprehended, and the difficulty of subsequent change would be prohibitive. Vestiges of the use of 5 and of 20 are familiar; the former, for example, in the Roman numerals IV, VI, etc., the latter in such expressions as "three score and ten" and in the French *quatre-vingt.* Increasing maturity of a tribe or race, as of an individual, is accompanied by gain in the command of larger and larger numbers, the rate of progress being very dependent, however, on a fortunate choice of notation. However great the capacity for inventing number-words, it soon becomes necessary to employ some system which shall lead to a regular development of higher from lower names. The selection of a point at which dependent names, and later, symbols shall begin, is one of the most important steps in the history of mathematics. It is difficult for us to realize the extent of our indebtedness to the comparatively recent so-called Arabic — or more properly, Hindu — notation, in which numbers of whatever magnitude may be expressed by means of only ten symbols. In any case, however, the appreciation of large numbers soon becomes vague. To most of us the word *million* is nearly equivalent to an innumerable multitude.

PRIMITIVE GEOMETRY. — On the geometrical side data are naturally more meagre. The notions of a primitive society in regard to areas and perimeters and the ratio of a circumference to its diameter may quite escape discovery. On the other hand skill in making and reading maps is well known — as among the Esquimaux.

RELATION OF GREEK TO OLDER CIVILIZATIONS. — Mathematical science seems to have first assumed definite form in Greece, and it is of particular interest to study the indebtedness of the Greeks to the older civilizations referred to in the preceding chapter. Some degree of civilization doubtless existed further back than any records run, in China, in India, in Babylonia, and in Egypt. But of these only the latter two exerted a determining influence on the general evolution of European science, India making minor though fundamental contributions at a much later stage. Babylonia and Egypt exchanged ideas with each other, and, after unnumbered centuries, furnished Greece with a certain nucleus of scientific knowledge of which the Greeks made enormous use. In practical engineering the achievements of the older civilizations were marvellous, but for the creation of real science as systematized, organized knowledge, containing within itself the seeds of infinite growth, they were quite unequal.

Babylonian Arithmetic. — In Babylonian arithmetic whole numbers were expressed in general by only three of the so-called cuneiform or wedge-shaped characters employed on the tablets, 1 = 𒁹, 10 = 𒌋, 100 = 𒁹 𒀸, but the numbers known to have been used run into the hundred thousands, this naturally implying a highly developed command of the fundamental operations by means of which large numbers are made to depend upon smaller ones. The use of the words for thousand and ten thousand in characterizing an indefinite multitude is illustrated in many scriptural passages, for example: "Saul hath slain his thousands, and David his ten thousands" (1 Sam. xviii. 7); "a thousand thousands ministered unto him, and ten thousand times ten thousand stood before him" (Dan. vii. 10). With such expressions may be compared: "I will make thy seed as the dust of the earth"; "He telleth the number of the stars; he calleth them all by their names" (Gen. xiii. 16; Ps. cxlvii. 4). The number 40 also plays a special rôle in such expressions as the "forty years in the wilderness," the "forty days and forty nights" of rain which caused the flood.

Of remarkable interest in the Babylonian inscriptions is the occurrence, side by side with a decimal system, of a number system based on 60, employed for mathematical and astronomical purposes. A table of squares of the natural numbers presents, for example, nothing novel for the first seven numbers, after which follow, however, the equivalent of

1 4 is the square of 8
1 21 is the square of 9
1 40 is the square of 10
2 1 is the square of 11

Just as in our notation, for example, 325 means three times the square of ten plus twice ten, plus five, so this table must mean: —

once sixty plus four
once sixty plus twenty-one
once sixty plus forty
twice sixty plus one, etc.,

necessarily implying the representation of 60 by 1 in the second place.

In a table of cubes, the perfect cube 4096 is represented similarly by 1 8 16, that is $1 \times 60^2 + 8 \times 60 + 16 = 4096$. The origin of this sexagesimal system has been ingeniously attributed to the blending of two civilizations, one possessing a system based on 10, the other a system based on 6, — a combination suggested by the command of the Persian king that the Ionian troops wait 60 days at the bridge over the Ister; by the splitting of the river by Cyrus into 360 rivulets, etc. Fractions were employed to a limited extent with denominators 60, and $3600 (= 60 \times 60)$. The great step of completing the number system by a character for zero seems not to have been successfully made, though there are indications of an approach to it in later Babylonian times. There is evidence of a mystical or magical use of numbers. Each god, for example, was designated by a number from 1 to 60 according to his rank.

A rational system of weights and measures was introduced, the unit of weight depending on that of length, as in the modern metric system.

Babylonian Astronomy. — In connection with astronomical observations the Babylonians invented a method of measuring time by means of the water clock or clepsydra. From a vessel kept full, water was allowed to escape very slowly into a second vessel in which it could be weighed. To equal weights of water corresponded equal intervals of time.

Starting the flow at the moment the upper edge of the sun first appeared in the east and stopping as soon as the whole sun was visible, the amount of water collected was compared with that escaping from sunrise to sunrise, and the sun's diameter thus determined as $\frac{1}{720}$ of its whole path in the sky. The time required for traversing the whole path — *i.e.* the day — was then divided into 12 double hours, in one of which the sun's disk advanced by its own diameter multiplied by 60. Their use of the number 60 as a base led also to the further subdivision of the hour into 60 minutes of 60 seconds each. The year was reckoned as 365 days, and even the unequal rate of the sun's motion at different periods was recognized.

Particularly noteworthy in connection with Chaldean astronomy is the discovery of a period of 6585 days, — a little more than 18 years, — for the recurrence of eclipses. This would appear to have been based on a long series of observations, but to have taken no account of the region of visibility of eclipses of the Sun. The periods of the planets in their orbits were approximately determined, but there is no evidence of a systematic geometrical theory of celestial motions.

As to accuracy of direct observation it is said that in later Babylonian times angles were measured to within 6 minutes and time to less than a minute. Quantities obtained indirectly by observations extended over long periods, as the length of the lunar month, were naturally determined with correspondingly greater precision.

A list of eclipses of the moon from 747 B.C. was known to Ptolemy, while an astrological work prepared about 3700 B.C. contains evidence of a long series of pre-existing observations. To the Romans the Chaldeans were known as star-gazers, and the art of augury or divination was much cultivated, making some of the earliest known use of geometrical forms. Herodotus ascribes the origin of the sun-dial to Babylonia.

BABYLONIAN GEOMETRY. — In geometry the elementary use of the circle quickly leads to the discovery that a chord equal to the radius subtends one-sixth of the four right angles at the centre, and is thus one side of a regular inscribed hexagon, a figure found on Babylonian monuments. A failure to distinguish between the length of the arc and that of its chord led to the first approximation to the ratio of a circumference to its diameter, $\pi = 3$, which occurs in the Old Testament where King Solomon's molten sea is said to be "ten cubits from the one brim to the other: it was round all about, . . . and a line of thirty cubits did compass it round about."

There is some evidence of a knowledge of the fact that a triangle of sides 3, 4, and 5 has a right angle, and the trisection of the right angle was accomplished. The circle was divided into 360 degrees. The sun-dial and its division into degrees are very clearly mentioned in the books of Kings and Isaiah. Parallels, triangles, and quadrilaterals were used.

We may summarize what we know as to the main features of Babylonian mathematical science as follows: —

In astronomy, records of observations extending over many centuries, the determination of an 18-year eclipse period, the approximate determination of the year as 365 days, a good system of measuring time, the identification of Mercury, Venus, Mars, Jupiter, and Saturn as planets;

In arithmetic, a well-developed sexagesimal system, tables of squares and cubes, arithmetic and geometric progressions, the use of large numbers;

In geometry, the identification of the right triangle of sides 3, 4, and 5, the inscribed hexagon, the division of the circumference into 360 degrees, the crude approximation for the ratio of circumference to diameter, $\pi = 3$.

Mathematical Science in Egypt. — Josephus asserts that the Egyptians learned arithmetic from Abraham, who brought it with astronomy from Chaldea, and that the Egyptians in their turn taught the Greeks. The indebtedness of Greek science to Egyptians must at any rate have been very considerable. The pyramids are monumental evidence of appreciation of geometric form and of a relatively high development of engineering construction nearly 4000 years before the Christian era. Their builders must have had precise geometrical and astronomical notions. In nearly all of the pyramids, for example, the slope of the lateral faces is 52°, and the direction of their base-edges is nearly uniform. The regular inscribed hexagon was known. After an earlier year of 12 months of 30 days each, the

Egyptians added 5 days at the end of each year. According to their legend the god Thot won these days at play from the moon goddess. An edict of 238 B.C. introduced the leap-year, but the innovation was afterwards forgotten. The Egyptian records number more than 350 solar, and more than 800 lunar eclipses before the Alexandrian period.

THE AHMES PAPYRUS. — Our most important source of information in regard to early Egyptian mathematics is the so-called Ahmes manuscript, dating from some time between 1700 and 2000 B.C. "Direction for attaining knowledge of all dark things" are the opening words of this oldest known mathematical treatise. Rules follow for computing the capacity of barns and the area of fields. The text consists, however, rather of actual examples than of rules, the inferring of these being left to the reader. Reference is made to writings some 500 years older, presumably based in their turn on centuries of tradition.

In the computations fractions are used as well as whole numbers, but fractions other than $\frac{2}{3}$ are expressed in terms of fractions with unit numerators. The problem of decomposing other fractions into a limited number of such reciprocals is interestingly treated, examples occurring of considerable complexity. It would appear that such decompositions, effected by special devices or hit upon accidentally, were gradually tabulated as records of mathematical experiment.

The problems discussed by Ahmes include a class equivalent to our algebraic equations of the first degree with one unknown quantity, — the first known appearance of this important idea. Thus, for example: —

"Heap (or quantity) its $\frac{2}{3}$, its $\frac{1}{2}$, its $\frac{1}{7}$, its whole makes 33." In our notation $\frac{2}{3}x + \frac{x}{2} + \frac{x}{7} + x = 33.$

The solution requires the number to be found which multiplying $1 + \frac{2}{3} + \frac{1}{2} + \frac{1}{7}$ shall produce 33. The result appears in the sufficiently intricate form

$$14 + \tfrac{1}{4} + \tfrac{1}{97} + \tfrac{1}{56} + \tfrac{1}{679} + \tfrac{1}{776} + \tfrac{1}{194} + \tfrac{1}{388}.$$

Again: "Rule for dividing 700 loaves among four persons, $\frac{2}{3}$ for one, $\frac{1}{2}$ for the second, $\frac{1}{3}$ for the third, $\frac{1}{4}$ for the fourth, . . . Add $\frac{2}{3}$, $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ that gives $1 + \frac{1}{2} + \frac{1}{4}$. Divide 1 by $1 + \frac{1}{2} + \frac{1}{4}$ that gives $\frac{1}{2} + \frac{1}{14}$. Make $\frac{1}{2} + \frac{1}{14}$ of 700 that is 400." Thus, to modernize this solution, the four persons A, B, C, and D receive on one round $1 + \frac{1}{2} + \frac{1}{4} = \frac{7}{4}$ loaves; the number of rounds is $\frac{700}{1 + \frac{1}{2} + \frac{1}{4}}$ or $\frac{1}{1 + \frac{1}{2} + \frac{1}{4}} \times 700 = 400$, from which the respective shares are readily obtained.

Certain problems show an acquaintance with arithmetic and geometric progressions. Thus, for example, a series is given of the numbers 7, 49, 343, 2401, 16807, the successive powers of 7, accompanied by the words person, cat, mouse, barley, measure. Almost 4000 years later this was interpreted to mean: 7 persons have each 7 cats, each cat catches 7 mice, each mouse eats 7 stalks of barley, each stalk can yield 7 measures of grain; what are the numbers and what is their sum?

Special symbols are used for addition, subtraction, and equality. The Egyptian seems never to have had a multiplication table. Multiplication by 13, for example, was accomplished by repeated doubling, and then by adding to the number itself, its products by 4 and by 8.

Herodotus reports from the fifth century B.C. that the Egyptians reckoned with stones, a practise independently developed in many lands, notably in the form of the abacus. This little computing machine of beads on wires was invented independently in different parts of the ancient world. In China and other parts of the Orient it is still widely and very skilfully employed.

The handbook of Ahmes is also rich on the geometrical side. It contains information in regard to weights and measures, and treats of the conversion from one denomination into another. As in case of the progressions, geometrical problems are given, depending on the use of formulas not derived in the text itself. They include computation of areas of fields bounded either by straight lines or circular arcs, including in the former case only isosceles triangles, rectangles, and trapezoids. An isosceles triangle of base 4 and side 10 is said to have as its area $\frac{4}{2} \times 10 = 20$, the actual area being of course $\frac{4}{2} \times \sqrt{100-4} (= 19.6$ approximately). It is interesting that this and similar crude methods continued in use by surveyors for many centuries, even after Euclid had given geometrical science its modern form. Another problem amounts to finding two squares having a given total area and their sides in a given ratio, being thus equivalent to solving the equations

$$x^2 + y^2 = 100 \qquad\qquad x : y = 1 : \tfrac{3}{4}$$

By trial $x = 1$, $y = \frac{3}{4}$, give $x^2 + y^2 = (\frac{5}{4})^2$.
Since $100 = (\frac{5}{4})^2 \times 8^2$, the trial values must be multiplied by 8, so that $x = 8$ and $y = 6$.

The classical problem of "squaring the circle" is attempted, the result being equivalent to the approximation $\pi = \frac{256}{81} = 3.16$, as against the actual 3.14 — an excellent result for the time.

Other computations deal with the capacity of storehouses — of unknown shape — for grain. A remarkable group of problems deals with a certain geometrical ratio in pyramids equivalent to a modern cosine or cotangent, and of interest in connection with the uniform slope of the great pyramids.

EGYPTIAN LAND MEASUREMENT. — Greek writers emphasize the methods of land measurement of the Egyptians consequent on the obliteration of boundaries by floods of the Nile. Herodotus relates that Sesostris had so divided the land among all Egyptians that each received a rectangle of the same size, and was taxed accordingly. Whoever lost any of his land by the action of the river must report to the king, who would then send an overseer to measure the loss, and make a proportionate abatement of the tax. Thus arose geometry (*geometria* = earth measurement). Diodorus, for example, says: "The Egyptians claim to have introduced alphabetical writing and the observation of the stars, likewise the theorems of geometry, and most of the arts and sciences." The priests "occupy themselves busily with geometry and arithmetic, for as the river annually changes the land, it causes many controversies as to boundaries between neighbors. These cannot be easily adjusted unless a geometer ascertains the real facts by direct measurement. Arithmetic serves them in domestic affairs and in connection with the theorems of geometry; it is also of no slight advantage to those who occupy themselves with the stars. For if the position and motions of the stars have been carefully observed by any people it is by the Egyptians; they preserve records of particular observations for an incredibly long series of years. . . . The motions and times of revolution and stationary points of the planets, also the influence of each on the development of living things and all their good and evil influences have been very carefully observed by them."

Egyptian Geometry. — In a passage written about 420 B.C., the Greek mathematician, Democritus, boasts that "In constructing lines according to given conditions no one has ever surpassed me, not even the so-called rope-stretchers of the Egyptians." The exact orientation of the Egyptian temples required the determination of the meridian and of a right angle. Both processes were naturally an important part of the mathematical lore of the priesthood. The first step was accomplished by observation of the stars. It is believed that the second step was the function of the "rope-stretchers," the name being due to their dependence on a rope of length 12, divided by two knots into sections of 3, 4, and 5. When the two ends of the rope are joined and the three sections drawn taut by the knots, the angle opposite the section 5 is a right angle. The geometrical knowledge thus attributed to the Egyptians of a special case of the Pythagorean proposition does not, of course, imply knowledge of the proposition itself, or even the ability to prove the particular case, which was probably known only empirically. Egyptian architecture made use of geometrical figures as wall decoration and even employed the principle of proportionality, by dividing a blank wall-space into squares before applying the design. The idea of perspective drawing seems, however, not to have been attained.

The existence of such a problem book as that of Ahmes may be considered as fairly implying also the existence of comparable treatises of a more theoretical character, but other evidence of this is lacking.

The main features of Egyptian mathematical science are then as follows: about 2000 B.C. a well-developed use of whole numbers and fractions; a method of solving equations of the first degree with one unknown quantity; an approximate method for finding the circumference of a circle of given radius; approximate methods for finding areas of isosceles triangles and trapezoids; the rudiments of a theory of similar figures.

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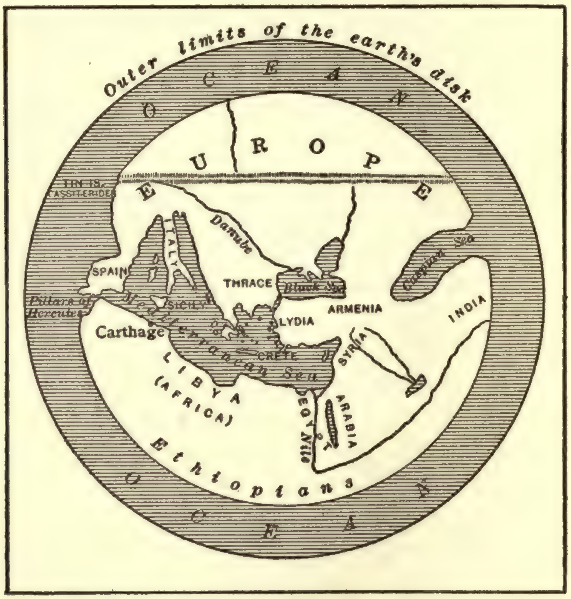

Outer limits of the earth's disk
O C E A N
E U R O P E
TIN IS.
CASSITERIDES
Danube
SPAIN
ITALY
THRACE
Black Sea
Caspian Sea
Pillars of Hercules
SICILY
Carthage
Mediterranean Sea
LYDIA
ARMENIA
INDIA
CRETE
SYRIA
LIBYA
(AFRICA)
EGYPT
Nile
ARABIA
Ethiopians
O C E A N

MAP OF THE WORLD BY HECATÆUS (517 B.C.)

(From Breasted's *Ancient Times.* Courtesy of Messrs. Ginn & Co.)

Hecatæus, a geographer of Miletus, travelled widely, including a journey up the Nile, and wrote a geography of the world. In this book, as in the Map . . . the Mediterranean Sea was the centre and the lands about it were all those known to the author. . . . After the Unknown Historian of the Hebrews [about 850 B.C.] he was the first historical writer of the early world.
— *Breasted.*

CHAPTER III

THE BEGINNINGS OF SCIENCE IN GREECE

Except the blind forces of Nature nothing moves in this world which is not Greek in its origin. — *Sir Henry Sumner Maine.*

A spirit breathed of old on Greece and gave birth to poets and thinkers. There remains in our classical education I know not what of the old Greek soul — something that makes us look ever upward. And this is more precious for the making of a man of science than the reading of many volumes of geometry. — *Poincaré.*

Number, the inducer of philosophies,
The synthesis of letters. — *Æschylus.*

Mathematics, considered as a science, owes its origin to the *idealistic* needs of the Greek philosophers, and not as fable has it, to the *practical* demands of Egyptian economics. . . . Adam was no zoölogist when he gave names to the beasts of the field, nor were the Egyptian surveyors mathematicians. — *Hankel.*

Geographical Boundaries. — From the twilight of civilization and the first faint suggestions of science in Chaldea and Egypt, we pass to the more brilliant dawn of science and civilization in Greece. Geographically we shall be concerned not merely with Greece itself, but, as time passes, with other Hellenic countries, especially the Ionian shores and islands of western Asia Minor, and the Greek colonies in southern Italy, Sicily, and, after its conquest by Alexander the Great, northern Egypt. Greece and its civilization seem immeasurably closer to us both in time and in spirit than do ancient Babylonia and Egypt. In these more remote civilizations science had been cultivated chiefly as a tool, either for immediate practical applications or as a part of the professional lore of a conservative priesthood. In Greece, on the other hand, for the first time in the history of our race, human thought achieved freedom, and real science became possible.

> Mathematics as a science commenced when first some one, probably a Greek, proved propositions about *any* things or about *some* things, without specification of definite particular things. — *Whitehead.*

INDEBTEDNESS OF GREECE TO BABYLONIA AND EGYPT. — It is plain, nevertheless, that Greek civilization and Greek science owed much to Egypt and Chaldea. Herodotus has been quoted already, and Theon of Smyrna (second century A.D.) says: —

> In the study of the planetary movements the Egyptians had employed constructive methods and drawing, while the Chaldeans preferred to compute, and to these two nations the Greek astronomers owed the beginnings of their knowledge of the subject.

Again in the third century A.D. Porphyry observes: —

> From antiquity the Egyptians have occupied themselves with geometry, the Phœnicians with numbers and reckoning, the Chaldeans with theorems.

THE GREEK POINT OF VIEW. — It is not, however, so much the achievements of the Greeks in positive science which compel our attention and admiration as it is the remarkable spirit which they displayed toward man and the universe. Here for the first time we meet with a new point of view, and while Shelley's well-known dictum, "We are all Greeks, our laws, our literature, our religion, our art have their roots in Greece," must be dismissed as incorrect as well as extravagant, and even Sir Henry Maine's maxim, which stands at the head of this chapter, is undoubtedly an exaggeration, these famous sayings serve well to illustrate the fact that with the Greeks came into the world a new spirit and a new interpretation of Nature.

In a striking essay entitled "What we owe to Greece," Butcher has portrayed with extraordinary clearness those characteristics of the Greeks which lifted them above all of their predecessors and above most if not all of those that have come after them: —

The Greeks before any other people of antiquity possessed the love of knowledge for its own sake. To see things as they really are, to discern their meaning and adjust their relations, was with them an instinct and a passion. Their method in science and philosophy might be very faulty and their conclusions often absurd, but they had that fearlessness of intellect which is the first condition of seeing truly. . . . Greece, first smitten with the passion for truth, had the courage to put faith in reason and in following its guidance to take no account of consequences. 'Those,' says Aristotle, 'who would rightly judge the truth must be arbitrators and not litigants.' 'Let us follow the argument wheresoever it leads' may be taken not only as the motto of the Platonic philosophy but as expressing one side of the Greek genius. . . . .

At the moment when Greece has come into the main current of the world's history, we find a quickened and stirring sense of personality and a free people of intellectual imagination. The oppressive silence with which Nature and her unexplained forces had brooded over man is broken. Not that the Greek temper is irreverent or strips the universe of mystery. The mystery is still there and felt . . . but the sense of mystery has not yet become mysticism. . . . Greek thinkers are not afraid lest they should be guilty of prying into hidden things of the gods. They hold frank companionship with thoughts that had paralyzed Eastern nations into dumbness or inactivity, and in their clear gaze there is no ignoble terror. . . . Know thyself, is the answer which the Greek offers to the sphinx's riddle. . . . But to the Greeks, 'know thyself' meant not only to know man but the less pleasing task to know foreigners. . . . The people of ancient India did not care to venture beyond their mountain barriers and to know their neighbors. The Egyptians, though in certain branches of science they had made progress, — in medicine, in geometry, in astronomy, — had acquired no scientific distinction for they kept to themselves, but the Greeks were travellers. . . . Aristotle thought it worth his while to analyze and describe the constitutions of 58 states, including in his survey not only Greek states but those of the barbarian world. . . .

It was the privilege of the Greeks to discover the sovereign efficacy of reason. . . . And it was Ionia that gave birth to the idea which was foreign to the East but has become the starting-point of modern science, the idea that Nature works by fixed laws. . . . Again, in history the Greeks were the first who combined science and art, reason and imagination. . . . The application of a clear and fearless intellect to every domain of life was one of the services rendered by Greece to the world. It was connected with an awakening of the lay spirit. In the East the priests had generally held the keys of knowledge. . . . To Greece then we owe the love of science, the love of art, the love of freedom. . . . And in this union we recognize the distinctive features of the West. The Greek genius is the European genius in its first and brightest bloom.

SOURCES. — The sources of our information as to the details of the scientific ideas of the Greeks are exceedingly meagre, some of the most important historical and scientific treatises being known to us only by title or by detached quotations, or indirectly through Arabic translations. Among specific ancient sources of information in regard to Greek mathematical science the following may be mentioned: —

About 330 B.C., Eudemus, a disciple of Aristotle, wrote a history of geometry of which a summary by Proclus has been preserved.

About 70 B.C., Geminus of Rhodes wrote an Arrangement of Mathematics with historical data. This has also been lost, but quotations are preserved in some of the later authors.

About 140 A.D., Theon of Smyrna wrote Mathematical Rules necessary for the Study of Plato.

About 300 A.D., Pappus' Collections contain much information in regard to the previous development of geometry.

In the fifth century A.D., Proclus published a commentary on Euclid's Elements with valuable historical data.

THE CALENDAR. — The Greek calendar was based at an early period on the lunar month, the year consisting of 12 months of 30 days each. About 600 B.C. a correction was made by Solon, making every two years contain 13 months of 30 days and 12 of 29 days each, giving thus 369 days per year. In the following century a much closer approximation — $365\frac{1}{4}$ days — was attained by confining the thirteenth month to three years out of eight. This arrangement naturally failed, however, to meet the

Greek desire that the months begin regularly at or near new moon, and Aristophanes makes the Moon complain:

CHORUS OF CLOUDS

"The Moon by us to you her greeting sends,
But bids us say that she's an ill-used moon,
And takes it much amiss that you should still
Shuffle her days, and turn them topsy-turvy;
And that the gods (who know their feast-days well,)
By your false count are sent home supperless,
And scold and storm at her for your neglect."

About 400 B.C., Meton the Athenian observed that 19 years consist of almost exactly 235 lunar months, and accordingly proposed a new calendar with 125 months of 30 days and 110 of 29 days, corresponding to an average year of 365 days, 6 hours and 19 minutes — only about 30 minutes too long. Of this Meton's cycle the traditional rule for determining the date of Easter still preserves traces. On account of so much confusion in the official calendar the almanacs of the time even designated the dates for agricultural operations by means of the constellations visible at the corresponding time.

TIME MEASUREMENT. — While sun and moon suffice for large-scale measurement of time, the approximate determination of its subdivisions early became important, and this problem has been solved with continually increasing precision to our own day. Early time measurement depended either on some form of sun-dial as a natural means, or on an apparatus analogous to the hour-glass as an artificial method.

In Isaiah xxxviii. 8, in connection with a promise of prolonged life to Hezekiah, it is said

> And this shall be a sign unto thee from the Lord, that the Lord will do this thing that he hath spoken; behold, I will bring again the shadow of the degrees, which is gone down in the sun-dial of Ahaz, ten degrees backward. So the sun returned ten degrees, by which degrees it was gone down.

The first sun-dial of which a description is preserved belongs to the time of Alexander the Great, and consisted of a hollow hemisphere with its rim horizontal and a bead at the centre to cast the shadow. Curves drawn on the concave interior divided the period from sunrise to sunset into twelve parts, these lengths being thus proportionate to the lengths of the daylight period.

The use of the clepsydra, or water clock, in Greece dates from the fifth century B.C. It consisted there of a spherical bottle with a minute outlet for the gradual escape of water. Its use in regulating public speaking is illustrated by Demosthenes' demand when interrupted, "You there: stop the water."

For the sake of conformity with the sun-dial division of each day and each night into twelve equal parts, the rate of flow in the clepsydra required continual adjustment. Ingenious improvements were made in the mechanism in course of time, but in considering the work of the Greek astronomers, the impossibility of what we should consider accurate time measurement must not be forgotten.

GREEK ARITHMETIC. — In Greek arithmetic the earliest known numerals are merely the initials of the respective number words. Two other systems came into use later. In one of these the numbers from 1 to 24 are represented by the 24 letters of the Ionian alphabet; in the other the letters represent numbers, but no longer in consecutive order. This use of letters for numbers was not confined to Greece, but appears to have originated there. The Greeks had no zero, and never discovered the immense advantage of a position-system, such as that by which we are able to express all numbers by only ten symbols. Fractions occur not infrequently. The change from the earlier notation to that with 24 characters was a disastrous one. There were not only more characters to memorize, but computation became materially more complicated. These disadvantages far more than offset the superior compactness, the sole merit of the new system. The special importance of such compactness for coins has led to the suggestion that they were the medium through which this notation was introduced.

A simple numerical computation of late date in the Greek alphabetic numerals and its modern equivalent are

$$\begin{array}{lllll} \multicolumn{5}{l}{\overline{\sigma\ \xi\ \epsilon}} \\ \hline \multicolumn{5}{l}{\sigma\ \xi\ \epsilon} \\ \hline \overset{\delta}{\mathrm{M}} & \overset{\alpha}{\mathrm{M}} & {}_{,}\beta & {}_{,}\alpha & \\ \overset{\alpha}{\mathrm{M}} & {}_{,}\beta & {}_{,}\gamma & \overline{\chi} & \overline{\tau} \\ & {}_{,}\alpha & \overline{\tau} & \overline{\kappa\epsilon} & \\ \hline & \overset{\zeta}{\mathrm{M}} & \multicolumn{3}{l}{\overline{\sigma\ \kappa\ \epsilon}} \end{array} \qquad \begin{array}{lll} 265 & & \\ 265 & & \\ \hline 40\,000, & 12\,000, & 1000 \\ & & \\ 12\,000, & 3\,600, & 300 \\ 1\,000, & 300, & 25 \\ \hline 70\,225 & & \end{array}$$

— *Gow.*

Division was an exceedingly laborious process of repeated subtraction.

Probably nothing in the modern world would have more astonished a Greek mathematician than to learn that, under the influence of compulsory education, the whole population of Western Europe, from the highest to the lowest, could perform the operation of division for the largest numbers. — *Whitehead.*

Approximate square roots were found by the later Greeks. Theon in the fourth century A.D. for example gives the following rule: —

When we seek a square-root, we take first the root of the nearest square-number. We then double this and divide with it the remainder reduced to minutes and subtract the square of the quotient, then we reduce the remainder to seconds and divide by twice the degrees and minutes (of the whole quotient). We thus obtain nearly the root of the quadratic.

The reckoning board, or abacus, — known in so many different forms throughout the world, — came into very early use, but actual evidence in regard to its form is meagre. A sharp distinction was made between the art of calculation (*logistica*), and the science of numbers (*arithmetica*). The former was deemed unworthy the attention of philosophers, and to their attitude may be fairly attributed the fact that Greek mathematics was always weak on the analytical side, and seemed in a few centuries to reach the limit of its possible development.

GREEK GEOMETRY. — It was in geometry that Greek mathematics chiefly developed, and for several fundamental reasons. The Greek mind had a strong predilection for formal logic, a keen æsthetic appreciation of beauty of form, and, on the other hand, with no adequate symbolism for arithmetic or algebra, a distinct disdain, at any rate among the educated, for the commercialized mathematics of computation. The history of Greek mathematics is therefore to a great extent the history of geometry. Formal geometry as distinguished from the solving of particular geometrical problems, had, indeed, no previous existence, and we have to do with the beginnings of elementary geometry as we now know it.

THE IONIAN PHILOSOPHERS. — The sense of curiosity, the feeling of wonder, the spirit of inquiry, — these are the common elements of philosophy and science. It is thus not strange that the earliest names in science are likewise the earliest in philosophy.

In the childhood and youth of the race specialization has not begun, all knowledge lies invitingly open to the expanding mind. We have seen how much had been accumulated in Egypt and Babylonia of knowledge and skill in observing and recording the phenomena of the heavens, in irrigation and in measurement of land. Much of the same general character was doubtless true of the Phœnicians, the Trojans, the Cretans, and other precursors of the Greeks. But nothing deserving the name of science has come down to us from the Ægean or Greek civilization before the time of Thales of Miletus, chief of the Ionian philosophers, and one of the "seven wise men of Greece."

THALES. — The ancient and fragmentary register of Greek mathematicians, or history of Greek geometry before Euclid, attributed to Eudemus, begins:

> As it is now necessary to consider also the beginnings of the arts and sciences in the present period, we report that, according to the evidence of most, geometry was invented by the Egyptians, taking its origin from the measurement of land. This last was necessary

> for them on account of the inundation of the Nile, which obliterated every man's boundaries. It is however, nothing wonderful that the invention of this as of the other sciences has grown out of necessity, as everything in its beginnings proceeds from the incomplete to the complete. A regular transition takes place from perception to thoughtful consideration, from this to rational knowledge. Just as now with the Phœnicians an exact knowledge of numbers took its rise in the needs of trade and commerce, so geometry began with the Egyptians for the reason mentioned. Thales, who went to Egypt, first brought this science into Greece. Much he discovered himself, of much however he transmitted the beginnings to his successors. Some things he made more general, some more comprehensible.

The significance packed into this terse quotation may well be emphasized. The mathematics of the Chaldeans, the Egyptians, the Phœnicians, was merely a tool, crudely shaped to meet vital concrete needs; it had little possibility of development. The Greek intellect, seizing upon the fragmentary knowledge of these practical races, refined from it the germs of a new pure science, making the knowledge "more general" and "more comprehensible," and at the same time discovering much that was new. On the other hand, inclining in its zeal for pure science to the opposite extreme of disregard for the concrete applications, Greek science eventually reached its own limit of possible growth. In the long run scientific progress must depend on due appreciation of the complementary importance of both pure and applied science.

Thales was of Phœnician descent, and was born about 624 B.C. in Miletus, a city of Ionia, at that time a flourishing Greek colony in what is now Asia Minor. As an engineer he was employed to construct an embankment for the river Halys. As a merchant he dealt in salt and oil, and, visiting Egypt, learned there something of the wisdom of the Egyptian priesthood. He occupied himself with the study of the stars as well as of geometry, and in particular,

> announced to the inhabitants of Miletus that night would enter upon the day, the sun hide himself, the moon place herself in front, so that his light and radiance would be intercepted.

Herodotus says that there was a war between the Lydians and the Medes, and after various turns of fortune

> in the sixth year a conflict took place, and on the battle being joined, it happened that the day suddenly became night. And this change, Thales of Miletus had predicted to them, definitely naming this year, in which the event really took place. The Lydians and the Medes, when they saw the day turned into night, ceased from fighting, and both sides were desirous of peace.

This eclipse is supposed to have taken place in 585 B.C. The prediction of the year of an eclipse gained Thales a great reputation with his contemporaries, though his designation with six others as "wise men of Greece" appears to have had a primarily political significance. None of the other six at any rate had any scientific standing. He taught that the year has 365 days; that the equinoxes divide the year unequally; that the moon is illuminated by the sun. The mathematical attainments attributed to Thales include the following theorems of elementary geometry; the angles at the base of an isosceles triangle are equal; when two straight lines cut each other the opposite angles are equal; the first proof that the circle is bisected by its diameter; the inscription of the right triangle in the semicircle; the measurement of height by shadow, involving the principle of similar triangles.

Plutarch relates that Niloxenus, conversing with Thales concerning King Amasis, says: —

> Although he also admires you on account of other things, he prizes above everything the measurement of the pyramids, in that you have without any trouble and without needing an instrument, merely placed your staff at the end of the shadow cast by the pyramid, showing from the two triangles formed by the contact of the solar rays that one shadow has the same relation to the other as the pyramid to the staff.

Some writers even attribute to Thales a knowledge that the sum of the angles of a triangle is two right angles, also of the idea of a circle as a locus of a point having a certain property, but conclusive evidence can hardly be adduced. Even the implied knowledge of similar triangles is doubtful. In connection with his shadow measurements it is interesting that his scholar Anaximander, born 611 B.C., introduced the sun-dial into Greece.

While our knowledge of Thales and his work is extremely meagre, the mathematical results above mentioned have considerable significance in connection with the comparison between Greece and Egypt. The Egyptian standpoint was fundamentally practical, specific, inductive; the Greek shows already its characteristic tendencies to abstract generalization, to logical proof, and to the methods of deductive science. Most of the facts ascribed to Thales may well have been known to the Egyptians. For them these facts would have remained unrelated; for the Greeks they were the beginnings of an extraordinary development of the science of geometry.

MILESIAN COSMOLOGY. — The cosmological ideas of the Milesian philosophers were sufficiently ingenious and picturesque. To Thales the earth is a circular disk floating in an ocean of water. This water is the fundamental element of the whole. Ice, snow, and frost turn readily into water, even rocks wear away and disappear in it. Man himself seems capable of turning into it, while the waters of sea and land shrink into solid residues. By evaporation of the water air is formed, its agitation causes earthquakes. The stars between their setting and rising pass behind the earth.

The following passages (Fairbanks' translation) indicate the estimation in which Thales was held by later Greek philosophers.

> As to the quantity and form of this first principle or element, there is a difference of opinion; but Thales, the founder of this sort of philosophy, says that it is water (accordingly he declares that the earth rests on water), getting the idea I suppose because he saw that the nourishment of all beings is moist, and that warmth itself is generated from moisture and persists in it (for that from which all things spring is the first principle of them); and getting the idea also from the fact that the germs of all beings are of a moist nature, while water is the first principle of the nature of what is moist. . . . 'Some say that the earth rests on water. I have ascertained that

the oldest statement of this character is the one credited to Thales, the Milesian, to the effect that it rests on water, floating like a piece of wood or something else of that sort.' . . . And Thales, according to what is related of him, seems to have regarded the soul as something endowed with the power of motion, if indeed he said that the loadstone has a soul because it moves iron. . . . Some say that soul is diffused throughout the whole universe; and it may have been this which led Thales to think that all things are full of gods. — *Aristotle.*

Of those who say that the first principle is one and movable, to whom Aristotle applies the distinctive name of physicists, some say that it is limited; as for instance Thales of Miletos . . . who seems also to have lost belief in the gods. These say that the first principle is water, and they are led to this result by things that appear to the senses; for warmth lives in moisture and dead things wither up and all germs are moist and all nutriment is moist . . . . Thales is the first to have set on foot the investigation of nature by the Greeks; although so many others preceded him, he so far surpassed them as to cause them to be forgotten. It is said that he left nothing in writing except a book entitled Nautical Astronomy. — *Theophrastus.*

It is said that Thales of Miletos, one of the seven wise men, was the first to undertake the study of Physical Philosophy. He said that the beginning (the first principle) and the end of all things is water. All things acquire firmness as this solidifies, and again, as it melts, their existence is threatened; to this are due earthquakes and whirlwinds and movements of the stars . . . . Thales was the first of the Greeks to devote himself to the study and investigation of the stars and was the originator of this branch of science; on one occasion he was looking up at the heavens and was just saying he was intent on studying what was overhead, when he fell into a well; whereupon a maid-servant named Thratta laughed at him and said: 'In his zeal for things in the sky he does not see what is at his feet.' And he lived in the time of Krœsos. — *Hippolytus.*

Thales of Miletos regards the first principle and the element as the same thing. . . . So we call earth, water, air, fire, elements. . . . Thales declared that the first principle of things is water. The Physicists, followers of Thales, all recognize that the void is really a void. The earth is one and spherical in form. It is in the midst of the universe. Thales and Democritus find in water the cause of earthquakes. . . . Thales thinks that the Etesian winds blowing against Egypt raise the mass of the Nile, because its outflow is beaten back by the swelling of the sea which lies over its mouth. — *Ætius.*

ANAXIMANDER. — A second native of Miletus, Anaximander (about 611–545 B.C.) had a different interpretation of nature, holding that the fundamental stuff, out of which all things are made, is something between air and water. He believed the earth to be balanced in the centre of the world, because being in the centre and having the same relation to all parts of the circumference, it ought not to tend to fall in one direction rather than in any other. This point of view, not easily taken by the layman, illustrates the natural tendency of the Greek philosopher to emphasize geometrical symmetry.

Among those who say that the first principle is one and movable and infinite is Anaximander of Miletos, son of Praxiades, pupil and successor of Thales. He said that the first principle and element of all things is infinite, and he was the first to apply this word to the first principle; and he says that it is neither water nor any other one of the things called elements, but the infinite is something of a different nature from which came all the heavens and the worlds in them; and from what source things arise, but that they return of necessity when they are destroyed. . . . Evidently when he sees the four elements changing into one another, he does not deem it right to make any one of these the underlying substance, but something else besides them. — *Theophrastus.*

The earth is a heavenly body, controlled by no other power and keeping its position because it is the same distance from all things. The form of it is curved, cylindrical, like a stone column. It has two faces. One of these is the ground beneath our feet and the other is opposite to it. The stars are the circle of fire, separated from the fire about the world, and surrounded by air. There are certain breathing-holes like the holes of a flute through which we see the stars; so that when the holes are stopped up there are eclipses. The moon is sometimes full and sometimes in other phases, as these holes are stopped up or open. The circle of the sun is 27 times that of the moon. . . . Man came into being from another animal, namely the fish, for at first he was like a fish. — *Hippolytus* (*on Anaximander*).

Anaximander, collecting data from the Ionian sailors frequenting Miletus, constructed a map of the earth, and speculated on the relative distances of the heavenly bodies.

Herodotus relates that

> during the reign of Cleomenes, Aristagoras, prince of Miletus, arrived at Sparta; the Lacedæmonians affirm, that desiring to have a conference with their sovereign, he appeared before him with a tablet of brass in his hand, on which was inscribed every known part of the habitable world, the seas, and the rivers.

ANAXIMENES. — A third Ionian Greek, often associated with those just mentioned, is Anaximenes (sixth century B.C.), like them a native of Miletus. For him the stars are fixed upon the celestial vault, and pass behind the northern (highest) part of the earth on setting. Air, not water, is the first cause of all things, the others being formed by its compression or rarefaction. The heat of the sun is due to its rapid motion, but the stars are too remote to give out heat.

> Anaximenes arrived at the conclusion that air is the one movable, infinite, first principle of all things. For he speaks as follows: 'Air is the nearest to an immaterial thing; for since we are generated in the flow of air, it is necessary that it should be infinite and abundant, because it is never exhausted.' (A fragment accredited to Anaximenes.)
>
> Most of the earlier students of the heavenly bodies believed that the sun did not go underneath the earth but rather around the earth and this region, and that it disappeared from the view and produced night because the earth was so high toward the north. . . . Anaximenes and Anaxagoras and Democritus say that the breadth of the earth is the reason why it remains where it is. . . . Anaximenes says that the earth was wet, and when it dried it broke apart, and that earthquakes are due to the breaking and falling of hills. — *Aristotle.*

The school of Thales and his successors in this Ionian outpost of Greek civilization was soon succeeded by developments of still greater importance in the more remote Italian colonies.

PYTHAGORAS AND HIS SCHOOL. — The register of mathematicians proceeds: — "After these Pythagoras transformed the occupation with this branch into a true science, by considering the foundation of it from a higher standpoint, and investigated its theorems in a more abstract and intellectual way. It is he also who invented the theory of the irrational and the construction of the cosmical bodies." These few words like those quoted of Thales are full of meaning. The Egyptian priests knew geometrical facts, the raw material of mathematical science; Thales adapted this material to building purposes, Pythagoras began the systematic foundations of the structure. Both in name and in substance mathematics as a science begins with Pythagoras.

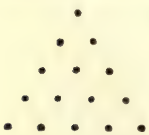
Pythagoras founded in the Greek cities of southern Italy a school which had much of the character of a fraternity or secret society, this with political tendencies ultimately arousing hostility which proved destructive to it. Beyond these undisputed facts his life and work are obscured by a great mass of tradition and myth, even the date of his birth being doubtful. A native of the island of Samos not far from Miletus, he appears to have been much affected by Egyptian influences during a residence in that country. A visit to Babylon even is alleged, but with doubtful authority. The etiquette of the Pythagorean school required that all discoveries should be attributed to the "Master" and not revealed to outsiders. To Pythagoras himself must probably be ascribed the so-called Pythagorean theorem, this forming the necessary basis for the theory of the irrational mentioned in the register. A similar inference may be drawn in regard to the regular polyhedra. On the other hand, Pythagoras appears to have interested himself in the theory of numbers, particularly in connection with music and geometry. He is said to have first introduced weights and measures among the Greeks.

The attribution of particular results or beliefs to individuals of this period is however very doubtful on account of the fact that Pythagoras left no writings whatever, that his school was essentially a secret society, and that in later centuries it became the custom to credit its founder with all sorts of knowledge which he could not possibly have possessed.

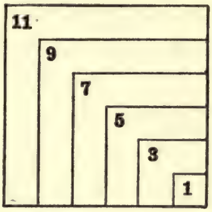
Pythagoras makes the classification, arithmetic (numbers absolute), music (numbers applied), geometry (magnitudes at rest), astronomy (magnitudes in motion), this fourfold division or "quadrivium" continuing in vogue for some two thousand years. The distinction between abstract and concrete arithmetic had been emphasized among the Greeks in comparatively early times. Arithmetic and geometry were distinguished on one side from mechanics, astronomy, optics, surveying, music, and computation on the other. The aim of Greek arithmetic "was entirely different from that of the ordinary calculator, and it was natural that the philosopher who sought in numbers to find the plan on which the Creator worked, should begin to regard with contempt the merchant who wanted only to know how many sardines, at 10 for an obol, he could buy for a talent."

The limited mathematics of the practical Egyptians had consisted of numerical cases. It was an easy step for Pythagoras to make number in a somewhat mystical sense the central element in his philosophy.

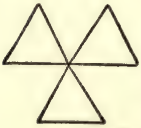
Pythagorean Arithmetic. — In pure arithmetic or number theory as we should call it, the Pythagoreans enunciated such dicta as, for example, "Unity is the origin and beginning of all numbers but not itself a number." Prime and composite numbers were also distinguished, and theorems of considerable algebraic complexity discovered. There is naturally no algebraic symbolism, but "unknown" and "given" quantities are employed in the modern sense. Odd and even numbers received special names, and besides the series of squares and cubes and the arithmetic and geometric progressions previously known, other series were derived from these, for example, the triangular numbers: 1, 3, 6, 10, 15, etc., by successive addition of the natural numbers. The reason for the name triangular will be clear if one counts the dots in the triangle formed by taking one, two, three or more rows beginning at the top of the figure.



The series of squares is formed by adding the odd numbers successively; $1 + 3 = 4$, $1 + 3 + 5 = 9$, etc. The series 2, 6, 12, 20, 30, etc. is formed by adding the even numbers, or again by multiplying adjacent natural numbers. If we construct a series of squares or parallelograms with a common angle and sides of length 1, 2, 3, 4, 5, etc. the figure which must be added to any one to produce the next larger was called by the Greeks a *gnomon*, the area of which would be represented by one of the series of odd numbers, — an interesting and typical example of the Greek habit of combining geometry with number-theory. As products of two numbers were associated with areas — "square" or "oblong" — so products of three factors were interpreted as volumes. A later Pythagorean calls the cube the "geometrical harmony" — an expression embodying the association of mathematics with music. The cube has indeed 6 faces, 8 vertices, 12 edges; 6, 8, and 12 are in harmonic progression, that is, 8 is the harmonic mean between 6 and 12.



→ Pythagorean Geometry. — In geometry the Pythagoreans formulated definitions of the fundamental elements, line, surface, angle, etc. They are credited with a number of theorems depending on the application of one surface to another,[1] and implying a knowledge of methods of determining area and of the properties of parallel lines. They developed a fairly complete theory of the triangle, including the fundamental proof that the sum of the angles of a triangle is two right angles, by a method not very different from our own. The theory of the "cosmical bodies" mentioned in the register is of special interest. Any solid angle must have at least three faces. If three equal equilateral triangles have a common vertex they will when cut or folded so that their edges are brought together, form a solid angle, and a fourth equal triangle will complete a regular tetrahedron. Similarly, if we start with four triangles, we may



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[1] Some of these are equivalent to the solution of the quadratic equation.

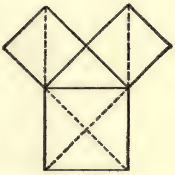
build up with four others a regular octahedron, or starting with five, an icosahedron with 20 faces. Six triangles, however, will fill the angular space about a point, and thus not permit the formation of a regular polyhedron. Using squares instead of triangles, we obtain only the cube; using pentagons (angle 108°), the regular dodecahedron — 12 faces, 3 at each vertex. The Egyptians must have been familiar with the cube, the regular tetrahedron, and the octahedron. To these, with the icosahedron, the Pythagoreans associated the four cosmical elements — earth, air, fire, and water. Their discovery of an additional body, the regular dodecahedron, formed by 12 pentagons, made a break in the correspondence, and the need was met by the addition of the universe, or, according to others, the ether, as a fifth term in the cosmical series. This correspondence was not merely symbolical, but physical, the earth being supposed to consist of cubical particles, etc. We cannot infer that the impossibility of a sixth regular polyhedron was known. That only these five regular polyhedra are possible was in fact first proved by Euclid. There is a tradition that the Pythagorean discoverer of the dodecahedron was drowned at sea on account of the sacrilege of announcing his discovery publicly. A later commentator records a similar tradition that the discoverer of the irrational perished by shipwreck, since the inexpressible should remain forever concealed, and that he who touched and opened up this picture of life was transported to the place of creation and there washed in eternal floods.

The regular polygons were naturally studied, and in particular the decomposition of them into right triangles of 45° and 30°–60°. With the pentagon the attempt naturally failed, but the five-pointed star formed by drawing diagonals was a special emblem of the Pythagoreans. With the inscribed pentagon connects itself naturally the division of a line in extreme and mean ratio, or, as it was later characterized, the "golden section." This division, by which the square on the greater segment of a line is equivalent to the rectangle whose sides are the other segment and the whole line, occurs repeatedly in Greek architecture of the fifth century, with fine effect, and must have been systematically employed.

As to the celebrated theorem which bears the name of Pythagoras, he may well have learned from the Egyptian rope-stretchers that a right angle is formed by taking sides of lengths 3 and 4 and separating the other ends a distance 5, while his study of numbers would easily have led to the discovery that in the series of squares the adjacent 9 and 16 make 25. It would naturally be investigated whether a similar relation could be verified for other right triangles. In the most familiar case of the isosceles right triangle it soon appears that the length of the equal sides being taken as 1, the length of the hypotenuse could be only approximately expressed. It cannot indeed be exactly expressed by any whole number, or fraction; it is *irrational.*

> If it is true as Whewell says, that the essence of the triumphs of science and its progress consists in that it enables us to consider evident and necessary, views which our ancestors held to be unintelligible and were unable to comprehend, then the extension of the number concept to include the irrational, and we will at once add, the imaginary, is the greatest forward step which pure mathematics has ever taken.
>
> — *Hankel.*

In this case the proof of the Pythagorean theorem is easily effected by a simple graphical construction, involving merely the drawing of diagonals of squares. The smaller triangles in the figure are evidently all equal. The larger square contains four of them, the smaller squares, two each. It seems possible that this was the Pythagorean method, but as to how the proof was accomplished in other cases we have no information, the simpler proof of Euclid having completely superseded the earlier. On the other hand, for the corresponding arithmetical problem of finding three whole numbers which can be the sides of a right triangle, Pythagoras is said to have given a correct solution, equivalent in our notation to



$$(2a+1)^2 + (2a^2+2a)^2 = (2a^2+2a+1)^2,$$

a denoting any positive integer. How this method was discovered remains a matter of conjecture.

We may recognize here the characteristic elements of the inductive method, first, observation of the particular fact that in a certain right triangle, with sides 3, 4, and 5, the sum of the squares on the two sides is equal to that on the hypotenuse; second, the formation of the hypothesis that this may be true also for right triangles in general; third, the verification of the hypothesis in other particular cases. Then follows the deductive confirmation of the hypothesis as a law for all right triangles.

PYTHAGOREAN PHYSICAL SCIENCE. — It has been already noted that one of the most fundamental principles of the Pythagorean school was the significance attached to number in connection with all sorts of phenomena, the regular motions of the heavenly bodies, the musical tones, etc. There is a tradition that Pythagoras, walking one day, meditating on the means of measuring musical notes, happened to pass near a blacksmith's shop, and had his attention arrested by hearing the hammers as they struck the anvil produce sounds which had a musical relation to each other. It was found that vibrating cords emitted tones dependent in a simple way on their length; for example, cords of lengths 2, 3, and 4 giving a tone, its fifth and its octave respectively. The monochord used in studying these numerical relations is said to have been the first apparatus of experimental physics. It was even supposed that each of the various heavenly bodies and the sphere of the fixed stars had a characteristic tone, these all uniting to produce the so-called "music of the spheres."

TERRESTRIAL MOTION; PHILOLAUS, HICETAS. — The universe was believed to consist of the four elements, — earth, air, fire, water, — to be a sphere with a spherical earth at its centre, and to have life. Pythagoras identified the morning and evening stars, and attributed the moon's light to reflection. It is of peculiar interest that later Pythagoreans, in particular Philolaus, about 400 B.C., attributed the apparent daily motion of the heavenly bodies from east to west not to their own actual motion but to a motion of the earth in the opposite direction. This latter motion, however, was thought of, not as a rotation, but as an orbital motion about a so-called "central fire." Just as the moon revolved about the earth, always turning the same face towards the latter, so the earth might revolve about the central fire which would be forever invisible to the inhabitants of the other side of the earth. While we say that the moon rotates about its axis in the same time in which it revolves about the earth, to the ancients such a motion was not considered to include rotation at all. A further essentially arbitrary assumption introduced between the earth and the central fire a counter-earth (*antichthon*), which was required to make up the supposed number of the heavenly bodies, and which would hide the central fire from dwellers in the antipodes. Aristotle, criticising this theory, says of the Pythagoreans: —

> They do not with regard to the phenomena seek for their reasons and causes, but forcibly make the phenomena fit their opinions and preconceived notions. . . . When they anywhere find a gap in the numerical ratios of things, they fill it up in order to complete the system. As ten is a perfect number and is supposed to comprise the whole nature of numbers, they maintain that there must be ten bodies moving in the universe, and as only nine are visible, they make the antichthon the tenth.

All the other heavenly bodies describe orbits, each in its own hollow sphere about the central fire, the generally adopted order, based on the apparent rate of motion among the stars, being Moon, Sun, Venus, Mercury, Mars, Jupiter, Saturn. Pythagorean speculations as to relative distances of the different planets were naturally mystical notions merely. The sun was said to move around the central fire in an "oblique circle," *i.e.* the ecliptic. The moon was believed to be inhabited by plants and animals. The moon might be eclipsed either by the earth or by the counter-earth. This remarkable system, admitting the earth to move and not to be the centre of the universe, was not generally or long accepted, but had a share in securing the acceptance of the theories of Copernicus nearly 2000 years later. One at least of the Pythagoreans made the great further step, somewhat loosely described by Cicero in the words: —

> Hicetas of Syracuse, according to Theophrastus, believes that the heavens, the sun, moon, stars, and all heavenly bodies are standing still, and that nothing in the universe is moving except the earth, which, while it turns and twists itself with the greatest velocity round its axis, produces all the same phenomena as if the heavens were moved and the earth were standing still.

The activity of the Pythagorean school continued to be important until about 400 B.C., that is, until the rise of the Athenian school under Plato and his successors. It had not only created the science of mathematics; it had developed, however vaguely and imperfectly, the idea of a world of physical phenomena governed by mathematical laws.

Dr. Allman says of Pythagoras: —

> In establishing the existence of the regular solids he showed his deductive power; in investigating the elementary laws of sound he proved his capacity for induction; and in combining arithmetic with geometry . . . he gave an instance of his philosophic power.
>
> These services, though great, do not form, however, the chief title of the Sage to the gratitude of mankind. He resolved that the knowledge which he had acquired with so great labour, and the doctrine which he had taken such pains to elaborate, should not be lost; and . . . devoted himself to the formation of a society *d'élite*, which would be fit for the reception and transmission of his science and philosophy; and thus became one of the chief benefactors of humanity, and earned the gratitude of countless generations.

In medicine, we meet before the fifth century only with the anatomist Alcmaeon (508 B.C.) of the early medical school at Crotona, in Italy, and in natural philosophy (besides Thales and others already mentioned) with Xenophanes, who, like Pythagoras, held that fossils are in fact what they appear to be, and not mere "freaks of nature," as was generally believed.

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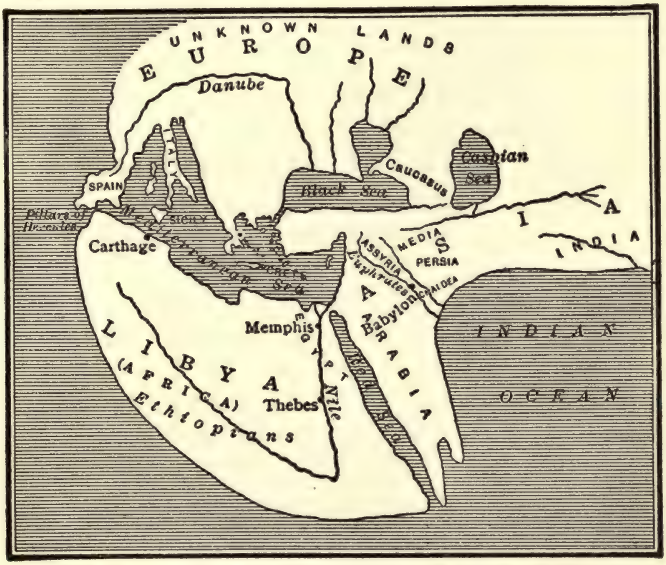

UNKNOWN LANDS
EUROPE
Danube
ITALY
SPAIN
Black Sea
Caucasus
Caspian Sea
Pillars of Hercules
SICILY
Mediterranean Sea
CRETE
Carthage
ASIA
INDIA
MEDIA
PERSIA
ASSYRIA
Euphrates
CHALDEA
Babylon
ARABIA
Memphis
EGYPT
Nile
Red Sea
Thebes
LIBYA
(AFRICA)
Ethiopians
INDIAN OCEAN

MAP OF THE WORLD ACCORDING TO HERODOTUS

(From Breasted's *Ancient Times.* Courtesy of Messrs. Ginn & Co.)

From long journeys in Egypt and other Eastern Countries Herodotus returned with much information regarding these lands. His map showed that the Red Sea connected with the Indian Ocean, a fact unknown to his predecessor, Hecatæus. [See p. 34.]

—*Breasted.*

CHAPTER IV

SCIENCE IN THE GOLDEN AGE OF GREECE

Our science, in contrast with others, is not founded on a single period of human history, but has accompanied the development of culture through all its stages. Mathematics is as much interwoven with Greek culture as with the most modern problems in engineering. She not only lends a hand to the progressive natural sciences, but participates at the same time in the abstract investigations of logicians and philosophers. — *Klein.*

There still remain three studies suitable for freemen. Calculation in arithmetic is one of them; the measurement of length, surface, and depth is the second; and the third has to do with the revolutions of the stars in reference to one another . . . there is in them something that is necessary and cannot be set aside . . . if I am not mistaken, [something of] divine necessity. — *Plato.*

Literature and Art. — The fifth century B.C. witnessed that astonishing flowering of the Greek genius in literature and military glory which has made it ever since famous. The battles of Marathon and Salamis had flung back the Asiatic hosts which threatened to overrun and enslave Europe, and had transformed the Greeks from a group of jealous and parochial city states into a great democratic nation. Trade prospered, wealth increased, and for about a century letters, art, and science flourished as never before and never since. History began to be written by Herodotus and Thucydides. The drama was developed by Æschylus, Sophocles, and Euripides to such a pitch that even to-day, after the lapse of nearly 2500 years, crowds listen with eager interest to the Œdipus of Sophocles and the Iphigenia of Euripides, while the poetry of Pindar and the wit of Aristophanes have never lost their charm. In architecture and the plastic arts the Parthenon and its sculptures still testify to Greek supremacy.

In science, also, great names testify to memorable deeds. No such perfection, to be sure, was attained in science as in literature and in sculpture, but vast progress was made in mathematical science beyond anything hitherto accomplished, and the foundations were securely laid for a rational interpretation of man and of nature. Literature, architecture, sculpture, and the drama require no special apparatus or reagents. Mathematical science also is not dependent upon such externals, being in this respect like literature and art, and we find geometry and arithmetic at the outset moving forward far more rapidly than natural or physical science.

PARMENIDES. — The recognition of the spherical shape of the earth and its division into zones are attributed not only to the Pythagoreans, but also to Parmenides of Elis, who lived in the early part of the fifth century. He introduced a system of concentric spheres analogous to that soon to be so highly developed by Eudoxus. He identified the evening and the morning stars, and attributed the moon's brightness to reflected light. He regarded the sun as consisting of hot and subtle matter detached from the Milky Way, the moon chiefly of the dark and cold.

EMPEDOCLES. — Passing over the guesses of Heraclitus and Parmenides at the riddle of existence and of man and nature, we may pause for a moment to examine the speculations of Empedocles (about 455 B.C.). A native of Agrigentum in southern Sicily, Empedocles was regarded as poet, philosopher, seer, and immortal god. He appears to have been a close observer of nature, understanding the true cause of solar eclipses and believing the moon to be twice as far from the sun as from the earth. The latter is held in place by the rapidly rotating heavens "as the water remains in a goblet which is swung quickly round in a circle." Aristotle attributes to Empedocles that analysis of the universe into the four "elements," earth, air, fire, and water, which until comparatively recent times was universally accepted as fundamental. It is, nevertheless, not only misleading but absurd to hold with Gomperz ("Greek Thinkers," I, 230) that Empedocles' theory of the four elements "takes us at a bound into the heart of modern chemistry." The facts seem rather to be that Empedocles put together and hospitably accepted and clarified the theories of his various predecessors. He is the first sanitarian of whom we have any record, for Empedocles is credited with having cut down a hill of his native city and thus cured a plague by letting in the north wind, and to have done a similar service to the neighboring "parsley" city of Selinus (Selinunte) by simply draining a local marsh.

The following is a fragment from the writings of Empedocles: —

> So all beings breathe in and out; all have bloodless tubes of flesh spread over the outside of the body, and at the openings of these the outer layers of skin are pierced all over with close-set ducts, so that the blood remains within, while a facile opening is cut for the air to pass through. Then whenever the soft blood speeds away from these, the air speeds bubbling in with impetuous wave, and whenever the blood leaps back the air is breathed out; as when a girl, playing with a clepsydra of shining brass, takes in her fair hand the narrow opening of the tube and dips it in the soft mass of silvery water, the water does not at once flow into the vessel, but the body of air within pressing on the close-set holes checks it till she uncovers the compressed stream; but then when the air gives way the determined amount of water enters. And so in the same way when the water occupies the depths of the bronze vessel, as long as the narrow opening and passage is blocked up by human flesh, the air outside, striving eagerly to enter, holds back the water inside behind the gates of the resounding tube, keeping control of its end, until she lets go with her hand. Then, on the other hand, the very opposite takes place to what happened before; the determined amount of water runs off as the air enters. Thus in the same way when the soft blood, surging violently through the members, rushes back into the interior, a swift stream of air comes in with hurrying wave, and whenever it [the blood] leaps back, the air is breathed out again in equal quantity.
>
> — *Fairbanks.*

ANAXAGORAS. — (500–428 B.C.) For the student of science Anaxagoras, a native of Clazomene in Asia Minor, is more important than Empedocles. Turning aside from wealth and civic distinction in his enthusiasm for science, he seems to have occupied himself with the problem of squaring the circle, a problem attacked even by the Egyptians with some degree of success, and destined to exercise great influence on the development of Greek geometry. The beginnings of perspective are also attributed to him, in connection with studies of the stage. He was particularly interested in a great meteorite — the appearance of which he was afterwards said to have predicted — supposing it to have fallen from the sun, and inferring that the latter was a "mass of red-hot iron greater than the Peloponnesus," not very distant from the earth. Like the Pythagoreans he assigned as the order of distances: — moon, sun, Venus, Mercury, Mars, Jupiter, Saturn. The earth's axis was inclined, in order that there might be variations of climate and habitability. He explained the moon's phases correctly, also solar and lunar eclipses, but he misinterpreted the Milky Way as due to the shadow cast by the earth. His theory of the nature and origin of the cosmos, viz. that it was material and had come by the combination and differentiation of primitive elementary substances or "seeds" of matter, was repugnant to those holding the polytheistic dogmas of his time and brought him into popular disfavor. Convicted of impiety, he died in exile, 428 B.C. By his insistence upon the importance of minute invisible "seeds" or particles of matter he paved the way for the "atomism" of Leucippus and Democritus.

THE ATOMISTS. — A very little observation of external nature shows that disintegration is forever going on. Ice turns to water, water to vapor, rocks to sand and sand to dust — in other words, masses to particles. Furthermore, dust vanishes and vapor disappears, while clouds and fogs, rain and snow, make their appearance without obvious cause, and dust accumulates from invisible sources. What is more reasonable than to suppose that visible things — rocks and ice and water — become gradually resolved into invisible particles, and that these in their turn condense into new visible substances at some later time? For these or similar ideas the material "seeds" of Anaxagoras had, as stated above, paved the way, when later emphasized by Leucippus and his more famous pupil Democritus. Of the life of Leucippus almost nothing is known, but he was probably a contemporary of Empedocles and Anaxagoras, and possibly a pupil of Zeno. Leucippus assumed the existence of empty space as well as of matter, and held that of atoms all things are constituted. Space is infinite in magnitude, atoms infinite in number and indivisible, with only quantitative differences. Atoms are always in activity, and worlds are produced by atoms variously shaped and weighted, falling in empty space and giving rise to an eddying motion by mutual impact.

DEMOCRITUS OF ABDERA was a pupil and associate of Leucippus, whose theories of empty space and material atoms he developed and made so famous that his own name alone is often associated with them. Of his life, his works, and his death little is certainly known, but he may be regarded as marking the culmination and conclusion of the Ionian school; and his reputation, both in antiquity and in mediæval times, was immense. Like contemporary and preceding philosophers, his writings were in verse, and Cicero is said to have deemed his style worthy of comparison with that of Plato. His somewhat boastful comparison of his own geometrical power with that of the Egyptian rope-stretchers has been quoted.

Democritus appears to have agreed closely in his interpretation of nature with Leucippus, and regarded empty space and atoms as cosmic elements. He also held that by the motion of the atoms was produced the world with all that it contains. Soul and fire are of one nature, their atoms small, smooth, and round. By inhaling them life is maintained. Hence the soul perishes with and in the same sense as the body, — a doctrine which made Democritus odious to later generations. Dante, for example, places him far down in hell as "ascribing the world to chance."

The atomic theory of perception held that from every object "images" of that object are being given off in all directions, some of which enter the organs of sense and cause "sensations." Democritus further held that sensations are the only sources of our knowledge. He was regarded as one of the extreme sceptics of antiquity, as *e.g.* in this saying, "We know nothing: not even if there is anything to know." Galileo, himself of a highly sceptical turn of mind, refers with approval to Democritus, and it is probably on this side, *i.e.* by exemplification of the critical spirit, that Democritus rendered his greatest service. His positive contributions to science, even in atomism, were apparently neither novel nor important. Democritus explained the Milky Way as composed of a vast number of small stars, but to his disciple, Metrodorus of Chios, it was a former path of the Sun.

THE BEGINNINGS OF RATIONAL MEDICINE. HIPPOCRATES OF COS. — Before the middle of the fifth century B.C., science in the healing art had no existence. Excepting among a few of the more enlightened, sacrifices and other appeals to the gods still characterized medicine as a priestly rather than a scientific profession, while the prevailing ignorance of anatomy and physiology made rational treatment of the sick difficult if not impossible. Alcmæon, in the previous century, had taken some steps in the right direction, proving for example that the sperm does not originate, as was currently believed, in the spinal marrow, and that the brain is the organ of mind, and advancing a naturalistic theory of disease which seems to foreshadow that of his great successor Hippocrates.

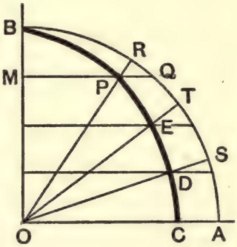
Two island centres of medical lore (they can hardly be called medical schools), both of the cult of Asclepias, existed in the southeastern Ægean, viz. Cos and Cnidus, and on the former was born, in 460 B.C., Hippocrates, "the Father of Medicine," in the next century already characterized by Aristotle as "the Great." Of his life, education, practice, and writings comparatively little is certainly known. Many of the writings attributed to him are of doubtful authenticity and are more safely assigned to the Hippocratic "school." Enough remain, however, especially when added to the references by later authors to him and to his sayings and to his methods of practice, to make it clear that in every respect Hippocrates was worthy of the lofty reputation with which his name has come down to us after five and twenty centuries.

And yet it is not for the practical arts of medicine or any of its basic sciences that Hippocrates did his most famous work. It was rather in his attitude toward health and disease that his real greatness lay. For, as far as we know, it was Hippocrates who first insisted on regarding disease as a natural rather than a supernatural process, and Hippocrates who first urged that careful observation and study of the patient which entitles him to rank as the original "clinician" of medical science. Again, it was Hippocrates who first insisted on the existence and importance of those processes of self-repair which are to-day recognized as fundamental properties of living matter, — processes summed up in that famous phrase of his which has come down to us through the Latin of the middle ages, — *vis medicatrix naturæ,* — "the healing power of nature," one of the finest and truest of the tenets of scientific medicine to-day. Finally, by advancing his famous *theory of the four humors,* a theory which with minor modifications was for some two thousand years afterwards the prevailing theory of pathology, or the nature of disease, among the most enlightened, Hippocrates still further established his right to be regarded as the "father" of medicine, and the first (and only) medical man ever authoritatively entitled "the Great." This theory — crude enough to-day — held that health consists in the right mixture, and disease in the wrong mixture, of four "humors" (juices) of the body, viz. blood, phlegm, yellow bile, and black bile. Here again the great merit of Hippocrates' idea was that it directed attention to the body itself, and hence to *natural* rather than *supernatural* phenomena.

The tone of the Hippocratic writings is well illustrated by the titles of those accepted as probably genuine, *e.g.* On Airs, Waters, and Places; On Epidemics; On Regimen in Acute Diseases; On Fractures; On Injuries of the Head; etc. The so-called Hippocratic Oath is rightly described by Gomperz as "a monument of the highest rank in the history of civilization." That this oath is still administered to graduates about to enter on the practice of medicine, is sufficient evidence of the high character and far-sighted wisdom of its originator. (See Appendix.)

THE SOPHISTS. — In the fifth century B.C. political events following war with Persia made Athens supreme in Greece — the finest and richest city in the world. Its citizens aspired to success in public life, and sought training to that end from the sophists. While science was not generally cultivated as a leading subject in the educational system thus developed,[1] mathematics could not fail to be esteemed as a means of discipline, and several of the sophists made notable contributions to its development.

Hippias of Elis is the first sophist to be mentioned for important mathematical work. About 420 B.C. Hippias invented a curve called the *quadratrix,* serving for the solution of two of the three celebrated problems of Greek geometry; viz. the quadrature of the circle and the trisection of an angle. By means of straight line and circle constructions, the solution of the quadratic equation had been accomplished, though without algebraic symbolism, or any recognition of negative or imaginary results. The trisection problem, like that of duplicating the cube, was equivalent to the solution of the cubic equation, and could therefore not be accomplished by line and circle methods. The quadratrix was generated by the intersection P of two moving straight lines, one MQ always parallel to its initial position OA, the other OR revolving uniformly about a centre O. By means of this curve the trisection problem is reduced to that of trisecting a straight line, which is elementary.[2] The curve meets the perpendicular lines OA and OB at C and B respectively so that $OC:OB = 2:\pi$, where π is the ratio of the circumference of a circle to its diameter. To this quadrature solution the name of the curve is due.



Dinostratus showed that the assumptions $OC:OB > 2:\pi$ and $OC:OB < 2:\pi$ both lead to contradictions, therefore $OC:OB = 2:\pi$ — a good example of the Greek *reductio ad absurdum.* The study of a problem not capable of solution by elementary means thus led to the invention of this new curve, the first of which we have any definite record.

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[1] See Freeman, "Schools of Hellas."

[2] To trisect any angle as AOR, draw MQ parallel to OA and divide OM into three equal parts by lines parallel to OA, meeting the curve in D and E respectively. The radii OS and OT will then trisect the angle AOQ, by the definition of the curve.

THE CRITICISM OF ZENO. — The Stoic philosopher Zeno, teaching in Athens about this time, though not himself a mathematician, represents an important phase of philosophical criticism of mathematics. Every manifold, he says, is a number of units, but a true unit is indivisible. Each of the many must thus be itself an indivisible unit, or consist of such units. That which is indivisible however can have no magnitude, for everything which has magnitude is divisible to infinity. The separate parts have therefore no magnitude, etc. Again, as to the possibility of motion, he maintains that before the body can reach its destination it must reach the middle point, before it can arrive there it must traverse the quarter, and so on without end. Motion is thus impossible; so the tortoise, if he have any start, cannot be overtaken by the swift runner Achilles, for while Achilles is covering that distance the tortoise will have attained a second distance, and so on. Such specious criticism was naturally, and in a measure justly, evoked by misguided efforts of certain mathematicians to show that a line consists of a multitude of points, etc. These or similar controversies as to the interpretation of the infinite and the infinitesimal have persisted till our own day, resembling in that respect the classical problems of circle squaring and angle trisection to which reference has been made above. The more or less mystical statements about the new discoveries of the Pythagoreans also invited sceptical epigrams.

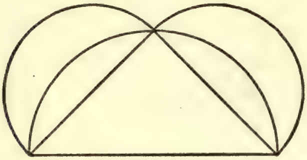
> Zeno was concerned with three problems. . . . These are the problem of the infinitesimal, the infinite, and continuity. . . . From him to our own day, the finest intellects of each generation in turn attacked these problems, but achieved, broadly speaking, nothing. . . .
>
> — *B. Russell.*

Aristotle accordingly solves the problem of Zeno the Eleatic, which he propounded to Protagoras the Sophist. Tell me, Protagoras, said he, does one grain of millet make a noise when it falls, or does the ten-thousandth part of a grain? On receiving the answer that it does not, he went on: Does a measure of millet grains make a noise when it falls, or not? He answered, it does make a noise. Well, said Zeno, does not the statement about the measure of millet apply to the one grain and the ten-thousandth part of a grain? He assented, and Zeno continued, Are not the statements as to the noise the same in regard to each? For as are the things that make a noise, so are the noises. Since this is the case, if the measure of millet makes a noise, the one grain and the ten-thousandth part of a grain make a noise.

CIRCLE MEASUREMENT: ANTIPHON AND BRYSON; HIPPOCRATES OF CHIOS. — Two of the sophists, Antiphon and Bryson, made an interesting contribution to the problem of squaring the circle, by means of the inscribed and circumscribed regular polygons. Antiphon started with a regular polygon inscribed in a circle, and constructed by known elementary methods an equivalent square. By doubling the number of sides repeatedly he obtained polygons which become more and more nearly equivalent to the circle, — the first correct attack on this formidable problem. Bryson took the important further step of employing both inscribed and circumscribed polygons, making however the not unnatural assumption that the area of the circle may be considered the arithmetical mean between them.

Another great step in the development of the theory of the circle was accomplished by Hippocrates of Chios, who had relations with the now dispersed Pythagoreans during the latter half of the fifth century and came to Athens in later life after financial reverses. He is said in the register of mathematicians to have written the first Elements or textbook of mathematics, in which he made effective use of the *reductio ad absurdum* as a method of relating one proposition to another.

To Hippocrates is due the theorem that the areas of circles are proportional to the squares on their diameters. He appears to have employed geometrical figures with letters at the vertices, in the modern fashion. From the theorem in regard to areas of circles follows naturally a general theorem for similar segments and sectors of circles. His work on lunes is remarkable. Starting with an isosceles right triangle, he describes a semicircle on each of the three sides. By the theorem just quoted the semicircle on the hypotenuse is equal in area to the sum of the other two. If the larger semicircle is taken away from the entire figure, two equal lunes remain; if the two smaller semicircles are taken away, the triangle remains. Therefore the two lunes are together equivalent to the triangle, and the area of each may be determined. The gulf between rectilinear and curvilinear figures has at last been successfully crossed. A second attempt employs three equal chords instead of two, and incidentally the theorem that the square on the side of a triangle is greater than the sum of the squares on the other two sides when the angle opposite the first side is greater than a right angle. Other interesting and still more complicated attempts are preserved.



A third classical problem was that of the so-called "duplication of the cube." One of the older Greek tragedians attributed to King Minos the words referring to a tomb erected at his order:

> Too small thou hast designed me the royal tomb,
> Double it, yet fail not of the cube.

At a somewhat later period it is related that the Delians, suffering from a disease, were bidden by the oracle to double the size of one of their altars, and invoked the aid of the Athenian geometers. Hippocrates transformed the problem of solid geometry into one in two dimensions by observing that it is equivalent to that of inserting two geometrical means between given extremes. In our modern algebraic notation, the continued proportion $x : y = y : z = z : a$ leads to the equations $y^2 = xz$, $z^2 = ya$, whence, eliminating z, $y^3 = ax^2$, $y = a^{\frac{1}{3}}x^{\frac{2}{3}}$; y and z are the desired means between x and a, and by putting $a = 2x$ the problem is solved. No such algebraic notation existed at this time, however, and the geometrical methods invented by later Greek mathematicians were necessarily very complicated, as will appear below.

PLATO AND THE ACADEMY. — One of the greatest names in the history of philosophy is that of Plato, and yet with Plato philosophy enters upon a new phase in which it almost parts company with science. Before Plato philosophy was almost wholly devoted to inquiries or speculations touching the earth, the heavens, and the universe, and hence was substantially "nature" or "natural" philosophy. But with Plato and ever since his time the larger part of philosophy has been devoted to observation and speculation upon the human mind and its products, and has accordingly often been called "mental" or "moral" as contrasted with "natural" philosophy. It is therefore Thales and Pythagoras, Democritus and Aristotle, rather than Plato and his disciples, who are the protagonists of science as the word is used to-day.

As a disciple of Socrates, Plato found it expedient to leave Athens after the death of his master, and during the following eleven years he travelled widely in the Mediterranean world, doubtless familiarizing himself with the learning of Egypt and of the Greek Ptolemies. After having been sold as a slave, redeemed and set free, Plato returned to his native city, and established himself as a philosopher. While primarily a philosopher rather than a mathematician, Plato, unlike his master Socrates, — who desired only enough mathematics for daily needs, — rated highly the importance of mathematics and rendered services of the greatest value in its development. This was doubtless due in part to the influence of Archytas, a friend of the Pythagoreans, with whom he had associated during his prolonged exile.

The register proceeds: "Plato . . . caused mathematics in general, and geometry in particular, to make great advances, by reason of his well known zeal for the study, for he filled his writings with mathematical discourses, and on every occasion exhibited the remarkable connection between mathematics and philosophy."

"Let no one ignorant of geometry enter under my roof" was the injunction which confronted Plato's would-be disciples. His respect for mathematics finds interesting expression in the remarks he puts into the mouth of Socrates in the Dialogues, and to him it is largely indebted for its place in higher education.

In the Laws he advises the study of music or the lyre to last from the age of 13 years to 16, followed by mathematics, weights and measures, and the astronomical calendar until 17. For a few picked boys on the other hand in the Republic, he recommends before they are 18, abstract and theoretical mathematics, theory of numbers, plane and solid geometry, kinetics, and harmonics. Of arithmetic he says, "Those who are born with a talent for it are quick at learning, while even those who are slow at it have their general intelligence much increased by studying it." "No branch of education is so valuable a preparation for household management and politics and all arts and crafts, sciences and professions, as arithmetic; best of all by some divine art, it arouses the dull and sleepy brain, and makes it studious, mindful, and sharp."

The geometrical Greek view of numbers, exemplified in our use of square and cube in algebra, is well illustrated by Theætetus, who says to Socrates that his teacher

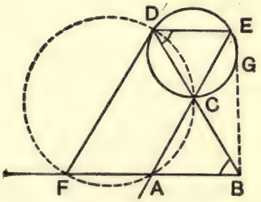
> was giving us a lesson in roots, with diagrams, showing us that the root of 3 and the root of 5 did not admit of linear measurement by the foot (that is, were not rational). He took each root separately up to 17. There as it happened he stopped, so the other pupil and I determined, since the roots were apparently infinite in number, to try to find a single name which would embrace all these roots. We divided all numbers into two parts. The number which has a square root we likened to the geometrical square, and called 'square and equilateral' (*e.g.* 4, 9, 16). The intermediate numbers, such as 3 and 5 and the rest which have no square root, but are made up of unequal factors, we likened to the rectangle with unequal sides, and called rectangular numbers.

Under Plato's influence mathematics first acquired its unified significance, as distinguished from geometry, computation, etc. Accurate definitions were formulated, questions of possibility considered, methods of proof criticized and systematized, logical rigor insisted upon. The philosophy of mathematics was begun. The point is the boundary of the line; the line is the boundary of the surface; the surface is the boundary of the solid.

Such axioms as "Equals subtracted from equals leave equals" date from this period. The analytical method is developed, connecting that which is to be proved with that which is already known. Another principle carefully observed is to isolate the problem by removing all non-essential elements, and a third consists in proving that assumptions inconsistent with that which is to be proved are impossible.

The Analytic Method. — The analytic method, proceeding from the unknown to the known, depends for its validity on the reversibility of the steps; the synthetic method on the contrary proceeds from the known to the unknown, with unimpeachable validity. It was characteristic of the Greek geometers to aim at this form for their demonstrations, even if the results had been first obtained analytically. The two methods are well illustrated by the following: —

A circle is given and two external points A and B. It is required to draw straight lines AC and BC meeting the circle in C, D, and E so that DE shall be parallel to AB. It is shown that if the construction can be made, the tangent to the circle at D will meet AB (produced if necessary) in a point F which will lie on a new circle passing through A, C, and D. This *analysis* of consequences is the desired clue on which the following *synthesis* of the construction is then based. Starting again with A, B and the circle, we locate F so that $BA \times BF = BC \times BD =$ square of the tangent BG from B. Then drawing a tangent from F to the circle, D is determined and with it the required line DE.



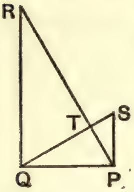
A solution of the "duplication of the cube" problem is also attributed to Plato, though the mechanical process employed is so much at variance with his usual teachings that the correctness of the attribution is seriously questioned.

$SPQR$ is a frame in which SPQ and PQR are always right angles, while PQ may be varied, and SQ and PR can be revolved about Q

and P respectively. They are to be so revolved if possible that they shall cross at right angles at T, and that ST and TR shall be respectively equal to the lengths between which mean proportionals are to be inserted. Then by similar triangles

$$ST : PT = PT : QT = QT : RT$$

PT and QT are the required mean proportionals. If ST is taken equal to twice TR the special case of the duplication of the cube is represented.



To Plato is attributed a systematic method for finding numbers which may be sides of right triangles, his method being essentially an extension of the Pythagorean already described. Plato's Timæus dialogue is indeed an important source of our information in regard to Pythagorean mathematics. Plato speaks with emphatic scorn of the shameful ignorance of mensuration on the part of his countrymen.

> He is unworthy of the name of man who is ignorant of the fact that the diagonal of a square is incommensurable with its side.

While predominantly interested in geometry, Plato's arithmetical attainments were considerable for his time. He made, for example, a correct statement about the 59 divisions of 5040.

> Arithmetic has a very great and elevating effect, compelling the soul to reason about abstract number, and if visible or tangible objects are obtruding upon the argument, refusing to be satisfied.
>
> —*Plato,* Republic.

> . . . It would be proper then, Glaucon, to lay down laws for this branch of science and persuade those about to engage in the most important state-matters to apply themselves to computation, and study it, not in the common vulgar fashion, but with the view of arriving at the contemplation of the nature of numbers by the intellect itself, — not for the sake of buying and selling as anxious merchants and retailers, but for war also, and that the soul may acquire a facility in turning itself from what is in the course of generation to truth and real being. — *Plato,* Republic.

> But the mathematical doctrines concerning the parts and elements of the Universe are put forward by Plato, not so much as assertions concerning physical facts, of which the truth or falsehood is to be determined by a reference to nature herself. They are rather propounded as examples of a truth of a higher kind than any reference to observation can give or can test, and as revelations of principles such as must have prevailed in the mind of the Creator of the universe; or else as contemplations by which the mind of man is to be raised above the region of sense, and brought nearer to the Divine Mind. — *Whewell.*

PLATONIC COSMOLOGY. — The spherical figure of the earth was now generally accepted in Greece, and the older fanciful cosmogonies gradually disappeared. To Plato, whose interest in physical science was indeed but secondary, the earth was a sphere at the centre of the universe, requiring no support. He supposes the distances of the heavenly bodies to be proportional to the numbers: Moon 1, Sun 2, Venus 3, Mercury 4, Mars 8, Jupiter 9, Saturn 27, — these numbers being obtained by combining the arithmetic and geometric progressions, 1, 2, 4, 8 and 1, 3, 9, 27.

> Plato accepts as a principle that the heavenly bodies move with a uniform and regular circular motion; he then proposes to the mathematicians this problem: 'What are the uniform and regular circular motions which may properly be taken as hypotheses in order that we may save the appearances presented by the planets?'

His general conception of the world as expressed in the Timæus and in the tenth book of the Republic is decidedly mystical. In the latter a soul returning to its body after 12 days in the other world relates its experiences in imaginative language: —

> Everyone had to depart on the eighth day and to arrive at a place on the fourth day after, whence they from above perceived extended through the whole heaven and earth a light as a pillar, mostly resembling the rainbow, only more splendid and clearer, at which they arrived in one day's journey; and there they perceived in the neighborhood of the middle of the light of heaven, the extremities of

> the ligatures of heaven extended; for this light was the band of heaven, like the hawsers of triremes, keeping the whole circumference of the universe together.

Aristotle sums up Plato's theories — not too clearly — in the words:

> In a similar manner the *Timæus* shows how the soul moves the body because it is interwoven with it. For consisting of the elements and divided according to the harmonic numbers, in order that it might have an innate perception of harmony and that the universe might move in corresponding movements, He bent its straight line into a circle, and having by division made two doubly joined circles out of the one circle, He again divided one of them into seven circles in such a manner that the motions of the heavens are the motions of the soul.

Plato probably had no real knowledge of those deviations of the planets from uniform circular motion, which were to engross the attention of succeeding philosophers and astronomers. His system is consistently geocentric, and assumes a stationary earth. According to Plutarch: —

> Theophrastus states that Plato, when he was old, repented of having given the earth the central place in the universe which did not belong to it,

this presumably indicating an inclination towards the theories of the later Pythagoreans.

Plato adopts the Pythagorean or Empedoclean hypothesis of the four elements, the component particles being assumed to have respectively the shapes of the cube (earth), icosahedron (water), octahedron (air), and tetrahedron (fire).

> All the heavenly bodies are looked on as divine beings, the first of all living creatures, the perfection of whose minds is reflected in their orderly motions.

Summing up an extended discussion of Plato's astronomical theories, Dreyer says: —

There is absolutely nothing in his various statements about the construction of the universe tending to show that he had devoted much time to the details of the heavenly motions, as he never goes beyond the simplest and most general facts regarding the revolutions of the planets. Though the conception of the world as Cosmos, the divine work of art, into which the eternal ideas have breathed life, and possessing the most godlike of all souls, is a leading feature in his philosophy, the details of scientific research had probably no great attraction for him, as he considered mathematics inferior to pure philosophy in that it assumes certain data as self-evident, for which reason he classes it as superior to mere opinion but less clear than real science.

Through his widely read books he helped greatly to spread the Pythagorean doctrines of the spherical figure of the earth and the orbital motion of the planets from west to east.

The conjunction of philosophical and mathematical activity such as we find, beside Plato, only in Pythagoras, Descartes and Leibnitz, has always borne the finest fruits for mathematics. To the first we owe scientific mathematics in general. Plato discovered the analytical method, through which mathematics was raised above the standpoint of the Elements, Descartes created analytic geometry, our own celebrated countryman Leibnitz the infinitesimal calculus, — and these are the four greatest steps in the development of mathematics. — *Hankel.*

ARCHYTAS. — To Archytas, a late Pythagorean, with whom Plato had had close relations, was due the earliest solution of the duplication problem. This very interesting and somewhat elaborate solution involves a combination of three services, a cone of revolution, a cylinder having the vertex of the cone in the circumference of its base, and a surface generated by revolving a semicircle about an axis passing through one end of its diameter. It shows remarkable mastery of elementary geometry, both plane and solid, and an interesting tendency to employ a wider range of methods, including motion, which might, but for adverse tendencies, have had important results in connecting mathematics with its possible applications to mechanics, etc. The influence of Plato in avoiding such connections and associating geometry with abstract logic and philosophy, undoubtedly had compensating advantages in promoting elegance and scientific rigor, — crystallizing out a more refined product. Archytas is said also to have invented the screw and the pulley and to have been the first to give a systematic treatment of mechanics, employing geometrical theorems for this purpose.

MENÆCHMUS: CONIC SECTIONS. — Even more interesting in its foreshadowing of future mathematical developments are the solutions of the duplication problem by Menæchmus. The problem which we should express in modern algebraic notation by the continued proportion $a:x::x:y::y:b$, Menæchmus, without any such notation or any system of coordinate geometry, shows to be equivalent to that of determining the intersection either of a parabola and a hyperbola, corresponding to the two proportions

$$a:x::x:y \text{ and } a:x::y:b,$$

or to the intersection of two parabolas, in case the second proportion is replaced by $x:y::y:b$. The construction of either parabola or the hyperbola naturally required some mechanical device.

The Greeks of this period distinguished three types of the cone formed by the rotation of the right triangle about one of its sides, according as the angle formed by that side with the hypotenuse was less than, equal to, or greater than half a right angle. A plane perpendicular to an element would cut a cone of the first kind in an ellipse, the second in a parabola, the third in a hyperbola. These curves were named accordingly sections of the acute-angled, the right-angled, the obtuse-angled cone.

> The discovery of the conic sections . . . first threw open the higher species of form to the contemplation of geometers. But for this discovery, which was probably regarded . . . as the unprofitable amusement of a speculative brain, the whole course of practical philosophy of the present day, of the science of astronomy, of the theory of projectiles, of the art of navigation, might have run in a different channel; and the greatest discovery that has ever been made in the history of the world, the law of universal gravitation, with its in-

numerable direct and indirect consequences and applications to every department of human research and industry, might never to this hour have been elicited. — *Sylvester.*

Many of Plato's followers and disciples in the Academy continued the development of mathematics. To Xenocrates, for example, is attributed the determination of the number of all possible syllables as 1,002,000,000,000, a result obtained by some unknown method. This whole period is one of great productivity and importance in the history of mathematics. New theorems and new methods are discovered, former methods are critically scrutinized, loci problems are investigated, these and the study of the three classical problems leading to the introduction of new curves and a general extension of geometrical knowledge. Geometry, with emphasis, indeed, on its philosophical side, predominates over the theory of numbers, and even the latter is given so geometrical a form that mathematics is unified.

A NEW COSMOLOGY. — Eudoxus of Cnidos (408?–355 B.C.) was a student both of Archytas and, for a time, of Plato. He was not only mathematician and astronomer, but also physician. In mathematics he is almost a new creator of the science, developing the theory of proportion, making a special study of the "golden section," already mentioned in connection with the regular polygons, and obtaining important results in solid geometry. In the words of the register, "Eudoxus of Cnidos . . . first increased the number of general theorems, added to the three proportions three more, and raised to a considerable quantity the learning begun by Plato on the subject of the (golden) section, to which he applied the analytical method."

To him was formerly attributed the proof that the volume of a pyramid is one third that of the prism having the same base and altitude, as well as the corresponding theorem for cones and cylinders. A recently discovered manuscript of Archimedes shows, however, that for this Democritus deserves the credit. The method of exhaustion, so-called, employed in proving these theorems was expressed in the auxiliary theorem: "When two volumes are unequal, it is possible to add their difference to itself so many times that the result shall exceed any assigned finite volume." This exceedingly useful and important principle, avoiding the difficulties of infinitesimals, was expressed in several approximately equivalent forms, and was already implied in the work of Antiphon and Bryson. A solution of the duplication problem which gained Eudoxus the appellation "godlike" has been entirely lost.

There appear to have been no astronomical instruments at this time except the simple gnomon and sun-dial, but the more obvious irregularities of the planetary motions were beginning to attract attention, and under Eudoxus led to the development of a new and important theory. Nearest to the central earth is the moon, carried on the equator of a sphere revolving from west to east in 27 days. The poles of this sphere are themselves carried on a second sphere, which turns in about $18\frac{1}{2}$ years about the axis of the zodiac. The angle between the axes of these two spheres corresponds with the moon's variation in latitude. A third outer sphere gives the daily east to west motion. Similarly there are three spheres for the sun. For each of the five planets a fourth sphere is necessary to account for the stations and retrogressions of its apparent orbital motion — thus making with the single sphere of the stars 27 spheres, all having their common centre at the centre of the earth.

How far these spheres were regarded as having concrete existence, how far they merely expressed in convenient geometrical form the observed relations and motions, we cannot determine from extant evidence. The amount of observational data available was entirely inadequate to serve as a basis for any quantitatively correct theory. The third sphere of the sun was based on an erroneous hypothesis as to its motion. For Mercury, Jupiter and Saturn the theory was reasonably adequate, for Venus less so, and for Mars quite defective.

Calippus, a follower of Eudoxus, endeavored with some degree of success to remedy these defects by adding a fifth sphere for each of the refractory planets, and at the same time a fourth and fifth for the sun, in order to account for the recently discovered inequality in the length of the four seasons.

Reviewing the development of this interesting theory, Dreyer says: —

> But with all its imperfections as to detail, the theory of homocentric spheres proposed by Eudoxus demands our admiration as the first serious attempt to deal with the apparently lawless motions of the planets. . . . Scientific astronomy may really be said to date from Eudoxus and Calippus, as we here for the first time meet that mutual influence of theory and observation on each other which characterizes the development of astronomy from century to century. Eudoxus is the first to go beyond mere philosophical reasoning about the construction of the universe; he is the first to attempt systematically to account for the planetary motions. When he has done this the next question is how far this theory satisfies the observed phenomena, and Calippus at once supplies the observational facts required to test the theory, and modifies the latter until the theoretical and observed motions agree within the limits of accuracy attainable at that time. Philosophical speculation unsupported by steadily pursued observations is from henceforth abandoned: the science of astronomy has started on its career.

Eudoxus made the first known proposal for a leap-year, and for a star catalogue. A marble celestial globe in the national museum at Naples is perhaps a copy of one made by him.

ARISTOTLE, 384–322 B.C., "the master of those who know," the son of a physician, a student in Plato's Academy, and tutor of Alexander the Great, exercised a mighty and lasting influence on the development of Greek science and philosophy. His tendencies were mainly non-mathematical, but the theorem that the sum of the exterior angles of a plane polygon is four right angles is ascribed to him. He distinguishes sharply between geodesy as an art and geometry as a science; he considers the plane sections of the circular cyclinder; he recognizes the physical reason for the adoption of ten as the base number of arithmetic; he designates unknown quantities by letters. Continuity — an idea so important in modern mathematical and physical science — he defines by saying: —

> A thing is continuous when of any two successive parts, the limits, at which they touch, are one and the same, and are, as the word implies, held together.

ARISTOTLE'S MECHANICS. — In mechanics Aristotle seems almost to recognize the principle of virtual velocities. He discusses the composition of motions at an angle with each other. He enunciates the correct relation between the length of the arms of a lever and the loads which will balance each other upon it. He even deals with the central and tangential components of circular motion.

He asks such questions as: "Why are carriages with large wheels easier to move than those with small?" "Why do objects in a whirlpool move toward the center?" etc.

He is convinced that the speed of falling bodies is proportional to their weight — a belief credulously accepted until Galileo's experiment nineteen centuries later. He illustrates his discussions by geometrical figures, and states correctly: —

> If α be a force, β the mass to which it is applied, γ the distance through which it is moved, and δ the time of the motion, then α will move $\frac{1}{2}\beta$ through 2γ in the time δ, or through γ in the time $\frac{1}{2}\delta$.

He adds erroneously, however: —

> It does not follow that $\frac{1}{2}\alpha$ will move β through $\frac{1}{2}\gamma$ in the time δ, because $\frac{1}{2}\alpha$ may not be able to move β at all; for 100 men may drag a ship 100 yards, but it does not follow that one man can drag it one yard.

Of the bearing of Aristotle's physical theories Duhem says: —

> Incapable of any alteration, inaccessible to any violence, the celestial essence could manifest no other than its own natural motion, and that was uniform rotation about the centre of the universe.

Aristotle is the author of eight books on Physics, four on the Heavens, and four on Meteorology. In physics he explains the rainbow, attributes sound to atmospheric motion, and discusses refraction mathematically. While he undertakes to deal with motion, space and time — *i.e.* with the subject-matter of mechanics — his treatment is too metaphysical to have much real value. He declares for example that: —

The bodies of which the world is composed are solids, and therefore have three dimensions. Now, three is the most perfect number, — it is the first of numbers, for of *one* we do not speak as a number, of *two* we say both, *three* is the first number of which we say *all*. Moreover, it has a beginning, a middle, and an end.

Francis Bacon in the seventeenth century remarks of Aristotle: —

> Nor let any one be moved by this; that in his books Of Animals, and in his Problems and in others of his tracts, there is often a quoting of experiments. For he had made up his mind beforehand; and did not consult experience in order to make right propositions and axioms, but when he had settled his system to his will, he twisted experience round, and made her bend to his system; so that in this way he is even more wrong than his modern followers, the Schoolmen, who have deserted experience altogether.

ARISTOTELIAN ASTRONOMY. — Only the second of the four books on the Heavens is devoted to astronomy. He considers the universe to be spherical, the sphere being the most perfect among solid bodies, and the only body which can revolve in its own space. Rotation from east to west is more honorable than the reverse. He holds that the stars are spherical in form, that they have no individual motion, being merely carried all together by their one sphere.

> 'Furthermore, since the stars are spherical, as others maintain and we also grant, because we let the stars be produced from that body, and since there are two motions of a spherical body, rolling along and whirling, then the stars, if they had a motion of their own, ought to move in one of these ways. But it appears that they move in neither of these ways. For if they whirled (rotated), they would remain at the same spot and not alter their position, and yet they manifestly do so, and everybody says they do. It would also be

reasonable that all should be moved in the same motion, and yet among the stars the sun only seems to do so at its rising or setting, and even this one not in itself but only owing to the distance of our sight, as this when turned on a very distant object from weakness becomes shaky. This is perhaps also the reason why the fixed stars seem to twinkle, while the planets do not twinkle. For the planets are so near that the eyesight reaches them in its full power, but when turned to the fixed stars it shakes on account of the distance, because it is aimed at too distant a goal; now its shaking makes the motion seem to belong to the star, for it makes no difference whether one lets the sight or the seen object be in motion. But that the stars have not a rolling motion is evident; for whatever is rolling must of necessity be turning, while of the moon only what we call its face is visible.' — *Dreyer.*

Aristotle adopts the system of spheres of Eudoxus and Calippus, but seems to suppose these spheres to be concrete, and not a merely geometrical device for interpreting the phenomena or determining the positions. In order however to secure what he conceives to be the necessary relation between the motions of the spheres, he is obliged to increase their total number from 33 to not less than 55. The earth is fixed at the centre of the universe. That the earth is a sphere is shown logically, and is also evident to the senses. During eclipses of the moon, namely, the boundary line, which shows the shadow of the earth, is always curved. . . . If we travel even a short distance south or north, the stars over our heads show a great change, some being visible in Egypt, but not in more northern lands, and stars are seen to set in the south which never do so in the north. It seems therefore not incredible that the vicinity of the pillars of Hercules is connected with that of India, and that there is thus but one ocean.

The bulk of the earth he considers to be "not large in comparison with the size of the other stars." The estimated circumference of 400,000 stadia — about 39,000 miles — is the earliest known estimate of the size of the earth, and is of unknown origin, but may quite likely be due to Eudoxus. While the heavens proper are characterized by fixed order and circular motion, the space below the moon's sphere is subject to continual change, and motions within it are in general rectilinear — a theory destined long to block progress in mechanics. Of the four elements, earth is nearest the centre, water comes next, fire and air form the atmosphere, fire predominating in the upper part, air in the lower. In this region of fire are generated shooting stars, auroras, and comets, the latter consisting of ignited vapors, such as constitute the Milky Way.

Against any orbital motion of the earth Aristotle urges the absence of any apparent displacement of the stars. Reviewing his astronomical theories, Dreyer says: —

> His careful and critical examination of the opinions of previous philosophers makes us regret all the more that his search for the causes of phenomena was often a mere search among words, a series of vague and loose attempts to find what was 'according to nature' and what was not; and even though he professed to found his speculations on facts, he failed to free his discussion of these from purely metaphysical and preconceived notions. It is, however, easy to understand the great veneration in which his voluminous writings on natural science were held for so many centuries, for they were the first, and for many centuries the only, attempt to systematize the whole amount of knowledge of nature accessible to mankind; while the tendency to seek for the principles of natural philosophy by considering the meaning of the words ordinarily used to describe the phenomena of nature, which to us is his great defect, appealed strongly to the mediæval mind, and, unfortunately, finally helped to retard the development of science in the days of Copernicus and Galileo.

At times Aristotle shows consciousness that his theories are based on inadequate knowledge of facts.

> 'The phenomena are not yet sufficiently investigated. When they once shall be, then one must trust more to observation than to speculation, and to the latter no farther than it agrees with the phenomena.'
>
> 'An astronomer' he says 'must be the wisest of men; his mind must be duly disciplined in youth; especially is mathematical study necessary; both an acquaintance with the doctrine of number,

and also with that other branch of mathematics, which, closely connected as it is with the science of the heavens, we very absurdly call geometry, the measurement of the earth.'

Aristotle's writings include not merely works on scientific subjects, but treatises of the very first importance On Poetry, On Rhetoric, On Metaphysics, On Ethics, and On Politics. Besides his scientific works mentioned above, there are others entitled On Generation and Destruction, On the Parts of Animals, On Generation of Animals, Researches about Animals, On the Locomotion of Animals. One of the most important of his many services to science is the encyclopedic character of his writings, since from time to time he reviews in them the opinions of his predecessors whose works are sometimes known to us chiefly through his references to them. While standing thus upon the shoulders of the past, he shows at the same time both vast learning and much originality. He may be truly called the founder of zoölogy.

Of Aristotle's contributions to science, the greatest was unquestionably that spirit of curiosity, of inquiry, of scepticism, and of veracity which he brought to bear on everything about him and within him. His observations are often poor, his conclusions often erroneous, but his interest, his curiosity, his zeal are indefatigable.

THEOPHRASTUS. — One of Aristotle's principal pupils, and his successor in his School, was Theophrastus (372–287 B.C.) notable in the history of science chiefly as an early student of plants, and writer of the most important treatises of antiquity on botany. These were two large works, one of ten books and the other of eight, On the History of Plants, and On the Causes of Plants, respectively. In these, more than 500 species of plants are described, chiefly with reference to their medicinal uses. It is especially interesting to note that Theophrastus recognized the existence of sex in plants, though he does not appear to have known the sex organs.

EPICURUS AND EPICUREANISM. — A few words may be said of another philosopher of the fourth century, a follower to some extent of Democritus and the forerunner and exemplar of the Roman Lucretius. This was Epicurus (342–270 B.C.), who, born in Samos and educated in Athens and Asia Minor, became a famous teacher and the head of a remarkable community "such as the ancient world had never seen." The mode of life in this community was not that of the so-called "epicures" of to-day, but very plain, — water the general drink, and barley bread the general food. The magnetic personality of Epicurus held the community together, and his chief work was a treatise on Nature in thirty-seven books. Epicureanism is of interest in the history of science chiefly because of its effect on its Roman exponent, the poet Lucretius. Much of it was even a negation of science and the scientific spirit.

HERACLIDES. ROTATION OF THE EARTH. — To Heraclides of Pontus in the fourth century B.C. belongs the distinction of teaching that the earth turns on its own axis from west to east in 24 hours. He had been connected with the Pythagoreans, and with the schools of Plato and Aristotle. His work is known to us only indirectly, none of his own writings having survived. He is said also to have advanced the hypothesis that Venus and Mercury revolve about the sun, being therefore at a distance from the earth sometimes greater than the sun, sometimes less. Geminus writing in the first half of the first century B.C. of the different fields and points of view of astronomers and physicists, remarks: —

> For why do sun, moon and planets appear to move unequally? Because, when we assume their circles to be excentric, or the stars to move on an epicycle, the appearing anomaly can be accounted for, and it is necessary to investigate in how many ways the phenomena can be represented, so that the theory of the wandering stars may be made to agree with the etiology in a possible manner. Therefore also a certain Heraclides of Pontus stood up and said that also when the earth moved in some way and the sun stood still in some way, could the irregularity observed relatively to the sun be accounted for. In general it is not the astronomer's business to see what by its nature is immovable and of what kind the moved things are, but framing hypotheses as to some things being in motion and others being fixed,

he considers which hypotheses are in conformity with the phenomena in the heavens. He must accept as his principles from the physicist, that the motions of the stars are simple, uniform, and regular, of which he shows that the revolutions are circular, some along parallels, some along oblique circles.

This contrast between the physical phenomena and the mathematical theory which corresponds with them, without being true or perhaps even possible in all respects, is of continued and increasing importance in the history of science, as a larger stock of facts was accumulated and as theories still imperfect were more frequently subjected to critical comparison with observed data, instead of being accepted on purely philosophical or metaphysical grounds. Heraclides is not credited with any conception of orbital or progressive motion of the earth.

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CHAPTER V

GREEK SCIENCE IN ALEXANDRIA

There is an astonishing imagination, even in the science of mathematics. . . . We repeat, there was far more imagination in the head of Archimedes than in that of Homer. — *Voltaire.*

If the Greeks had not cultivated Conic Sections, Kepler could not have superseded Ptolemy; if the Greeks had cultivated Dynamics, Kepler might have anticipated Newton. — *Whewell.*

If we compare a mathematical problem with an immense rock, whose interior we wish to penetrate, then the work of the Greek mathematicians appears to us like that of a robust stonecutter, who, with indefatigable perseverance, attempts to demolish the rock gradually from the outside by means of hammer and chisel; but the modern mathematician resembles an expert miner, who first constructs a few passages through the rock and then explodes it with a single blast, bringing to light its inner treasures. — *Hankel.*

The Museum at Alexandria. — The subjugation of Greece by Alexander the Great in 330 B.C. checked the further development of Greek civilization on its native soil. After Alexander's death in 323, his vast empire was divided among his generals, and Alexandria, the new Egyptian capital, fell to the lot of Ptolemy. The city as such was then barely ten years old, but very soon became, under the rule of the Ptolemies, the centre of the learned world. By 300 B.C. the Museum (Seat of the Muses) was founded, becoming in effect a veritable university of Greek learning. To this were attached a great library, a dining hall, and lecture-rooms for professors. Here for the next 700 years Greek science had its chief abiding place. The fame of Alexandria soon outshone and eventually eclipsed that of Athens, while Romans journeyed from Rome — never important in ancient times as a scientific centre — to study at Alexandria the healing art, anatomy, mathematical science, geography, and astronomy. Neither Athens, Rome, Carthage, nor any other city of the ancient world can boast similar distinction as a home of science.

EUCLID. — Three centuries after Thales had introduced the rudiments of Egyptian mathematics into Greece, the focus of mathematical activity was again transferred to that ancient land, but its spirit and aims remained there still for centuries essentially Greek. Continuing the ancient register, Proclus writes: —

> Not much younger than these (the Aristotelians) is Euclid, who brought the elements together, arranged much of the work of Eudoxus in complete form, and brought much which had been begun by Theætetus to completion. Besides he supported what had been only partially proved by his predecessors with irrefragable proofs. . . . It is related that King Ptolemy asked him once if there were not in geometrical matters a shorter way than through the Elements: to which he replied that in geometry there is no straight path for kings. . . .

As a recent writer has well said: "There are royal roads in science; but those who first tread them are men of genius and not kings."

Euclid's period of activity was about 300 B.C.; his place of birth and even his race are unknown; he is said to have been of a mild and benevolent disposition, and to have appreciated fully the scientific merits of his predecessors. While we know next to nothing of his life and personality, his writings have had an influence and a prolonged vitality almost, if not quite, unparalleled.

EUCLID'S "ELEMENTS." — Scientifically, Euclid is attached to the Platonic philosophy. Thus he makes the goal of his Elements the construction of the so-called "Platonic bodies" *i.e.*, the five regular polyhedrons. This treatise, which served as the basis of practically all elementary instruction for the following 2000 years, is naturally his best-known work, and appears to have been accepted in the Greek world after many previous attempts as a finality. It consisted of thirteen books, of which only the first six are ordinarily included in modern editions. The whole is essentially a systematic introduction to Greek mathematics, consisting mainly of a comparative study of the properties and relations of those geometrical figures, both plane and solid, which can be constructed with ruler and compass. The comparison of unequal figures leads to arithmetical discussion, including the consideration of irrational numbers corresponding to incommensurable lines. The contents may be briefly summarized as follows: Book I deals with triangles and the theory of parallels: Book II with applications of the Pythagorean theorem, many of the propositions being equivalent to algebraic identities, or solutions of quadratic equations, which seem to us more simple and obvious than to the Greeks. It should be noted however that the geometrical treatment is relatively advantageous for oral presentation. Book III deals with the circle, Book IV with inscribed and circumscribed polygons. These first four books thus contain a general treatment of the simpler geometrical figures, together with an elementary arithmetic and algebra of geometrical magnitudes. In Book V, for lack of an independent Greek arithmetical analysis, a theory of proportion (which has thus far been avoided) is worked out, with the various possible forms of the equation $\frac{a}{b} = \frac{c}{d}$. The results are applied in Book VI to the comparison of similar figures. This contains the first known problem in maxima and minima, — the square is the greatest rectangle of given perimeter, — also geometrical equivalents of the solution of quadratic equations. The next three Books are devoted to the theory of numbers, including for example the study of prime and composite numbers, of numbers in proportion, and the determination of the greatest common divisor. He shows how to find the sum of a geometrical progression, and proves that the number of prime numbers is infinite.

If there were a largest prime number n then the product $1 \times 2 \times 3 \ldots \times n$ increased by 1 would always leave a remainder 1 when divided by n or by any smaller number. It would thus either be prime itself, or a product of prime factors greater than n, either of which suppositions is contrary to the hypothesis that n itself is the greatest prime number.

Book X deals with the incommensurable on the basis of the theorem: If two unequal magnitudes are given, and if one takes from the greater more than its half, and from the remainder more than its half and so on, one arrives sooner or later at a remainder which is less than the smaller given magnitude. Books XI, XII, and XIII are devoted to solid geometry, leading up to our familiar theorems on the volume of prism, pyramid, cylinder, cone, and sphere, but in every case without computation, emphasizing the habitual distinction between geometry and geodesy or mensuration . . . a distinction expressed by Aristotle in the form: "One cannot prove anything by starting from another species, for example, anything geometrical by means of arithmetic. Where the objects are so different as arithmetic and geometry one cannot apply the arithmetical method to that which belongs to magnitudes in general, unless the magnitudes are numbers, which can happen only in certain cases." Book XIII passes from the regular polygons to the regular polyhedrons, remarking in conclusion that only the known five are possible.

The extent to which Euclid's Elements represent original work rather than compilation of that of earlier writers cannot be determined. It would appear, for example, that much of Books I and II is due to Pythagoras, of III to Hippocrates, of V to Eudoxus, and of IV, VI, XI, and XII, to later Greek writers; but the work as a whole constitutes an immense advance over previous similar attempts.

Proclus (410–485 A.D.) is the earliest extant source of information about Euclid. Theon of Alexandria edited the Elements nearly 700 years after Euclid, and until comparatively recent times modern editions have been based upon his.

Like other Greek learning, Euclid has come down to later times through Arab channels. There is a doubtful tradition that an English monk, Adelhard of Bath, surreptitiously made a Latin translation of the Elements at a Moorish university in Spain in 1120. Another dates from 1185, printed copies from 1482 onward, and an English version from 1570. After Newton's time it found its way from the universities into the lower schools.

Different versions vary widely as to the axioms and postulates on which the work as a whole is based. It is believed that Euclid originally wrote five postulates, of which the fourth and fifth are now known as Axioms 11 and 12, — "All right angles are equal"; and the famous parallel axiom: — "If a straight line meets two straight lines, so as to make the two interior angles on the same side of it together less than two right angles, these straight lines will meet if produced on that side." The necessarily unsuccessful attempts which have since been made to prove this as a proposition rather than a postulate constitute an important chapter in the history of mathematics, leading in the last century to the invention of the generalized geometry known as non-Euclidean, in which this axiom is no longer valid.

INFLUENCE OF EUCLID. — The Elements of Euclid have exerted an immense influence on the development of mathematics, and particularly of mathematical pedagogy. Aside from their substance of geometrical facts, they are characterized by a strict conformity to a definite logical form, the formulation of what is to be proved, the hypothesis, the construction, the progressive reasoning leading from the known to the unknown, ending with the familiar Q.E.D. There is a careful avoidance of whatever is not geometrical. No attempt is made to develop initiative or invention on the part of the student; the manner in which the results have been discovered is rarely evident and is even sometimes concealed; each proposition has a degree of completeness in itself. This treatise translated into the languages of modern Europe has been a remarkable means of disciplinary training in its special form of logic. No other science has had any such single permanently authoritative treatise.

CRITICISM OF EUCLID. — On the other hand, its narrowness of aim, its deliberate exclusion of the concrete, its laborious methods of dealing with such matters as infinity, the incommensurable or irrational, its imperfect substitutes for algebra, as in the theory of proportion, have diminished its usefulness, and have in comparatively recent times (in English-speaking countries) led to the substitution of modernized texts. Still, no other mathematical treatise has had even approximately the deservedly far-reaching influence of Euclid. Its subject-matter is so nearly complete that its author's name is still a current synonym for elementary geometry.

> His elements are particularly admired for the order which controls them, for the choice of theorems and problems selected as fundamental (for he has by no means inserted all which he might give, but only those which are really fundamental), and for the varied argumentation, producing conviction now by starting from causes, now by going back to facts, but always irrefutable, exact and of most scientific character. . . . Shall we mention the constantly maintained invention, economy and orderliness, the force with which he establishes every point? If one adds to or takes from it, one will recognize that he departs thereby from science, tending towards error or ignorance. . . .

Elsewhere Proclus: —

> It is difficult in every science to choose and dispose in suitable order the elements from which all the rest may be derived. Of those who have attempted this some have increased their collection, others have diminished it; some have employed abridged demonstrations, others have expanded their presentation indefinitely, etc.
>
> In such a treatise it is necessary to avoid everything superfluous . . . to combine all that is essential, to consider principally and equally clearness and brevity, to give theorems their most general form, — for the detail of teaching particular cases only makes the acquisition of knowledge more difficult. From all these points of view, Euclid's Elements will be found superior to every other.

In a recent interesting discussion of Euclid's Elements, F. Klein (*Elementar-Mathematik vom Höheren Standpunkt aus. II*) says in substance: "A false estimation of the Elements finds its source in the general misunderstanding of Greek genius which long prevailed and still finds popular acceptance, namely that Greek culture was confined to relatively few fields, but in them reached a high degree of perfection and finality. The fact is, however, that the Greeks occupied themselves with the greatest versatility in all directions, and made in all directions wonderful progress. Nevertheless, from our modern standpoint, they fell short of the possibly attainable in all, and in some directions made only a beginning.

"In mathematics, for example, it has become a tradition that Greek geometry reached unique development, while in reality many other branches of mathematics were successfully cultivated. The development of Greek mathematics was particularly hampered by the lack of a convenient number-system and notation as a basis for an independent arithmetic, and by ignorance of negative and imaginary numbers. Euclid's intention in the Elements was by no means to write an encyclopedia of current geometry, which must have included conic sections and other curves, but rather to write for mature readers an introduction to mathematics in general, the latter being regarded in its turn, in the Platonic sense, as necessary preparation for general philosophic studies. Hence the emphasis on formal order and logical method, as well as the omission of all practical applications. He aims at the flawless logical derivation of all geometrical theorems from premises completely stated in advance."

Allowing for grave uncertainties of text, Klein's view is summed up as follows:

"(1) The great historical significance of Euclid's Elements consists in the fact that through it the ideal of a flawless logical treatment of geometry was first transmitted to future times.

"(2) As to the execution, much is very finely done, but much remains fundamentally imperfect from our present standpoint.

"(3) Numerous details of importance, especially at the beginning, remain completely doubtful on account of uncertainties of the text.

"(4) The whole development is often needlessly clumsy, as Euclid has no arithmetic ready to his hand.

"(5) In general the one-sided emphasis on the logical makes it difficult to understand the subject-matter as a whole, and its internal relations."

The Elements of the great Alexandrian remain for all time the first, and one may venture to assert, the *only* perfect model of logical exactness of principles, and of rigorous development of theorems. If one would see how a science can be constructed and developed to its minutest details from a very small number of intuitively perceived axioms, postulates, and plain definitions, by means of rigorous, one would almost say chaste, syllogism, which nowhere makes use of surreptitious or foreign aids, if one would see how a science may thus be constructed, one must turn to the Elements of Euclid. — *Hankel.*

Euclid always contemplates a straight line as drawn between two definite points, and is very careful to mention when it is to be produced beyond this segment. He never thinks of the line as an entity given once for all as a whole. This careful definition and limitation, so as to exclude an infinity not immediately apparent to the senses, was very characteristic of the Greeks in all their many activities. It is enshrined in the difference between Greek architecture and Gothic architecture, and between the Greek religion and the modern religion. The spire on a Gothic cathedral and the importance of the unbounded straight line in modern geometry are both emblematic of the transformation of the modern world. — *Whitehead.*

The universally admired perfection of the work of Euclid is revealed to the historians as the natural product of a long criticism which was developed in the constructive period of rational geometry, from Pythagoras to Eudoxus. Then commenced to appear the signification of those methods and principles by means of which the Greeks themselves attempted to interpret and conquer the paradoxes concerning infinity. These are the same difficulties which reappeared at the time the infinitesimal calculus was founded, and are now again asserting themselves in the most refined analysis. — *Enriques.*

OTHER WORKS OF EUCLID. — Besides the "Elements" Euclid wrote several other mathematical treatises, including one on Porisms, a special type of geometrical proposition; and one on Data, containing such theorems as the following:

Given magnitudes have a given ratio to each other.

When two lines given in position cut each other their point of intersection is given.

When in a circle of given magnitude a line of given magnitude is given, it bounds a segment which contains a given angle.

A work on Fallacies is designed to safeguard the student against erroneous reasoning. Still other treatises are devoted to Division of Figures, Loci, and Conic Sections; finally there are works on Phenomena, on Optics, and on Catoptrics dealing with applications of geometry.

The Phenomena gives a geometrical theory of the universe, the Optics is an unsuccessful attempt to deal with problems of vision on the hypothesis that light proceeds from the eye to the object seen. The fundamental assumptions are, for example: "Rays emitted from the eye are carried in straight lines, distant by an interval from one another," etc.

The Catoptrics deals in 31 propositions with reflections in plane, concave, and convex mirrors. It is remarked that a ring placed in a vase so as to be invisible from a certain position, may be made visible by filling the vase with water. The authenticity of this work is however questionable.

These two works constitute the earliest known attempt to apply geometry systematically to the phenomena of light-rays. The law of reflection is correctly applied. Just as geometry is based on a definite list of axioms, so Euclid makes his optics depend on eight fundamental facts of experience. For example, the light rays are straight lines. The figure inclosed by the rays is a cone with its vertex at the eye, while the boundary of the object corresponds to the base, etc. This work, though in very imperfect form, continued in use until Kepler's time.

ARCHIMEDES. — The second great name in the Alexandrian school and one of the greatest in the whole history of science is that of Archimedes. He was both geometer and analyst, mathematician and engineer. He enriched even the highly developed Euclidean geometry, made important progress in algebra, laid the foundations of mechanics, and even anticipated the infinitesimal calculus, reaching thus a level which was not surpassed for 2000 years. Born in Syracuse, probably 287 B.C., the greater part of his life was spent in his native city, to which he rendered on occasion invaluable services as a military engineer. According to Livy it was due to the efforts of Archimedes that the Romans under Marcellus were held in check during the protracted siege of Syracuse. On the fall of the city in 212 B.C. the venerable mathematician, absorbed in a geometrical problem, was killed by a Roman soldier, much to the regret of Marcellus, who appreciated and would have spared him. The conqueror carried out the wish of Archimedes by erecting a monument with a mathematical figure, and this was with some difficulty rediscovered and put in order by Cicero, during his official residence in Sicily, 75 B.C.

> Nothing afflicted Marcellus so much as the death of Archimedes, who was then, as fate would have it, intent upon working out some problem by a diagram, and having fixed his mind alike and his eyes upon the subject of his speculation, he never noticed the incursion of the Romans, nor that the city was taken. In this transport of study and contemplation, a soldier, unexpectedly coming up to him commanded him to follow to Marcellus, which he declined to do before he had worked out his problem to a demonstration; the soldier, enraged, drew his sword and ran him through. Others write, that a Roman soldier, running upon him with a drawn sword, offered to kill him; and that Archimedes, looking back, earnestly besought him to hold his hand a little while, that he might not leave what he was at work upon inconclusive and imperfect; but the soldier, nothing moved by his entreaty, instantly killed him. Others again relate, that as Archimedes was carrying to Marcellus mathematical instruments, dials, spheres, and angles, by which the magnitude of the sun might be measured to the sight, some soldiers seeing him, and thinking that he carried gold in a vessel, slew him. Certain it is, that his death was very afflicting to Marcellus; and that Marcellus ever after regarded him that killed him as a murderer; and that he sought for his kindred and honored them with signal favours. — *Plutarch.*

The known works of Archimedes include the following: two books on the Equilibrium of Planes, with an interpolated treatise on the Quadrature of the Parabola, two books on the Sphere and the Cylinder, the Circle Measurement, the Spirals, the book of Conoids and Spheroids, the Sand Number, two books on Floating Bodies, Choices. Unlike Euclid's Elements, these are for the most part original papers on new mathematical discoveries, which were also often communicated to his contemporaries in the form of letters. Pappus quotes Geminus as saying of Archimedes: "He is the only man who has known how to apply to all things his varied natural gifts and inventive genius."

ARCHIMEDES AND EUCLID. — In contrasting the limitations of Euclid's Elements with the broad range of Greek mathematics, Klein characterizes the work of Archimedes somewhat as follows:

(1) Quite in contrast to the spirit controlling Euclid's Elements, Archimedes has a strongly developed sense for numerical computation. One of his greatest achievements indeed is the calculation of the ratio π of the circumference of a circle to its diameter, by approximations with regular polygons. There is no trace of interest for such numerical results with Euclid, who merely mentions that the areas of two circles are proportional to the squares of the radii, two circumferences as the radii, regardless of the actual proportionality factor.

(2) A far-reaching interest in applications of all sorts is characteristic of Archimedes, including the most varied physical and technical problems. Thus he discovered the principles of hydrostatics and constructed engines of war. Euclid on the contrary does not even mention ruler or compass, merely postulating that a straight line can be drawn through two points, or a circle described about a point. Euclid shares the view of certain ancient schools of philosophy, — a view unfortunately extant in certain quarters, — that the practical application of a science is something mechanical and unworthy. The very greatest mathematicians, Archimedes, Newton, Gauss, have combined theory and applications consistently.[1]

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[1] Plutarch, however, says: "Archimedes possessed so high a spirit, so profound a soul, and such treasures of highly scientific knowledge, that though these inventions (used to defend Syracuse against the Romans) had now obtained him the re-

(3) Finally, Archimedes was a great investigator and pioneer, who in each of his works carries knowledge a step forward. This affects materially the form of presentation. In a most recently discovered manuscript, the procedure is essentially modern as contrasted with the rigid formalism of the Elements.

Circle Measurement. — In this Archimedes proves three theorems.

(1) Every circle is equivalent to a right triangle having the sides adjacent to the right angle equal respectively to the radius and circumference of the circle.

(2) The circle has to the square on its diameter approximately the ratio 11 : 14.

(3) The circumference of any circle is three times as great as the diameter and somewhat more, namely less than $\frac{1}{7}$ but more than $\frac{10}{71}$.

He proves the first theorem by showing that the assumption that the circle is either larger or smaller than the triangle leads to a contradiction. The second he bases on the third, at which he arrives by computing successively the perimeters of both inscribed and circumscribed polygons of 3, 6, 12, 24, 48 and 96 sides. All this is contrary to the spirit of Euclid and essentially modern in its method of successive approximation. The difficulty of the achievement in view of the imperfect arithmetical notation available can hardly be overrated.

Quadrature of the Parabola. — Of special interest is his quadrature of the parabola. A segment is formed by drawing any chord PQ of the parabola: it is known that if a line is drawn from the middle point R of the chord parallel to the axis of the parabola, the tangent at the point S where this line meets the curve will be parallel to the chord, and the perpendicular from S to the chord is greater than any other which can be drawn from a point of the

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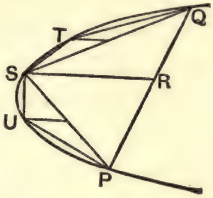
nown of more than human sagacity; he yet would not deign to leave behind him any commentary or writing on such subjects; but, repudiating as sordid and ignoble the whole trade of engineering, and every sort of art that lends itself to mere use and profit, he placed his whole affection and ambition in those purer speculations where there can be no reference to the vulgar needs of life; studies, the superiority of which to all others is unquestioned, and in which the only doubt can be whether the beauty and grandeur of the subjects examined, or the precision and cogency of the methods and means of proof, most deserve our admiration."

arc. The triangle formed by joining the same point S to the ends of the original chord being wholly contained within the segment, the area of the latter will be greater than that of the triangle and less than that of a parallelogram having the same base and altitude. Now the segment exceeds the triangle by two smaller segments, in each of which triangles STQ and SPU are again inscribed. It is a known property of the parabola that each of these triangles has one-eighth the area of the triangle PSQ. The area of each of the two smaller segments is therefore greater than one-eighth and less than one-fourth that of the triangle PSQ. The area of the original segment therefore is less than three-halves and greater than five-fourths that of triangle PSQ. The construction may evidently be repeated any number of times, and the ratio of the segment to the triangle will lie between numbers which converge towards four-thirds. Archimedes also succeeded in determining the area of the ellipse.



SPIRALS. — The discussion of spirals is based on the definition, "If a straight line moves with uniform velocity in a plane about one of its extremities which remains fixed, until it returns to its original position, and if at the same time a point moves with uniform velocity starting at the fixed point, the moving point describes a spiral." With the simple resources at his command, he also succeeds in obtaining the quadrature of this spiral, and in drawing a tangent at any point. In these quadratures he approximates the summation principle of the modern integral calculus.

Supplementing Euclid's treatment of the regular polyhedrons, Archimedes investigates the semi-regular solids formed by combining regular polygons of more than one kind. Of these he finds 13, ten of which have two kinds of bounding polygons, the others three kinds.

SPHERE AND CYLINDER. — In his important treatise on "The Sphere and the Cylinder" he derives three new theorems:

(1) That the surface of a sphere is four times the area of its great circle.

(2) That the convex surface of a segment of a sphere is equal to the area of a circle whose radius is equal to the straight line from the vertex of the segment to any point in the perimeter of its base.

(3) That the cylinder having a great circle of the sphere for its base and the diameter of the sphere for its altitude exceeds the sphere by one-half, both in volume and in surface. It was the figure for this last proposition which was at his wish carved upon his tombstone.

In attempting to solve the problem of passing a plane through a sphere so that the segments thus formed shall have either their surfaces or their volumes in an assigned ratio, he is led to a cubic equation; he appears to have given both a solution and a criterion for the existence of a positive root, but the work is lost.

In his Conoids and Spheroids he deals with the bodies formed by the revolution of the ellipse, parabola, and hyperbola, by means of plane cross-sections, ascertains the volume of these solids by comparing the portion between two neighboring planes with an inscribed and a circumscribed cylinder, — much in the modern manner.

> It is not possible to find in all geometry more difficult and more intricate questions or more simple and lucid explanations (than those given by Archimedes). Some ascribe this to his natural genius; while others think that incredible effort and toil produced these, to all appearance, easy and unlabored results. No amount of investigation of yours would succeed in attaining the proof, and yet, once seen, you immediately believe you would have discovered it; by so smooth and so rapid a path he leads you to the conclusion required.
>
> — *Plutarch.*

In other branches of mathematical science than geometry the work of Archimedes was relatively even more important.

The so-called Cattle Problem, for example, is a notable performance in the algebra of linear equations.

"The sun had a herd of bulls and cows, all of which were either white, gray, dun, or piebald; the number of piebald bulls was less than the number of white bulls by $(\frac{1}{2} + \frac{1}{3})$ of the number of gray bulls, it was less than the number of gray bulls by $(\frac{1}{4} + \frac{1}{5})$ of the number of dun bulls, and it was less than the number of dun bulls by $(\frac{1}{6} + \frac{1}{7})$ of the number of white bulls. The number of white cows was $(\frac{1}{3} + \frac{1}{4})$ of the number of gray cattle (bulls and cows), the number of gray cows was $(\frac{1}{4} + \frac{1}{5})$ of the number of dun cattle, the number of dun cows was $(\frac{1}{5} + \frac{1}{6})$ of the number of piebald cattle, and the number of piebald cows was $(\frac{1}{6} + \frac{1}{7})$ of the number of white cattle." The seven equations are insufficient to determine the eight unknown quantities. The solution attributed to Archimedes consists of numbers of nine figures each. 16

Again he succeeds in summing the series of squares: 1, 4, 9, 16, 25, 36, etc., to n terms, expressing the result in geometrical form. Both proof and formulation are of course much more complicated by reason of the entire lack of an algebraic symbolism, the same remark naturally applying also to the preceding cattle problem and to the cubic equation referred to above. This last was indeed to Archimedes not primarily an equation at all, but a proportion

$$a - x : b :: \tfrac{4}{9}\, a^2 : x^2.$$

In his Circle Measurement already outlined, he showed mastery of square root, and the comparison of irrational numbers with fractions, showing for example that

$$\tfrac{1351}{780} > \sqrt{3} > \tfrac{265}{153}.$$

How these fractions were obtained cannot be certainly determined, but it was presumably by a process analogous at least to the modern method of continued fractions, though such fractions themselves could not have been known to him.

In the Sand Counting, Archimedes undertakes to give a number which shall exceed the number of grains of sand in a sphere with a radius equal to the distance from the earth to the starry firmament. The treatise begins: "Many people believe, King Gelon, that the number of sand grains is infinite. I mean not the sand about Syracuse, nor even that in Sicily, but also that on the whole mainland, inhabited and uninhabited. There are others again who do not indeed assume this number to be infinite, but so great that no number is ever named which exceeds this. . . . I will attempt to show however by geometrical proofs which you will accept that among the numbers which I have named . . . some not only exceed the number of a sand-heap of the size of the earth, but also of that of a pile of the size of the universe." He assumes that 10,000 grains of sand would make the size of a poppy-seed, that the diameter of a poppy-seed is not less than one-fortieth of a finger-breadth, that the diameter of the earth is less than a million stadia, that the diameter of the universe is less than 10,000 diameters of the earth. To express the vast number which results from these assumptions — 10^{63} in our notation — he employs an ingenious system of units of higher order comparable with the modern use of exponents, an immense advance on current arithmetical symbolism.

MECHANICS OF ARCHIMEDES. — In mechanics Archimedes is a pioneer, giving the first mathematical proofs known. In two books on Equiponderance of Planes or Centres of Plane Gravities, he deals with the problem of determining the centres of gravity of a variety of plane figures, including the parabolic segment. A treatise on levers and perhaps on machines in general has been lost, as also a work on the construction of a celestial sphere. A sphere of the stars and an orrery constructed by him were long preserved at Rome. He describes an original apparatus for determining the angular diameter of the sun, discussing its degree of accuracy.

The lever and the wedge had been practically known from remote antiquity, and Aristotle had discussed the practice of dishonest tradesmen shifting the fulcrum of scales towards the pan in which the weights lay, but no previous attempt at exact mathematical treatment is known.

Archimedes assumes as evident at the outset:

(1) Magnitudes of equal weight acting at equal distances from their point of support are in equilibrium;

(2) Magnitudes of equal weight acting at unequal distances from their point of support are not in equilibrium, but the one acting at the greater distance sinks.

From these he deduces:

(3) Commensurable magnitudes are in equilibrium when they are inversely proportional to their distances from the point of support.

In a work on Floating Bodies, extant in a Latin version by Tartaglia, Archimedes defines a fluid as follows: "Let it be assumed that the nature of a fluid is such that, all its parts lying evenly and continuous with one another, the part subject to less pressure is expelled by the part subject to greater pressure. But each part is pressed perpendicularly by the fluid above it, if the fluid is falling or under any pressure." "Every solid body lighter than a liquid in which it floats sinks so deep that the mass of liquid which has the same volume with the submerged part weighs just as much as the floating body." The specific gravity of heavier bodies was of course employed in his solution of the crown problem, which with his achievements as a military engineer gave him a great reputation among his contemporaries. Vitruvius in his *De Architectura* says:

> Though Archimedes discovered many curious matters that evinced great intelligence, that which I am about to mention is the most extraordinary. Hiero, when he obtained the regal power in Syracuse, having, on the fortunate turn of his affairs, decreed a votive crown of gold to be placed in a certain temple to the immortal gods, commanded it to be made of great value, and assigned for this purpose an appropriate weight of the metal to the manufacturer. The latter, in due time, presented the work to the king, beautifully wrought; and the weight appeared to correspond with that of the gold which had been assigned for it.
>
> But a report having been circulated, that some of the gold had been abstracted, and that the deficiency thus caused had been supplied by silver, Hiero was indignant at the fraud, and, unacquainted with the method by which the theft might be detected, requested Archimedes would undertake to give it his attention. Charged with

> this commission, he by chance went to a bath, and on jumping into the tub, perceived that, just in the proportion that his body became immersed, in the same proportion the water ran out of the vessel. Whence, catching at the method to be adopted for the solution of the proposition, he immediately followed it up, leapt out of the vessel in joy, and returning home naked, cried out with a loud voice that he had found that of which he was in search, for he continued exclaiming, 'I have found it, I have found it!' — *Vitruvius.*
>
> Archimedes, who combined a genius for mathematics with a physical insight, must rank with Newton, who lived nearly two thousand years later, as one of the founders of mathematical physics. . . . The day (when having discovered his famous principle of hydrostatics he ran through the streets shouting Eureka! Eureka!) ought to be celebrated as the birthday of mathematical physics; the science came of age when Newton sat in his orchard. — *Whitehead.*

The recently discovered New Manuscript[1] of Archimedes throws a very interesting light on his methods of attacking problems in mechanics, as well as on his use of mechanical methods for geometrical problems. Naturally his mathematical methods are highly developed in comparison with the relatively simple problems of mechanics with which he deals.

> 'Certain things first became clear to me by a mechanical method, although they had to be demonstrated by geometry afterwards because their investigation by the said method did not furnish an actual demonstration. But it is of course easier, when we have previously acquired, by the method, some knowledge of the questions, to supply the proof than it is to find it without any previous knowledge. I apprehend that some, either of my contemporaries or of my successors, will, by means of the method when once established, be able to discover other theorems . . . which have not yet occurred to me.'
>
> Our admiration of the genius of the greatest mathematician of antiquity must surely be increased, if that were possible, by a perusal of the work before us. — *Heath.*

ARCHIMEDES AS AN ENGINEER. — His engineering skill, which has gained from an eminent German historian the appellation of

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[1] See Reviews by C. S. Slichter and D. E. Smith. Bulletin, American Mathematical Society, May, 1908, Feb. 1913.

"the technical Yankee of antiquity," may be inferred from Plutarch's account of the siege of Syracuse: —

> Now the Syracusans, seeing themselves assaulted by the Romans, both by sea and by land, were marvellously perplexed, and could not tell what to say, they were so afraid; imagining it was impossible for them to withstand so great an army. But when Archimedes fell to handling his engines, and set them at liberty, there flew in the air infinite kinds of shot, and marvellous great stones, with an incredible noise and force on the sudden, upon the footmen that came to assault the city by land, bearing down, and tearing in pieces all those which came against them, or in what place soever they lighted, no earthly body being able to resist the violence of so heavy a weight; so that all their ranks were marvellously disordered. And as for the galleys that gave assault by sea, some were sunk with long pieces of timber like unto the yards of ships, whereto they fasten their sails, which were suddenly blown over the walls with force of their engines into their galleys, and so sunk them by their over great weight.
>
> These machines (used in the defense of the Syracusans against the Romans under Marcellus) he (Archimedes) had designed and contrived, not as matters of any importance, but as mere amusements in geometry; in compliance with king Hiero's desire and request, some time before, that he should reduce to practice some part of his admirable speculation in science, and by accommodating the theoretic truth to sensation and ordinary use, bring it more within the appreciation of people in general. Eudoxus and Archytas had been the first originators of this far-famed and highly-prized art of mechanics, which they employed as an elegant illustration of geometrical truths, and as means of sustaining experimentally, to the satisfaction of the senses, conclusions too intricate for proof by words and diagrams. As, for example, to solve the problem, so often required in constructing geometrical figures, given the two extremes, to find the two mean lines of a proportion, both these mathematicians had recourse to the aid of instruments, adapting to their purpose certain curves and sections of lines. But what with Plato's indignation at it, and his invectives against it as the mere corruption and annihilation of the one good of geometry, — which was thus shamefully turning its back upon the unembodied objects of pure intelligence to recur to sensation, and to ask help (not to be obtained without base super-

visions and depravation) from matter; so it was that mechanics came to be separated from geometry, and, repudiated and neglected by philosophers, took its place as a military art.

One of his most famous inventions was the water-screw used for irrigation, in Egypt, and for pumping. On occasion of difficulty in the launching of a certain ship he successfully applied a cogwheel apparatus with an endless screw.

> Archimedes . . . had stated that given the force, any given weight might be moved, and even boasted, we are told, relying on the strength of demonstration, that if there were another earth, by going into it he could remove this. Hiero being struck with amazement at this, and entreating him to make good this problem by actual experiment, and show some great weight moved by a small engine, he fixed accordingly upon a ship of burden out of the king's arsenal, which could not be drawn out of the dock without great labor and many men; and, loading her with many passengers and a full freight, sitting himself the while far off with no great endeavor, but only holding the head of the pulley in his hand and drawing the cords by degrees, he drew the ship in a straight line, as smoothly and evenly, as if she had been in the sea. The king, astonished at this, and convinced of the power of the art, prevailed upon Archimedes to make him engines accommodated to all the purposes, offensive and defensive, of a siege . . . the apparatus was, in most opportune time, ready at hand for the Syracusans, and with it also the engineer himself.
>
> — *Plutarch.*

In astronomy his orrery has been mentioned; he also attempted to determine the length of the year more closely.

To the critical estimates already cited may be added as typical of countless others: —

> Whoever gets to the bottom of the works of Archimedes will admire the discoveries of the moderns less. — *Leibnitz.*
>
> His discoveries are forever memorable for their novelty and the difficulty which they presented at that time, and because they are the germ of a great part of those which have since been made, chiefly in all branches of geometry which have for their object the measure-

ment of the dimensions of lines and curved surfaces and which require the consideration of the infinite. — *Mach.*

The genius of Archimedes created the theory of the composition of parallel forces, of centres of gravity, and of equilibrium of floating bodies. But antiquity went no farther; not only were the first principles of dynamics unsuspected, but the statistical composition of concurrent forces was unknown, and the explanation of machines was confined to extension of the principles of the lever, which is the starting-point of the works of Archimedes, but may nevertheless have been recognized before him. — *Tannery.*

ALEXANDRIAN GEOGRAPHY: EARTH MEASUREMENT. — The far reaching conquests of Alexander and the resulting migrations and colonizations naturally gave a powerful stimulus to geography as a branch of descriptive knowledge. Chaldean records became accessible to the Alexandrian Greeks and a more accurate system of time-measurement was introduced. Until about this period it had been customary to make appointments at the time when a person's shadow should have a certain length.

ERATOSTHENES, — 275–194 B.C., librarian of the great library at Alexandria, making a systematic quantitative study of the data thus collected, laid the foundations of mathematical geography — a transformation quite analogous to that taking place in astronomy. After a historical review he gives numerical data about the inhabited earth, which he estimates to have a length of 78,000 stadia and a breadth of 38,000. In connection with this he gives also a remarkably successful determination of the circumference of the earth. This was based on his observation that a gnomon at Syene (Assouan) threw no shadow at noon of the summer solstice, while at Alexandria the zenith distance of the sun at noon was $\frac{1}{50}$ of the circumference of the heavens. Assuming the two places to lie on the same meridian and taking their distance apart as 5000 stadia, he infers that the whole circumference must be 250,000 stadia. He or some successor afterwards substituted 252,000, perhaps in order to obtain a round number, 700 stadia, for the length of one degree.

This result, subject to some uncertainty as to the length of the stadium, was a close approximation to the real circumference, but we may suppose that this degree of accuracy was to some extent a matter of accident. Posidonius, a noted Stoic philosopher, born in 136 B.C., stated that the bright star Canopus culminated just on the horizon at Rhodes, while its meridian altitude at Alexandria was "a quarter of a sign, that is, one forty-eighth part of the zodiac." This would correspond with a circumference of 240,000 stadia, the method being quite inferior in accuracy to that of Eratosthenes, on account of the impossibility of determining when a star is just on the horizon. Eratosthenes is also credited with measuring the obliquity of the ecliptic with an error of but about seven minutes.

A student of the Athenian Platonists and a man of extraordinary versatility, philosopher, philologian, mathematician, athlete, Eratosthenes wrote on many subjects. He may well have been responsible for the introduction of leap-year into the Egyptian calendar by the "Decree of Canopus" in 238 B.C.,

> in order that the seasons may continually render their service according to the present order and that it may not happen that some of the public festivals which are celebrated in the winter come to be observed sometimes in the summer. . . .

He invented a method and a mechanical apparatus for duplicating the cube.[1] Such a mechanical solution is naturally obnoxious to the principles of Plato and Euclid.

His so-called "sieve" is a method for systematically separating out the prime numbers by arranging all the natural numbers in order, and then striking out first all multiples of 2, then of 3, and so forth, thus sifting out all but the primes 1, 2, 3, 5, 7, 11, 13, 17, etc.

Apollonius of Perga, about 260–200 B.C., "the great geometer," was the last of this famous Alexandrian group of mathematicians, and owes his reputation to his important work on the conic sections. His predecessors had in general recognized only those sections formed from right circular cones by planes normal to an element. Archimedes, indeed, and Euclid obtained ellipses by passing other planes through right cones, but Apollonius first showed that any cone and any section could be taken, and introduced the names ellipse, parabola, and hyperbola. In the prefatory letter to Book I, Apollonius says to the friend to whom it is addressed: —

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[1] See Gow, p. 245.

> 'Apollonius to Eudemus, greeting. When I was in Pergamum with you, I noticed that you were eager to become acquainted with my Conics; so I send you now the first book with corrections and will forward the rest when I have leisure. I suppose you have not forgotten that I told you that I undertook these investigations at the request of Naucrates the geometer, when he came to Alexandria and stayed with me; and that, having arranged them in eight books, I let him have them at once, not correcting them very carefully (for he was on the point of sailing) but setting down everything that occurred to me, with the intention of returning to them later. Wherefore I now take the opportunity of publishing the needful emendations. But since it has happened that other people have obtained the first and second books of my collections before correction, do not wonder if you meet with copies which are different from this.' — *Gow.*

Of the eight books, the first four are devoted to an elementary introduction. In Book I he defines the cone as generated by a straight line passing through a point on the circumference of a circle and a fixed point not in the same plane; he fixes the manner in which sections are to be taken and defines diameters and vertices of the curves, also the latus rectum and centre, conjugate diameters and axes. The other branch of the hyperbola is taken due account of for the first time. In Book II asymptotes are defined by the statement: "One draws a tangent at a point of the hyperbola, measures on it the length of the diameter parallel to it, and connects the point thus determined with the centre of the hyperbola." Book III contains numerous theorems on tangents and secants and introduces foci with the definition: "A focus is a point which divides the major axis into two parts whose rectangle is one-fourth that of the latus rectum and the major axis," or the square on the minor axis. The focus of the parabola however is not recognized, nor has he any knowledge of the directrix of a conic section, these omissions being first filled by Pappus in the third century A.D. It is shown that the normal makes equal angles with the focal radii to the point of contact, and that the latter have a constant sum for the ellipse, a constant difference for the hyperbola. This book, he says in the letter quoted above, "contains many curious theorems, most of them are pretty and new, useful for the synthesis of solid loci. . . . In the invention of these, I observed that Euclid had not treated synthetically the locus . . . but only a certain small portion of it, and that not happily, nor indeed was a complete treatise possible at all without my discoveries." These three books, which are indeed based largely on the earlier work of Euclid and others, contain most of the properties of conic sections discussed in modern text-books on analytic geometry. Book IV discusses the intersections of conics, treating tangency correctly as equivalent to two ordinary intersections. In Book V Apollonius even undertakes the difficult problem of determining the longest and shortest lines which can be drawn from a given point to a conic, identifying this with the problem of drawing normals from a given point. He succeeds in discovering the points for which two such normals coincide, *i. e.* what we call the centre of curvature. Book VI deals with equal and similar conics, reaching the problem of passing through a given cone a plane which shall cut out a given ellipse. Book VII deals with conjugate diameters and the complementary chords parallel to them. Book VIII is lost. On the whole, in this remarkable work of some 400 propositions he achieved nearly all the results which are included in our modern elementary analytic geometry, even approximating the introduction of a system of coördinates by his use of lines parallel to the principal axes.

It is noteworthy that Fermat, one of the inventors of modern analytic geometry, was led to it by attempting to restore certain lost proofs of Apollonius on loci.

Of his other mathematical writings little more than the titles are known. Among these are one on burning mirrors, one on stations and retrogressions of the planets, and one on the use and theory of the screw. In astronomy he is believed to have suggested expressing the motions of the planets by combining uniform circular motions, an idea afterwards elaborated by Hipparchus and Ptolemy. How far his mathematical results were new, how far he merely compiled and coördinated the work of others, notably Euclid and Archimedes, cannot be precisely determined, but the proportion of original work is certainly very large.

On the arithmetical side he obtained a closer approximation than Archimedes for the value of π, invented an abridged method of multiplication, and employed numbers of higher order in the manner of Archimedes. This last experiment if followed out to its logical conclusions might have had fundamental significance for the future development of computation. In the words of Gow: —

> he, as well as Archimedes, lost the chance of giving to the world once for all its numerical signs. That honor was reserved by the irony of fate for a nameless Indian of an unknown time, and we know not whom to thank for an invention which has been as important as any to the general progress of intelligence.

APOLLONIUS AND ARCHIMEDES. — With Apollonius and Archimedes the ancient mathematics had accomplished whatever was possible without the resources of analytic geometry and infinitesimal calculus, which, though already foreshadowed, were not fully realized until the seventeenth century.

> It is not only a decided preference for synthesis and a complete denial of general methods which characterize the ancient mathematics as against our newer science (modern mathematics): besides this external formal difference there is another real, more deeply seated, contrast, which arises from the different attitudes which the two assumed relative to the use of the concept of *variability*. For while the ancients, on account of considerations which had been transmitted to them from the philosophic school of the Eleatics, never employed the concept of motion, the spatial expression for variability, in their rigorous system, and made incidental use of it only in the treatment of phoronomically generated curves, modern geometry dates from the instant that Descartes left the purely algebraic treatment of equations

and proceeded to investigate the variations which an algebraic expression undergoes when one of its variables assumes a continuous succession of values. — *Hankel.*

In one of the most brilliant passages of his *Aperçu historique* Chasles remarks that, while Archimedes and Apollonius were the most able geometricians of the old world, their works are distinguished by a contrast which runs through the whole subsequent history of geometry. Archimedes, in attacking the problem of the quadrature of curvilinear areas, established the principles of the geometry which rests on measurements; this naturally gave rise to the infinitesimal calculus, and in fact the method of exhaustions as used by Archimedes does not differ in principle from the method of limits as used by Newton. Apollonius, on the other hand, in investigating the properties of conic sections by means of transversals involving the ratio of rectilineal distances and of perspective, laid the foundations of the geometry of form and position. — *Ball.*

The works of Archimedes and Apollonius marked the most brilliant epoch of ancient geometry. They may be regarded, moreover, as the origin and foundation of two questions which have occupied geometers at all periods. The greater part of their works are connected with these and are divided by them into two classes, so that they seem to share between them the domain of geometry.

The first of these two great questions is the quadrature of curvilinear figures, which gave birth to the calculus of the infinite, conceived and brought to perfection successively by Kepler, Cavalieri, Fermat, Leibnitz and Newton.

The second is the theory of conic sections, for which were invented first the geometrical analysis of the ancients, afterwards the methods of perspective and of transversals. This was the prelude to the theory of geometrical curves of all degrees, and to that considerable portion of geometry which considers, in the general properties of extension, only the forms and situations of figures, and uses only the intersection of lines or surfaces and the ratios of rectilineal distances.

These two great divisions of geometry, which have each its peculiar character, may be designated by the names of *Geometry of Measurements* and *Geometry of Forms and Situations,* or Geometry of Archimedes and Geometry of Apollonius. — *Chasles* (*Gow*).

Medical Science at Alexandria. Beginnings of Human Anatomy. — Alexandria is famous in the history of medicine for many reasons. It was here that human, — as contrasted with comparative, — anatomy was first freely studied (probably favored by the Egyptian practice of disemboweling and embalming the dead) with the result that many of the grotesque errors of the earlier Greeks, including even Aristotle, were corrected. In this connection two names, and those of rivals, have come down to us as of chief importance, Herophilus and Erasistratus. The former, himself a student at Cos, was a close follower of the teachings of Hippocrates and regarded by the ancient world as his worthy successor. Erasistratus, on the contrary, opposed the Hippocratic doctrines. Both became distinguished anatomists. It is believed that the valves of the heart were first recognized and named by Erasistratus, who also studied and described the divisions, cavities and membranes of the brain, as well as the true origin and nature of the nerves. Herophilus likewise studied the brain, the pulmonary artery and the liver, besides giving to the duodenum the name (twelve-inch) which it still bears. Physiology, meanwhile, made little or no progress, and Cicero, two centuries later, still speaks of the arteries as "air tubes." It appears also that vivisection as well as anatomy was practised at Alexandria, and probably even upon human beings.

Pergamum, in Asia Minor, was for a time a rival centre of medical learning and medical education, but was eventually overshadowed by the more famous Alexandrian school. Of this last the most celebrated pupil was Galen (born 130 A.D.), the most noted medical man of the ancient Roman world. Galen was a native of Pergamum who, having first studied at home and at Smyrna, spent some years at Alexandria. He then returned to Pergamum, but soon went to Rome, where he became physician to the Emperor Commodus. Galen was an original and voluminous writer on anatomy. That his name is still constantly linked with that of Hippocrates is probably the best evidence of his importance in the history of medical science.

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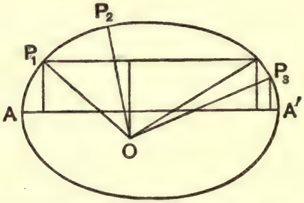


The three normals to the ellipse. Apollonius.

CHAPTER VI

THE DECLINE OF ALEXANDRIAN SCIENCE

> The century which produced Euclid, Archimedes and Apollonius was . . . the time at which Greek mathematical genius attained its highest development. For many centuries afterwards geometry remained a favorite study, but no substantive work fit to be compared with the *Sphere and Cylinder* or the *Conics* was ever produced. One great invention, trigonometry, remains to be completed, but trigonometry with the Greeks remained always the instrument of astronomy and was not used in any other branch of mathematics, pure or applied. The geometers who succeed to Apollonius are professors who signalised themselves by this or that pretty little discovery or by some commentary on the classical treatises.
>
> The force of nature could go no further in the same direction than the ingenious applications of exhaustion by Archimedes and the portentous sentences in which Apollonius enunciates a proposition in conics. A briefer symbolism, an analytical geometry, an infinitesimal calculus were wanted, but against these there stood the tremendous authority of the Platonic and Euclidean tradition, and no discoveries were made in physics or astronomy which rendered them imperatively necessary. It remained only for mathematicians, as Cantor says, to descend from the height which they had reached and "in the descent to pause here and there and look around at details which had been passed by in the hasty ascent." The elements of planimetry were exhausted, and the theory of conic sections. In stereometry something still remained to be done, and new curves, suggested by the spiral of Archimedes, could still be investigated. Finally, the arithmetical determination of geometrical ratios, in the style of the Measurement of the Circle, offered a considerable field of research, and to these subjects mathematicians now devoted themselves. — *Gow.*

IN the second century B.C. Hypsicles developed the theory of arithmetical progression and added two books of elements to Euclid's thirteen, but the chief mathematical work of this century was due to Hipparchus, a great astronomer, and Hero, an engineer.

ORBITAL MOTION OF THE EARTH. ARISTARCHUS. — Before dealing with Hipparchus and Hero, however, we have to consider the highly interesting and significant astronomical theories of Aristarchus of Samos (270 B.C.–?), who was the author of a treatise On the Dimensions and Distances of the Sun and Moon. He endeavored to determine these distances relatively by ascertaining or estimating the angular distance between the two bodies when the moon is just half illuminated, that is, when the lines joining sun, earth, and moon form a right angle at the moon — a method which may have been due to Eudoxus. The difficulties of this determination are so serious, however, that no high degree of accuracy could be attained, the actual result of Aristarchus $\frac{29}{30}$ of a right angle — against the true $\frac{539}{540}$ — corresponding to a ratio of about 1 to 19 of the two distances. Aristarchus had no trigonometry, and no other method of attacking this problem seems to have been known to the Greeks.

In his Sand Counting already mentioned, Archimedes says of Aristarchus,

> He supposes that the fixed stars and the sun are immovable, but that the earth is carried round the sun in a circle which is in the middle of the course; but the sphere of the fixed stars, lying with the sun round the same centre, is of such a size that the circle, in which he supposes the earth to move, has the same ratio to the distance of the fixed stars as the centre of the sphere has to the surface. But this is evidently impossible, for as the centre of the sphere has no magnitude, it follows that it has no ratio to the surface. It is therefore to be supposed that Aristarchus meant that as we consider the earth as the centre of the world, then the earth has the same ratio to that which we call the world, as the sphere in which is the circle, described by the earth according to him, has to the sphere of the fixed stars.

Aristarchus thus meets the objection that motion of the earth would cause changes in the apparent positions of the stars by assuming that their distances are so great as to render the motion of the earth a negligible factor. Another reference to Aristarchus, in Plutarch, mentions an opinion that he

> ought to be accused of impiety for moving the hearth of the world, as the man in order to save the phenomena supposed that the heavens stand still and the earth moves in an oblique circle at the same time as it turns round its axis.

How far this remarkable anticipation of the Copernican theory was a conviction rather than a mere fortunate speculation cannot be known, but at any rate it failed of that acceptance necessary to its permanence. In the next century the rotation of the earth on its axis was indeed taught by Seleucus, an Asiatic astronomer, but it was 1700 years before these daring theories were again advanced. Seleucus also observed the tides, saying "that the revolution of the moon is opposed to the earth's rotation, but the air between the two bodies being drawn forward falls upon the Atlantic Ocean, and the sea is disturbed in proportion."

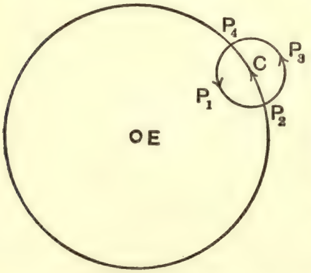
PLANETARY IRREGULARITIES. — The earlier theory of homocentric spheres, while accounting more or less successfully for the apparent motions of the heavenly bodies, had maintained each of them at a constant distance from the earth, and thus quite failed to explain the differences of brightness which were soon discovered, as well as the variations in the apparent size of the moon. The conception of motion in neither a straight line nor a circle was repugnant to the Greek philosophers, and the difficulty was therefore met, first by supposing the earth not to be exactly at the centre of the circular orbits about it, second by introducing subsidiary circles or epicycles.

EXCENTRIC CIRCULAR ORBITS. — The complete planetary system according to the excentric circle theory was therefore as follows. In the centre of the universe the earth, round which moved the moon in 27 days, and the sun in a year, probably in concentric circles. Mercury and Venus moved on circles, the centres of which were always on the straight line from the earth to the sun, so that the earth was always outside these circles, for which reason the two planets are always within a certain limited angular distance of the sun, from which the ratio of the radius of the excentric to the distance of its centre from the earth could easily be determined for either planet. Similarly, the three outer planets moved on excentric circles, the centres of which lay somewhere on the line from the earth to the sun, but these circles were so large as always to surround both the sun and the earth. — *Dreyer.*

It seems probable that Aristarchus was led through this theory to conceive of heliocentric orbits, and then to reflect that the earth, too, might revolve about the sun as easily as the sun and planets round the earth.

EPICYCLES. — Progress in observational astronomy increased the number and magnitude of planetary irregularities beyond the stationary points, retrograde motions, and variations, known to Aristarchus, and apparently far beyond possible explanation by the simple theory of excentric circles. The system was therefore superseded by, or combined with, that of epicycles, not necessarily as physically realized, but as at least a geometrical working hypothesis, which should conform to and explain the observed phenomena.

The system of epicycles consists in superimposing one circular motion upon another, and repeating the process to any needful extent. The motion of the moon about the earth, for example, is explained by assuming first a circle (later called the deferent) on which moves the centre of a second smaller circle called the epicycle, on which the moon itself travels. By varying the dimensions of both circles and the velocities of the two motions, the observed changes, both of position and brightness of the moon, may be more or less satisfactorily accounted for and even computed in advance. In particular, the apparent retrograde motions of the planets in certain parts of their orbits may be explained.



In the figure E denotes the earth, the large circle is the deferent of a planet, C the centre of the epicycle, P_1, P_2, P_3, P_4 different possible positions of the planet in its epicycle. The distance of P from E obviously varies; the apparent motion of P being compounded of a forward motion of C and a backward motion at P_1 is slower, at P_3 faster, than the average. By suitable adjustment of the dimensions and velocities there may be retrogression for a certain length of arc near P_1, bounded by stationary points where the two motions seem to an observer at E to neutralize each other.

How far this complicated scheme really departed from the original postulate of uniform circular motion is sufficiently indicated by Proclus' remark, "The astronomers who have presupposed uniformity of motions of the celestial bodies were ignorant that the essence of these movements is, on the contrary, irregularity." While in point of fact the theory of epicycles and that of excentric circles have much in common, the former gradually displaced the latter on account of its greater simplicity. Had Aristarchus worked out the earlier system in full detail, the history of astronomy might have been considerably modified.

At the Museum of Alexandria a school of observers of whom Aristillus and Timocharis were notable members instituted systematic astronomical observations with graduated instruments and made a small star catalogue. Thus was laid a foundation for the brilliant discoveries of Hipparchus and Ptolemy, while astronomy, which had in the work of Eudoxus assumed the character of true science, though with a too slender observational basis, now became an exact science, gradually shedding its encumbrances of speculation and vague generalization.

Hipparchus. Star Catalogue. — The next great astronomer and much the greatest of antiquity is Hipparchus, probably a native of Bithynia, but long resident at Rhodes, a city which rivalled Alexandria itself in its intellectual activity. All his works but one are lost, but his great successor and disciple, Ptolemy, has based his famous Almagest on the work of Hipparchus and it is possible to determine in a general way how much is to be credited to each. Having at his disposal the primitive star catalogue of Aristillus and Timocharis, Hipparchus was profoundly impressed —

as was Tycho Brahe centuries later — by the sudden appearance in 134 B.C. in the supposedly changeless starry firmament of a new star of the first magnitude. He accordingly set himself the heavy task of making a new catalogue, which ultimately included more than 1000 stars, for the part of the sky visible to him, and "remained, with slight alterations, the standard for nearly sixteen centuries." His list of constellations is the basis of our own.

PRECESSION OF THE EQUINOXES. — While this great piece of routine work was deliberately planned by Hipparchus, not so much as an end in itself as a necessary basis for future investigators, it nevertheless led to his most remarkable discovery, that of the precession of the equinoxes. In comparing, namely, the positions of certain stars with those observed about 150 years earlier, he detected a change of distance from the equinoctial point — where the celestial equator and the ecliptic meet — amounting in one case to about 2°. By an inspiration of genius, he interpreted this correctly as due to a slight progressive shifting of the equinoctial points, corresponding to a slow rotation of the earth's axis, by means of which the celestial pole in many thousand years describes a complete circle. His estimate of $36''$ per year was considerably below the actual value, which is about $50''$.

OTHER ASTRONOMICAL DISCOVERIES. PLANETARY THEORY. — Striving always for greater accuracy and completeness of data, he determined the length of the year within about six minutes. In attempting to explain the annual motion of the sun, he was aware that the change of direction is not uniform, and its distance from the earth, as shown by its apparent size, not constant. He determined the length of spring as 94 days, that of summer as $92\frac{1}{2}$, and by a somewhat complicated calculation arrived at the value $\frac{1}{24}$ as the eccentricity of the earth's position in the sun's orbit. These determinations were naturally very difficult and imperfect on account of the entire lack of accurate time measurement. Following Apollonius, Hipparchus devised a combination of uniform circular motions which should account for the observed facts within the limits of probable error of observation, and in this undertaking he was successful, the degree of accuracy of his theory corresponding to that of which his instruments were capable.

With the more complicated lunar theory he was naturally less successful. He is believed, however, to have discovered the more important irregularities of the moon's motion, supposing it to have a circular orbit in a plane making an angle of 5° with that of the sun's orbit — the ecliptic. The earth is not at the centre, but the latter revolves about the earth in a period of nine years.

Extending his study of eclipses to the ancient records of the Chaldeans, he made substantial improvements in the theory of both solar and lunar eclipses, and obtained a close approximation for the distance of the moon. He estimated the sun's radius at about twelve times that of the earth, its distance from the earth at about 2550 earth-radii, the moon's radius $\frac{29}{100}$ that of the earth, its distance about 60 earth-radii. The comparison of these figures with Ptolemy's and with the actual are (in earth-radii) —

	Sun's Radius	Sun's Distance	Moon's Radius	Moon's Distance
Hipparchus . . .	12	2550	.29	60
Ptolemy	5.5	1210	.29	59
Actual	109.	23,000	.273	$60\frac{1}{3}$

Hipparchus realized that he had no adequate method for determining these numbers for the sun.

The generally accepted order of the planets had now become Moon, Mercury, Venus, Sun, Mars, Jupiter, Saturn, — an order adopted very early in Babylonia, and received as a more or less probable hypothesis from this time until that of Copernicus. In attempting to deal with the motions of the other planets as he had done with that of the sun and moon, Hipparchus was soon baffled by lack of adequate data, and set himself steadfastly to supply the need, resigning to more fortunate future astronomers the task of interpretation.

Eudoxus, more than two centuries earlier, had developed a logical mathematical theory of the planetary motions. The more exact methods and data of Hipparchus brought out the entire inadequacy of existing theory to furnish anything better than a crude approximation to the motions of the planets, and showed the necessity both of a better theory and of more complete observational data. It is interesting to speculate on the consequences which might have resulted for astronomical science had the genius of Hipparchus adopted the daring heliocentric theories of Aristarchus instead of adhering to the traditional geocentric ideas.

INVENTION OF TRIGONOMETRY. — Not least important among the services of Hipparchus to science was his laying the foundations of trigonometry, by constructing for astronomical use a table of chords, equivalent to our tables of natural sines. He gave also a method for solving spherical triangles. It is said that he first indicated position on the earth by latitude and longitude — the germ of coördinate geometry — Eratosthenes having merely given the latitude by means of the height of the pole-star. For mapping the sky he used stereographic projection, for mapping the earth orthographic.

To sum up the chief work of Hipparchus: — he made very effective use of extant records of earlier astronomers with critical consideration of their value; he made a prolonged and systematic series of observations with the best available instruments; he worked out a consistent mathematical theory of the motions of the heavenly bodies so far as his data warranted; he made a new catalogue of 1080 stars, with the classification by magnitude still in use; he discovered the precession of the equinoxes; he laid the foundations of trigonometry.

Delambre, the great French historian of astronomy, says: —

> When we consider all that Hipparchus invented or perfected and reflect upon the number of his works and the mass of calculations which they imply, we must regard him as one of the most astonishing men of antiquity, and as the greatest of all in the sciences which are not purely speculative, and which require a combination of geometrical knowledge with a knowledge of phenomena, to be observed only by diligent attention and refined instruments.

In spite of these brilliant achievements, the position of Apollonius and Hipparchus had become relatively isolated under the prevalent

Stoic philosophy, which was attended with a reversion to primitive cosmical notions. Even in Hipparchus a somewhat critical attitude, excellent in its immediate results, has been regarded by some as foreshadowing the period of decadence which actually followed. Astronomy is to remain nearly stationary for sixteen centuries.

INVENTIONS. CTESIBUS AND HERO. — In the period of civil war following the death of Alexander and followed in turn by Roman conquest, much attention was naturally devoted to the invention and improvements of military engines. Compressed air came into use as a motive power and the foundations of pneumatics were laid.

Ctesibus, a barber of Alexandria, distinguished by his mechanical inventions, and his follower Hero (or Heron) who flourished in the latter part of the second century B.C., made notable inventions and some real contributions to mathematical science. The works attributed to Hero, on the basis of a great quantity of confused and doubtful material, include: — a Mechanics, treating of centres of gravity and of the lever, wedge, screw, pulley, and wheel and axle; various works on military engines and mechanical toys, a Pneumatics — the oldest work extant on the properties of air and vapor — describing many machines, among others a fire-engine, a water-clock, organs, and in particular a steam-engine which we may regard as a remote precursor of our modern steam turbine. Many of the machines depend for their action on the flow of water into a vacuum, which Hero, having no conception of atmospheric pressure, attributed to nature's "abhorrence" of a vacuum. He arrived at the important law for the lever and the pulley: "The ratio of the times is equal to the inverse ratio of the forces applied." The Dioptra, a treatise on a kind of rudimentary theodolite, discusses such engineering problems as finding differences of level, cutting a tunnel through a hill, sinking a vertical shaft to meet a horizontal tunnel, measuring a field without entering it, etc. The instrument employed is described as a straight plank, 8 or 9 feet long, mounted on a stand but capable of turning through a semicircle. It was adjusted by screws, turning cog-wheels. There was an eye-piece at each end and a water level at the side.

With it two poles, bearing disks, were used, exactly as by modern surveyors. A cyclometer for a carriage is also described, with a series of cog-wheels and an index.

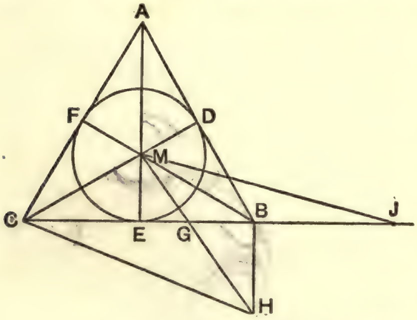
In optics he shows that under the law of equal angles of incidence and reflection, the path described by the ray is a minimum.

HERO'S TRIANGLE FORMULA. — His Geodesy, — also the Dioptra — contains the well-known formula for the area of a triangle

$$K = \sqrt{\frac{a+b+c}{2} \times \frac{a+b-c}{2} \times \frac{b+c-a}{2} \times \frac{c+a-b}{2}}$$

which, since it involves the multiplication of *four* lengths together, is heterodox from the Euclidean standpoint.

ABC is the given triangle of sides a, b, c, touching its inscribed circle at D, E, and F. Taking $BJ = AD$, we have $CJ = \frac{1}{2}(a+b+c)$ and area ABC = twice area CJM. Draw perpendiculars to CM at M and to CJ at B, meeting in H. A semicircle on the diameter CH will pass through both M and B. The sum of the angles CHB and CMB is 180°; the triangles BCH and MAD are therefore similar,



$$\text{so that } BC : BH = AD : MD, \text{ or } BC : BJ = BH : ME.$$

Also the triangles BGH and EGM are similar,

$$\text{so that } BH : ME = BG : EG \text{ and } BC : BJ = BG : EG,$$

whence $BC + BJ : BJ = BG + EG : EG$, that is,

$$CJ : BJ = BE : EG \text{ and } \overline{CJ}^2 : BJ \times CJ = CE \times BE : CE \times EG,$$

that is, $\overline{CJ}^2 : BJ \times CJ = BE \times CE : \overline{EM}^2$, which is equivalent to

$$BE \times CE : CJ \times EM = CJ \times EM : BJ \times CJ.$$

But $CJ \times EM = \frac{K}{2}$, $CE = \frac{1}{2}(a+b-c)$, etc.,

whence $4K^2 = (a+b+c)(a+b-c)(b+c-a)(c+a-b)$.

A triangle with sides 13, 14, 15 is selected as an illustration. Its area is

$$\sqrt{21 \times 6 \times 7 \times 8} = 84.$$

This work seems to have become a standard authority for generations of surveyors, and thus in course of time to have lost much of its identity by successive changes. The whole spirit of the work is rather Egyptian than Greek, that of the practical engineer as distinguished from that of the mathematician, thus in a measure a reversion to the aims of the Ahmes manuscript. "Let there be a circle with circumference 22, diameter 7. To find its area. Do as follows. $7 \times 22 = 154$ and $\frac{154}{4} = 38\frac{1}{2}$. That is the area."

Some of Hero's methods indicate knowledge of the new trigonometry of Hipparchus and of the principle of coördinates. Thus he finds areas of irregular boundary by counting inscribed rectangles, a process corresponding to the use of coördinate paper.

From Hero date such time-honored problems as that of the pipes. A vessel is filled by one pipe in time t_1, by another in time t_2. How long will it take to fill it when both pipes are used?

He defines spherical triangles and proves simple theorems about them: — for example, that the angle-sum lies between 180° and 540°. He determines the volume of irregular solids by measuring the water they displace. Having by a blunder introduced $\sqrt{-63}$ he confuses it with $\sqrt{63}$.

INDUCTIVE ARITHMETIC. NICOMACHUS. — As in the case of astronomy, progress in geometry now lags and finally ceases altogether. About 100 A.D. a final era of Greek mathematical science, predominantly arithmetical in character, begins with Nicomachus of Judea, whose work remained the basis of European arithmetic until the introduction of the Arabic arithmetic a thousand years later. He enunciates curious theorems about squares and cubes, for example: — In the series of odd numbers from 1, the first term is the first cube, the sum of the next two is the second, of the next three the third, etc., — doubtless simple observation and induction. He refers to proportion as very necessary to "natural science, music, spherical trigonometry and planimetry," and discusses various cases in great detail.

> Mathematics had passed from the study of the philosopher to the lecture-room of the undergraduate. We have no more the grave and orderly proposition, with its deductive proof. Nicomachus writes a continuous narrative, with some attempt at rhetoric, with many interspersed allusions to philosophy and history. But more important than any other change is this, that the arithmetic of Nicomachus is *inductive,* not deductive. It retains from the old geometrical style only its nomenclature. Its sole business is classification, and all its classes are derived from, and are exhibited in, actual numbers. But since arithmetical inductions are necessarily incomplete, a general proposition, though *prima facie* true, cannot be strictly proved save by means of an universal symbolism. Now though geometry was competent to provide this to a certain extent, yet it was useless for precisely those propositions in which Nicomachus takes most interest. The Euclidean symbolism would not show, for instance, that all the powers of 5 end in 5 or that the square numbers are the sums of the series of odd numbers. What was wanted, was a symbolism similar to the ordinary numerical kind, and thus inductive arithmetic led the way to algebra. — *Gow.*

PTOLEMY AND THE PTOLEMAIC SYSTEM. — With Claudius Ptolemy, in the second century of our era, Greek astronomy reaches its definitive formulation. In the 260 years which had elapsed since Hipparchus no progress of consequence had been made.

Of Hipparchus, from whom he inherited so much, Ptolemy writes: —

> It was, I believe, for these reasons and especially because he had not received from his predecessors as many accurate observations as he has left to us, that Hipparchus, who loved truth above everything, only investigated the hypotheses of the sun and moon, proving that it was possible to account perfectly for their revolutions by combinations of circular and uniform motions, while for the five planets, at least in the writings which he has left, he has not even commenced the theory, and has contented himself with collecting systematically the observations, and showing that they did not agree with the hypotheses of the mathematicians of his time. He explained in fact not only that each planet has two kinds of inequalities but also

that the retrogradations of each are variable in extent, while the other mathematicians had only demonstrated geometrically a single inequality and a single arc of retrograde motion; and he believed that these phenomena could not be represented by excentric circles nor by epicycles carried on concentric circles, but that, it would be necessary to combine the two hypotheses. — *Dreyer.*

The instruments used by Ptolemy for his astronomical observations included: — the "Ptolemaic rule," consisting of a rod with sights pivoted to a vertical rod, the angle at the junction being measured by the subtended chord; the armillary circle, a copper or bronze ring marked in degrees and mounted in the meridian plane on a post. A second movable ring is fitted into this with pegs diametrically opposite each other, by means of which the sun's midday height could be measured; the armillary sphere, similar in principle but somewhat more complicated; the astrolabe or astronomical ring for measuring either horizontal or vertical angles. Like the Chaldeans Ptolemy also used meridian quadrants of masonry. Time was still measured by the flow of water, with apparatus considerably improved by Ctesibus and Hero. The numerous observations of Ptolemy were made during the period 125–151 A.D. and he was in Alexandria in 139.

One of his observations he describes as follows:

In the 2d year of Antoninus, the 9th day of Pharmonthe, the sun being near setting, the last division of Taurus being on the meridian (that is, 5½ equinoctial hours after noon), the moon was in 3 degrees of Pisces, by her distance from the sun (which was 92 degrees, 8 minutes); and half an hour after, the sun being set, and the quarter of Gemini on the meridian, Regulus appeared, by the other circle of the astrolabe, 57½ degrees more forwards than the moon in longitude. — *Whewell.*

THE ALMAGEST. — In his celebrated *Syntaxis*, better known from Arabic translations as the Almagest, Ptolemy undertakes to present for the first time the whole astronomical science of his age.

In Book I he reviews the fundamental astronomical data thus: —

The earth is a sphere, situated in the centre of the heavens; if it were not, one side of the heavens would appear nearer to us than the other, and the stars would be larger there; if it were on the celestial axis but nearer to one pole, the horizon would not bisect the equator but one of its parallel circles; if the earth were outside the axis, the ecliptic would be divided unequally by the horizon. The earth is but as a point in comparison to the heavens, because the stars appear of the same magnitude and at the same distances *inter se,* no matter where the observer goes on the earth. It has no motion of translation, first, because there must be some fixed point to which the motions of the others may be referred, secondly, because heavy bodies descend to the centre of the heavens which is the centre of the earth. And if there was a motion, it would be proportionate to the great mass of the earth and would leave behind animals and objects thrown into the air. This also disproves the suggestion made by some, that the earth, while immovable in space, turns round its own axis, which Ptolemy acknowledges would simplify matters very much.[1]

Chapter IX explains the calculation of a table of chords. Starting with the chords of 60° and 72°, already known as sides of regular polygons, he devises ingenious geometrical methods for finding chords of differences and of half-angles. Thus he computes the chords for 12°, 6°, 3°, $1\frac{1}{2}°$, and $\frac{3}{4}°$. Hipparchus had already computed such a table, but Ptolemy completes it by showing that

$$\tfrac{2}{3}\text{ chord }1\tfrac{1}{2}° < \text{chord }1° < \tfrac{4}{3}\text{ chord }\tfrac{3}{4}°$$

and thence deriving close approximations for the chords of 1° and $\frac{1}{2}°$ and constructing a table for each half-degree up to 180°. His results are expressed in sexagesimal fractions of the radius (of which they are thus numerically independent) and are equivalent in accuracy to five decimals in our notation. He also employs our present method of interpolation skilfully. This chapter is the culmination of Greek trigonometry, which owed its further development to Indian and Arabic mathematicians.

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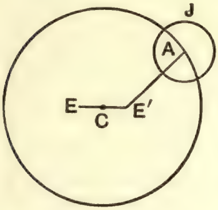
[1] "For Ptolemy more geometer and astronomer than philosopher, the astronomer who seeks hypotheses adapted to save the apparent movements of the stars knows no other guide than the rule of greatest simplicity: It is necessary as far as possible to apply the simplest hypotheses to the celestial movements, but if they do not suffice, it is necessary to take others which fit better." — *Duhem.*

In Books III, IV, and V, Ptolemy discusses the apparent motions and distances of the sun and moon by means of excentrics and epicycles, his method for determining the moon's distance being substantially the same as the modern. Book V describes the construction and use of his chief instrument, the astrolabe. Book VI deals with eclipses, using a value of π equivalent to our 3.1416. He determines the distance of the sun, following Hipparchus, by observing the breadth of the earth's shadow when the moon crosses it at an eclipse. Books VII and VIII contain a catalogue of 1028 stars based on that of Hipparchus, and a discussion of precession of the equinoxes, with a close determination of the unequal intervals between successive vernal and autumnal equinoxes. The remainder of the treatise is devoted to the planets, containing Ptolemy's chief original contributions.

While Ptolemy did not take advantage of the better data at his command to improve the theory of the sun's motion, he did make substantial progress with that of the moon, the discrepancies for which rarely exceed 10′, which represented about the maximum precision of his instruments. Hipparchus had assumed the moon to have a motion representable by one circle with the earth as a centre and by an epicycle with its centre upon this. Discrepancies between observed and computed positions led Ptolemy, bound as he was by the Aristotelian dictum that celestial bodies can move only in circular paths, to modify this by making the first circle excentric to the earth, the line joining the centres of the circle and the earth being itself assumed to revolve. This theory, while giving results of sufficient accuracy for the observations at certain positions of the moon, exaggerated considerably the variation of its distance from the earth, making this at times almost twice as great as at others.

For the five planets, or "wandering stars," he also assumed excentric deferents, and as a further means of accounting for discrepancies, an additional point, in line with the centres of earth and deferent, called the "equant," with respect to which the centre of the epicycle would have uniform angular velocity. The planes of the epicycles were slightly inclined to that of the ecliptic.

Thus in the figure, C is the centre of the circular deferent, E the earth and E' the equant. The center A of the epicycle travels at such a rate that the line $E'A$ has uniform angular velocity. The planet J travels in an epicycle about A. These assumptions afforded the needful freedom for a fairly close approximation to observed planetary motions, the mathematical computations involved becoming naturally quite elaborate. Ptolemy disclaimed the power of determining the distances or even the order of the planets.



That the system as a whole deserves our admiration as a ready means of constructing tables of the movements of sun, moon, and planets, cannot be denied. Nearly in every detail (except the variation of distance of the moon) it represented geometrically these movements almost as closely as the simple instruments then in use enabled observers to follow them, and it is a lasting monument to the great mathematical minds by whom it was gradually developed.

To the modern mind, accustomed to the heliocentric idea, it is difficult to understand why it did not occur to a mathematician like Ptolemy to deprive all the outer planets of their epicycles, which were nothing but reproductions of the earth's annual orbit transferred to each of these planets, and also to deprive Mercury and Venus of their deferents, and place the centres of their epicycles in the sun, as Heraclides had done. . . . The system of Ptolemy was a mere geometrical representation of celestial motions, and did not profess to give a correct picture of the actual system of the world. . . . For more than 1400 years it remained the Alpha and Omega of theoretical astronomy, and whatever views were held as to the constitution of the world, Ptolemy's system was almost universally accepted as the foundation of astronomical science. — *Dreyer.*

After Ptolemy we have no record of any important advance in astronomy for nearly 1000 years.

In reviewing Greek astronomy Berry says,

The Greeks inherited from their predecessors a number of observations, many of them executed with considerable accuracy, which were nearly sufficient for the requirements of practical life, but in the matter of astronomical theory and speculation, in which their best thinkers were very much more interested than in the detailed facts, they received virtually a blank sheet on which they had to write (at first with indifferent success) their speculative ideas. A considerable interval of time was obviously necessary to bridge over the gulf separating such data as the eclipse observations of the Chaldeans from such ideas as the harmonical spheres of Pythagoras; and the necessary theoretical structure could not be erected without the use of mathematical methods which had gradually to be invented. That the Greeks, particularly in early times, paid little attention to making observations, is true enough, but it may fairly be doubted whether the collection of fresh material for observations would really have carried astronomy much beyond the point reached by the Chaldean observers. When once speculative ideas, made definite by the aid of geometry, had been sufficiently developed to be capable of comparison with observation, rapid progress was made. The Greek astronomers of the scientific period, such as Aristarchus, Eratosthenes, and above all Hipparchus, appear moreover to have followed in their researches the method which has always been fruitful in physical science — namely, to frame provisional hypotheses, to deduce their mathematical consequences, and to compare these with the results of observation. There are few better illustrations of genuine scientific caution than the way in which Hipparchus, having tested the planetary theories handed down to him and having discovered their insufficiency, deliberately abstained from building up a new theory on data which he knew to be insufficient, and patiently collected fresh material, never to be used by himself, that some future astronomer might thereby be able to arrive at an improved theory.

Of positive additions to our astronomical knowledge made by the Greeks the most striking in some ways is the discovery of the approximately spherical form of the earth, a result which later work has only slightly modified. But their explanation of the chief motions of the solar system and their resolution of them into a comparatively small number of simpler motions was, in reality, a far more important contribution, though the Greek epicyclic scheme has been so remodelled, that at first sight it is difficult to recognize the relation between it and our modern views. The subsequent history will, however, show how completely each stage in the progress of astronomical science has depended on those that preceded.

When we study the great conflict in the time of Copernicus between the ancient and modern ideas, our sympathies naturally go out towards those who supported the latter, which are now known to be more accurate, and we are apt to forget that those who then spoke in the name of the ancient astronomy and quoted Ptolemy were indeed believers in the doctrines which they had derived from the Greeks, but that their methods of thought, their frequent refusal to face facts, and their appeals to authority, were all entirely foreign to the spirit of the great men whose disciples they believed themselves to be.

Other Works of Ptolemy. — In spite of his scientific attainments Ptolemy did not disdain to write an elaborate treatise on astrology. In a lost work on geometry, Ptolemy made the first known of the interminable series of attempts to give a formal proof of Euclid's parallel postulate, an attempt naturally foredoomed to failure.

In a great treatise on geography, hardly less important than the Almagest, Ptolemy gave a description of the known earth, locating not less than 5000 places by latitude and longitude. He even gave in addition to position the maximum length of day for 39 points in India, a land probably better known at this period than in the time of Mercator, near the end of the sixteenth century. Ptolemy reckoned longitude from the "Fortunate Isles," — the western boundary of the known world. Various methods of projection were discussed in connection with directions for map drawing.

Ptolemy also wrote on sound and on optics, dealing particularly in the latter with refraction, with what has been called "the oldest extant example of a collection of experimental measures in any other subject than astronomy." He discovered by careful experiment and induction the law that light-rays passing from a rarer to a denser medium are bent towards the perpendicular, and invented a simple apparatus for measuring angles of incidence and reflection.

Pappus. — The last two of the great Greek mathematicians were Pappus and Diophantus, who lived in Alexandria about 300 A.D.

The most important work of Pappus is his Collections, in eight books, of which all but the first and a part of the second are preserved. In this he comments fully on the most important Greek mathematical works known to him, making his treatise of the highest historical value, particularly in its careful summaries of books which have been lost. Book I and most of Book II are missing, the third reviews the various solutions of the duplication of the cube, adding Pappus' own, and discusses the regular inscribed polyhedrons; the fourth deals with several less simple geometrical matters, including the higher curves, spirals, conchoid, quadratrix, etc., the problem of describing a circle tangent to three given circles which touch each other; the fifth is also geometrical. In Book VI Pappus gives the mathematical basis for the Ptolemaic astronomy, — *i.e.* trigonometry and optics. Book VII contains his well-known theorems, sometimes mistakenly attributed to Gulden, that the volume of a solid of revolution is equal to the product of the area of the revolving figure and the length of the path of its centre of gravity, and that the surface generated is equal to the product of the perimeter and the length of the circular path described by its centre of gravity. In this final book he undertakes to deal with certain mechanical problems "more clearly and truly" than his predecessors have done. These include, for example, centre of gravity, inclined planes, the moving of a given weight by a given power with the help of cog-wheels, the determination of the diameter of a broken cylinder. The whole is somewhat weak on the arithmetical side.

With the political decline of Greece and the awakening to intellectual activity of great Semitic and Egyptian populations, mathematical science changed radically from the traditional deductive geometry, to an arithmetical and algebraic science in harmony with the aptitudes which have characterized these races. Thus Nicomachus as we have seen was of Jewish antecedents, Hero an Egyptian in his point of view and his scientific tendencies.

BEGINNINGS OF ALGEBRA. DIOPHANTUS. — Diophantus was active in Alexandria in the first half of the fourth century A.D., though we know so little about him that even his precise name is doubtful. His chief work is his Arithmetic, which is extant however only in somewhat mutilated form. It is the first known treatise on algebra, and is devoted to the solution of equations, employing algebraic symbols and analytical methods. Euclid had given the geometrical equivalent of the solution of a quadratic equation, and Hero could solve the same problem algebraically but lacked a satisfactory symbolism. The algebra of Diophantus was therefore not a sudden invention, but the result of gradual evolution during several centuries of increasing interest in arithmetical problems, and declining vogue of the abstract Euclidean geometry.

Writers on the history of algebra distinguish three classes or methods of algebraic expression: —

(*a*) the *rhetorical*, where no symbols are used, but every term and operation is described in full. This was the only method known before Diophantus, and was later in vogue in western Europe until the fifteenth century;

(*b*) the *syncopated*, which replaces common words and operations by abbreviations, but conforms to the ordinary rules of syntax. This was the style of Diophantus;

(*c*) the *symbolical* or modern, using symbols only, without words.

The syncopated method may be illustrated by the following passage from Heath's Diophantus: —

> Let it be proposed then to divide 16 into two squares. And let the first be supposed to be $1S$; therefore the second will be $16U - 1S$. Thus $16U - 1S$ must be equal to a square. I form the square from any number of N's minus as many U's as there are in the side of 16 U's. Suppose this to be $2N - 4U$. Thus the square itself will be $4S\ 16U - 16N$ etc.

In his Arithmetic, which is really a treatise on algebra, Diophantus represents the (single) unknown by the Greek *sigma* — all the other letters of the Greek alphabet standing for definite numbers — with successive powers to the sixth inclusive. If he requires two unknowns he admits only one at a time. His originality and power in the solution of problems are amply shown, though the solutions are rarely complete. For quadratic equations, for example, he gives but one root, even when both are positive. Negative numbers are for him unreal, and he also avoids the irrational. He admits fractional results however, and is indeed the first Greek for whom a fraction is a number rather than a mere ratio of two numbers. For the solution of pure equations his rule is: "If a problem leads to an equation containing the same powers of the unknown on both sides but not with the same coefficients, you must deduct like from like till only two equal terms remain. But when on one side or both some terms are negative, you must add the negative terms to both sides till all the terms are positive and then deduct as before stated."

His method for general quadratics is not given. He solves one cubic equation, also particular cases of the indeterminate equation $Ax^2 + Bx + C = Y^2$. The modern so-called Diophantine equations involving the solution in integers of one or more indeterminate equations, do not occur in his own extant work.

> In 130 indeterminate equations, which Diophantus treats, there are more than 50 different classes. . . . It is therefore difficult for a modern, after studying 100 Diophantic equations, to solve the 101st; and if we have made the attempt, and after some vain endeavours read Diophantus' own solution, we shall be astonished to see how suddenly he leaves the broad high-road, dashes into a side-path and with a quick turn reaches the goal, often enough a goal with reaching which we should not be content; we expected to have to climb a toilsome path, but to be rewarded at the end by an extensive view; instead of which, our guide leads by narrow, strange, but smooth ways to a small eminence; he has finished! He lacks the calm and concentrated energy for a deep plunge into a single important problem; and in this way the reader also hurries with inward unrest from problem to problem, as in a game of riddles, without being able to enjoy the individual one. Diophantus dazzles more than he delights. He is in a wonderful measure shrewd, clever, quick-sighted, indefatigable, but does not penetrate thoroughly or deeply into the root of the matter. As his problems seem framed in obedience to no obvious scientific necessity, but often only for the sake of the solution, the solution itself also lacks completeness and deeper signification. He is a brilliant performer in the *art* of indeterminate analysis invented by him, but the *science* has

nevertheless been indebted, at least directly, to this brilliant genius for few methods, because he was deficient in the speculative thought which sees in the True more than the Correct. That is the general impression which I have derived from a thorough and repeated study of Diophantus' arithmetic. — *Hankel.*

On the other hand Euler remarks: —

> Diophantus himself, it is true, gives only the most special solutions of all the questions which he treats, and he is generally content with indicating numbers which furnish one single solution. But it must not be supposed that his method was restricted to these very special solutions. In his time the use of letters to denote undetermined numbers was not yet established, and consequently the more general solutions which we are now enabled to give by means of such notation could not be expected from him. Nevertheless, the actual methods which he uses for solving any of his problems are as general as those which are in use to-day; nay, we are obliged to admit that there is hardly any method yet invented in this kind of analysis of which there are not sufficiently distinct traces to be discovered in Diophantus.

With the very important process of reducing problems to equations he is relatively successful and often highly ingenious. For example, "to find three numbers, so that the product of any two plus the sum of the same two shall be given numbers, for example, 8, 15, and 24." We should write: $xy + x + y = 8$; $yz + y + z = 15$; $zx + z + x = 24$.

Hence, by subtraction, $x(z - y) + z - y = 16$,

$$x + 1 = \frac{16}{z - y},\ z(x - y) + x - y = 9, z + 1 = \frac{9}{x - y}, \text{etc.}$$

He, on the other hand, takes $a - 1$ for one of the numbers and readily obtains $\frac{9}{a} - 1$ and $\frac{16}{a} - 1$ for the others, and $a = \frac{12}{5}$.

He employs tentative assumptions with great effect. For example, "To find a cube and its root such that if the same number be added to each, the sums shall also be a cube and its root." If

$2x$ is the original number and x the number added, (an arbitrary and presumably erroneous assumption), $8x^3 + x = 27x^3$, giving $19x^2 = 1$. The coefficient 19 not being a square, he now seeks to find two cubes whose difference is a square. If $(x + 1)^3 - x^3$ is equated to $(2x - 1)^2$ the special solution $x = 7$ is easily obtained. Returning to the original problem, the new assumption is made: — let x = number to be added, $7x$ = original number.

$$(7x)^3 + x = (8x)^3 \text{ whence } x = \tfrac{1}{13}$$

In another type to find a square between 10 and 11, he multiplies both by successive squares of integers until between the products (by 16) he finds a square, 169. The number required is $10\tfrac{9}{16}$. Such processes naturally give particular, not general, solutions.

His lost Porisms are believed to have "contained propositions in the theory of numbers most wonderful for the time." Summarizing his methods of dealing with equations we may say that: —

(1) he solves completely equations of the first degree having positive roots, showing remarkable skill in reducing simultaneous equations to a single equation in one unknown;

(2) he has a general method for equations of the second degree but employs it only to find one positive root;

(3) more remarkable than his actual solutions of equations are his ingenious methods of avoiding equations which he cannot solve.

How far his work was original, how far like Euclid in his Elements it was the result of compilation, cannot be definitely ascertained. As a whole it is somewhat uneven and makes rather the impression of great learning than of exceptional originality. He seems indebted in part to predecessors unknown to us. For him the earlier Greek distinction between computation and arithmetic has lost its force.

In reviewing the work of Pappus and Diophantus Gow says: —

> the Collections of Pappus can hardly be deemed really important. . . . But among his contemporaries, Pappus is like the peak of Teneriffe in the Atlantic. He looks back from a distance of 500 years, to find

his peer in Apollonius. . . . His work is only the last convulsive effort of Greek geometry, which was now nearly dead, and was never effectually revived. . . . It is not so with Ptolemy or Diophantus. The trigonometry of the former is the foundation of a new study which was handed on to other nations, indeed, but which has thenceforth a continuous history of progress. Diophantus also represents the outbreak of a movement which probably was not Greek in its origin, and which the Greek genius long resisted, but which was especially adapted to the tastes of the people who, after the extinction of Greek schools, received their heritage and kept their memory green. But no Indian or Arab ever studied Pappus or cared in the least for his style or his matter. When geometry came once more up to his level, the invention of analytical methods gave it a sudden push which sent it far beyond him and he was out of date at the very moment when he seemed to be taking a new lease of life.

A melancholy interest attaches to the fate of Hypatia, daughter of Theon an Alexandrian mathematician, herself a teacher of Greek philosophy and mathematics, who was torn to pieces by a Christian mob, doubtless as a representative of pagan (Greek) learning, at Alexandria in 415 A.D.

CONCLUSION AND RETROSPECT. — Intellectual interests in the Greek world (now really Roman) were by this time so completely alienated from mathematics, and indeed from science in general, that the brilliant work of Pappus and Diophantus aroused but slight and temporary interest. Geometry had reached within the possible range of the Euclidean method a relatively complete development. Algebra under Diophantus attained in spite of hampering notation a level not again approached for many centuries.

Little need be said of sciences other than those already dealt with. These, even more than mathematics and astronomy, shrank under Roman autocracy and Christian hostility. Only the works of Galen, Strabo, and Pliny need be mentioned, and with them we deal in the next chapter.

The torch of science now passes from the Greeks to the Indians of the far East after their conquest by Alexander, to be in turn surrendered to the Mohammedan conquerors of Alexandria A.D. 641. By them it is kept from extinction until in later ages it is once more fanned to ever increasing radiance in western Europe.

In attempting a retrospective estimate of Greek science it is fundamentally important to judge the whole background fairly. In science the Greeks had to build from the foundations. Other peoples had extensive knowledge and highly developed arts. Only among the Greeks existed the true scientific method with its characteristics of free inquiry, rational interpretation, verification or rectification by systematic and repeated observation, and controlled deduction from accepted principles.

> The Assyrians, Babylonians and Egyptians had certainly made great progress in the use of mechanical devices for moving heavy loads, in the construction of scales, and of pumps. Their measuring instruments were well developed, and acute observations were made, but of systematic, scientific investigation there is no evidence. The Greeks received many results and suggestions from Asia Minor, Mesopotamia, and Egypt, but their achievements are essentially their own.
> — *Wiedemann.*

In asking ourselves why these extraordinary beginnings seemed after a time to lose their power of continued development, we must not forget the effect of external conditions. It is conceivable indeed that scientific progress should continue from age to age, through the genius of individual teachers and students, regardless of political and social conditions. Such, however, is not the historic fact. For progress in science men of genius are indispensable, but in no country or age have they alone been able to make science flourish under conditions so unfavorable as were those of the early centuries of the Christian era.

Greek science, however, did not "fail," learned and elaborate as are the explanations that have been given of its alleged failure. Under "the chill breath of Roman autocracy" its growth was indeed checked, its animation suspended, for a full thousand years. Then in the Renaissance it renewed its vitality and has ever since been advancing more and more magnificently. This is not to say that criticisms as to the imperfections of the Greek scientific method are invalid, but rather to assert, as most critics must agree, that its merits outweighed its defects, and that the latter would not have proved disastrous but for the development of political, economic and military conditions under which the free Greek spirit could not continue its wonderful achievements.

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CHAPTER VII

THE ROMAN WORLD. THE DARK AGES

Among them [the Greeks] Geometry was held in highest honor: nothing was more glorious than Mathematics. But we have limited the usefulness of this art to measuring and calculating. — *Cicero.*

The Romans were as arbitrary and loose in their ideas as the Greeks, without possessing their invention, acuteness and spirit of system. — *Whewell.*

The Romans, with their limited peasant horizon and their short-sighted practical simplicity, cherished always for true science in their inmost hearts that peculiar mixture of suspicion and contempt which is so familiar today among the half educated. The arch dilettante Cicero boasts, even, that his countrymen, thank God! are not like those Greeks, but confine the study of mathematics and that sort of thing to the practically useful. — *Heiberg.*

The Roman World-Empire. — For several centuries, during the decline of Greek learning both in Greece itself and in Alexandria, two new and powerful States were developing; one having its centre at Carthage on the northern shore of Africa, almost opposite Sicily, the other — the Roman Empire — on the western shore of Italy in the valley of the Tiber. The latter, at first comparatively insignificant, rapidly rose to a position of world-wide power, conquering in turn Carthage, Greece, and the East and eventually extending over the greater part of the then known world, from Britain on the north to the Cataracts of the Nile on the south, from India in the east to the Pillars of Hercules in the west.

The Roman Attitude towards Science. — One of the most striking facts in the history of science is the total lack of any evidence of real interest in science or in scientific research among the Roman people itself or any people under Roman sway. Alexandrian science, even, though previously flourishing, languished and went steadily to its fall after the submission of that city to the Romans in the first century B.C. The truth seems to be that the Roman people, while highly gifted in oratory, literature, and history (as witness, for example, the works of Cicero, Virgil and Tacitus), were not interested and therefore not successful in scientific work. This is the more impressive when we reflect upon their marvellous military genius, and their preëminence in world-wide power, dominion and influence. In vain do we look for any Roman scientist or philosopher of such originality or range as Aristotle or Plato; for any Roman astronomer, like Aristarchus or Hipparchus or Ptolemy; for any Roman mathematician or inventor, like Archimedes; for any Roman natural philosopher, like Democritus; for any Roman pioneer in medicine, like Hippocrates, — for Galen was Roman neither by birth nor education, but only by adoption late in life.

Roman Engineering and Architecture. — There is however one marked feature of Roman civilization in which extraordinary ability was displayed and peculiar excellence achieved and in which the Romans were unquestionably far superior to all their predecessors and, until very recent times, to all their successors. This feature, which is one of the most characteristic, is the Roman genius for both military and civil engineering. It is only necessary to mention the surviving remains of Roman walls, fortresses, roads, aqueducts, theatres, baths, and bridges. Never before and never since has any empire built so many, so splendid, and so enduring monuments for the service of its peoples in peace and in war. The surface of southern Europe, western Asia and northern Africa is still covered after the lapse of twenty centuries with Roman remains which bid fair to resist decay and destruction for another two thousand years. Roman engineering is almost as distinguished as is Roman law. The Emperor Constantine in the fourth century wrote: "We need as many engineers as possible. As there is lack of them, invite to this study persons of about 18 years, who have already studied the necessary sciences. Relieve the parents of taxes and grant the scholars sufficient means." The land surveyors formed a well-organized gild, but they were merely practitioners of a traditional art, perpetuating the errors of their ancient Egyptian predecessors, not dreaming of new discoveries, nor even of imparting such knowledge as they had, — outside the ranks of their own gild.

SLAVE LABOR IN ANTIQUITY. — It must never be forgotten that throughout antiquity, and to a great extent even until very recent times, the labor question was wholly different from what it is to-day. Instead of the labor-saving machinery which is so extraordinary a feature of our time, but which was practically non-existent before the end of the eighteenth century, the slave was the machine for all heavy labor. It is not likely that he was ever a particularly cheap machine, but in the mass he was powerful, and it was probably largely by his labor that the fields were cultivated and irrigated, and that dams and ditches, walls and towers, roads and bridges and pyramids and temples, were built and fortified. It is notorious that the so-called "ships" of war, the galleys, were manned by slaves, even down to modern times. It is difficult to determine the efficiency of labor of this kind because we are generally ignorant as to the time factor, but whether from our modern point of view inefficient or not, the results were often remarkable and sometimes, as in the case of the Pyramids, stupendous.

JULIUS CÆSAR AND THE JULIAN CALENDAR. — Julius Cæsar himself undertook two great problems of practical mathematical science: — the rectification of the highly confused calendar, and a survey of the whole Roman empire. In the year 47 B.C. the accumulated calendar error amounted to not less than 85 days. Reform was accomplished by a decree making the year consist of 365 days with an additional day in February once in four years. The survey, of which the results were to be shown in a great fresco map, was not carried out until the reign of Augustus.

VITRUVIUS ON ARCHITECTURE. — The most famous ancient work on building and kindred topics, including building materials, is that entitled *De Architectura*, by Vitruvius, a Roman architect and engineer living (about 14 B.C.) in the age of Augustus. This celebrated work was the only one of importance on architecture known to the Middle Ages, and was the guide and text-book of the builders of that period as well as of those of the Renaissance. The book (now easily accessible in translation) is in part a compilation from earlier, and especially Greek, authors, and in part original. Vitruvius uses for π the value $3\frac{1}{8}$, — less accurate than that of Archimedes, but displaced later by the crude approximation 3. Of Vitruvius's life and work almost nothing is known, but no other ancient treatise of a similar technical nature has had in its own field so much influence on posterity.

FRONTINUS ON THE WATERWORKS OF ROME (c. 40–103 A.D.). At about the end of the first century of our era, Sextus Julius Frontinus, a Roman soldier and engineer, wrote a highly interesting and valuable account of the waterworks of Rome. Frontinus served as *prætor* under Vespasian; was afterwards sent to Britain as Roman governor of that island; was superseded by Agricola in 78 A.D. and was appointed in 97 A.D. *Curator Aquarum,* "an office never conferred except upon persons of very high standing."

ROMAN NATURAL SCIENCE AND MEDICINE. — Among the Roman workers and authors of importance in the history of natural science and medicine only a few require more than passing notice. This is the more remarkable when we reflect upon the vast extension of the Roman empire and the novel and hitherto unequalled opportunities afforded for observation and collection in natural history, and for the study of anthropology, geography, geology, meteorology, climatology, zoölogy, botany and the like, — not to mention military surgery, and the hygiene and sanitation of camp-life.

LUCRETIUS (98–55 B.C.) is to-day regarded not only as a great Roman poet but also as the most perfect exponent in his time of the natural philosophy of the Greeks who preceded him. He was a contemporary and a few years the junior of Cicero and Julius Cæsar. The first two books and the fifth of his *De Rerum Natura* (On the Nature of Things) are of interest to the modern scientific student, because of their dealing with problems of permanent importance to mankind. He was a disciple of Epicurus, and apparently also well acquainted with the works of Empedocles, Democritus, Anaxagoras, and many other of the great Greek writers such as Homer, Hippocrates, Thucydides, and especially Euripides. The title of his famous poem shows his interest in natural philosophy, and there is evidence that he was also a teacher and reformer. He is antagonistic to superstition and a strong advocate of rationalism, but he is neither irreverent nor revolutionary.

The following passages are typical: —

Water in summer time flows cool in wells,
Because the Earth then rarefied by heat,
Its proper stores most radiate to the air.
Hence more the Earth is drainéd of its heat,
And colder grow the currents under ground.
But when by cold in winter 'tis compressed,
Its heat escaping passes into wells. . . .

And now to tell by which of Nature's laws,
The stone called Magnet by the Greeks, — since first
'Mong the Magnesians found, — can iron draw.
Men gaze with wonder on the marvellous stone,
With pendent chain of rings, oft five or more,
Light hanging in the air suspensive, while
One from another feels the influence of the stone
That sends through all its wonder-working power.
Here many principles we must first lay down
And slow approach by long preparative,
Rightly to solve the rare phenomenon.
The more exact I then attentive ears. . . .

How different is fire from piercing frost!
Yet both composed of atoms toothed and sharp,
As proved by touch. Touch, O ye sacred powers —
Touch is the organ whence all knowledge flows;
Touch is the body's sense of things extern,
And of sensations that deep spring within;
Whether delightsome, as in genial act,
Or rude collision torturing from without;
How different, then, must forms of atoms be
Which such sensation varied can produce!

STRABO, — a Roman traveller, historian and geographer, lived somewhere between 63 B.C. and 24 A.D. His Geography is the most important work on that subject surviving from antiquity and, while apparently building on the foundation laid by Eratosthenes, is plainly an original work devoted largely to his own explorations and observations during years of travel and study in different countries, including Italy, Greece, Asia Minor, Egypt, and Ethiopia. He himself says: —

> Westward I have journeyed to the parts of Etruria opposite Sardinia; towards the South from the Euxine to the borders of Ethiopia, and perhaps not one of those who have written geographies has visited more places than I have between those limits.

His work is invaluable as a picture of the limited geographical knowledge of the time, but he had no such mathematical knowledge of geography as had his great predecessors, Eratosthenes, Hipparchus, and Ptolemy.

PLINY THE ELDER (23–79 A.D.), sometimes called Pliny the Naturalist, is another Roman of scientific attainments, whose great work entitled Natural History, although more an encyclopædia of miscellaneous information than a scientific treatise, is, nevertheless, like the works of Herodotus, a landmark in the history of civilization. It consists of thirty-seven books and is easily accessible in English. Pliny deals with the universe, God, nature, and natural phenomena; with earth, stars, earthquakes; with man, beasts, shells, fishes, insects, trees, fruits, gums, perfumes, timber, the diseases of plants, metals, stones, precious stones, etc. The author met his death in that eruption of Vesuvius which overwhelmed Pompeii in 79 A.D. and because of his scientific curiosity which led him to approach too near to the volcano.

GALEN (CLAUDIUS GALENUS) who flourished in the second century A.D. was born and partly educated at Pergamum in Asia Minor, where, after much travelling, and research, chiefly in anatomy and philosophy at Smyrna and at Alexandria, he also practised the healing art. Sent for by the Roman emperor, Lucius Verus, he was afterward physician to Marcus Aurelius and his son Commodus. His writings are voluminous, encyclopedic and anatomically important, though not especially original, and his name is often linked with that of Hippocrates, partly, no doubt, because after Galen we find no great name in anatomy until we come to Vesalius, some 1400 years later.

Late Roman Mathematical Science. — Two periods may be distinguished in ancient mathematical science, the first beginning with Pythagoras and ending with Hero. To these four to five centuries belong all the original works in geometry, astronomy, mechanics, and music. The period closes with the extension of the *pax Romana* over the Orient. The second extends to the sixth century, when Hellenism is proscribed by the new religion, the genius of invention is extinct, and men merely study the older works, commenting and coördinating. Astronomy gradually reverts to astrology, the mathematical geography well begun under Eratosthenes and Ptolemy becomes superficial and descriptive, with Strabo and even with Posidonius.

Whatever the eminence of the Romans in the practical arts of war, politics and engineering, their interest in abstract science was almost nil. On the other hand, commercial arithmetic, which had been studiously neglected by Greek mathematicians, now had the place of honor. The Roman numerals, clumsy as they seem to us, were superior to the Greek, and a useful system of finger-reckoning was developed, supplementing the skilful use of the abacus. If no abacus was at hand, the corresponding lines were quickly traced on sand or dust, small stones or *calculi* — whence our words calculation and calculus, — serving as counters. A complete Roman abacus — of which no example has come down to us — seems to have had eight long and eight short grooves. Of the former, one held five counters or buttons, each of the others four, each of the short grooves one, these last counting as five units each. The grooves with six counters served for computations with fractions. Geometry — but of Hero rather than of Euclid — was valued for its utility in surveying and architecture. Preparation for the engineering art included mathematics, optics, astronomy, history, and law. There were also teachers of mechanics and architecture. (See Vitruvius, above.)

Capella. — Early in the fifth century Martianus Capella wrote a compendium of grammar, dialectics, rhetoric, geometry, arithmetic, music, and astronomy, of great and lasting educational influence. His classification of these "seven liberal arts" maintained itself throughout the Middle Ages and is not yet wholly extinct. Gregory of Tours for example says: — "If thou wilt be a priest of God, then let our Martianus instruct thee first in the seven sciences."

Boethius (480–524) born at Rome on the eve of its fall in 476 is the author not only of the famous Consolations of Philosophy but also of works on Music and on Arithmetic which long served to represent Greek mathematics to the medieval world. In the course of his public-spirited career, Boethius interested himself in the reform of the coinage and in the introduction of water-clocks and sun-dials. His geometry consists merely of some of the simpler propositions of Euclid, with proofs of the first three only, and with applications to mensuration. Yet the intellectual poverty of the age was such that this remained long the standard for mathematical teaching. Boethius' Arithmetic begins: —

> By all men of old reputation who following Pythagoras' reputation have distinguished themselves by pure intellect it has always been considered settled that no one can reach the highest perfection of philosophical doctrines, who does not seek the height of learning at a certain crossway — the quadrivium.

For him the things of the world are either discrete (multitudes), or continuous (magnitudes). Multitudes are represented by numbers, or in their ratios by music; magnitudes at rest are treated by geometry, those in motion by astronomy. These four of the seven liberal arts form the *quadrivium;* grammar, dialectics and rhetoric, the *trivium.* A Christian in faith, a pagan in culture, Boethius has been called the "bridge from antiquity to modern times." (See page 50.)

The scholars of the time were almost without exception men whose first interests were theological. Mathematics, having no direct moral significance, seemed to them in itself unworthy of attention. On the other hand, they attached exaggerated importance to all sorts of mystical attributes of numbers and to the interpretation of scriptural numbers. Thus Augustine says the science of numbers is not created by men, but merely discovered, residing in the nature of things.

> Whether numbers are regarded by themselves or their laws applied to figures, lines or other motions, they have always fixed rules, which have not been made by men at all, but only recognized by the keenness of shrewd people.

Science and the Early Christian Church. — In the earlier centuries of our era the history of science gradually enters upon a new phase. The more highly developed civilization of Greece and Rome, weakened by corruption, has finally yielded to the attacks on the one hand of barbarous or semicivilized races, — Goths, Vandals, Huns, and Arabs, — and on the other hand to a moral revolution of humble Jewish origin. These changes were adverse to the development, or even the survival, of Greek science. The destructive relation of the northern barbarians to scientific progress may be easily imagined. The policy of official Christianity was based on antecedent antipathy for the unmoral intellectual attitude and the degenerate character which the early Christians found in close association with Greek learning, and on a too literal interpretation of the Jewish scriptures, with their primitive Chaldean theories of cosmogony and the world.

Justin Martyr, in the second century, says that what is true in the Greek philosophy can be learned much better from the Prophets. Clement of Alexandria (d. 227) calls the Greek philosophers robbers and thieves who have given out as their own what they have taken from the Hebrew prophets. Tertullian (160–220) insists that since Jesus Christ and his gospel, scientific research has become superfluous. Isidore of Seville in the seventh century declares it wrong for a Christian to occupy himself with heathen books, since the more one devotes himself to secular learning, the more is pride developed in his soul. Lactantius early in the fourth century includes in his "Divine Institutions" a section,

> 'On the false wisdom of the philosophers,' of which the 24th chapter is devoted to heaping ridicule on the doctrine of the spherical

figure of the earth and the existence of antipodes. It is unnecessary to enter into particulars as to his remarks about the absurdity of believing that there are people whose feet are above their heads, and places where rain and hail and snow fall upwards, while the wonder of the hanging gardens dwindles into nothing when compared with the fields, seas, towns, and mountains, supposed by philosophers to be hanging without support. He brushes aside the argument of philosophers that heavy bodies seek the centre of the earth, as unworthy of serious notice; and he adds that he could easily prove by many arguments that it is impossible for the heavens to be lower than the earth, but he refrains because he has nearly come to the end of his book, and it is sufficient to have counted up some errors, from which the quality of the rest may be imagined.

It was natural that Augustine (354–430), . . . should express himself with . . . moderation, as befitted a man who had been a student of Plato as well as of St. Paul in his younger days. With regard to antipodes, he says that there is no historical evidence of their existence, but people merely conclude that the opposite side of the earth, which is suspended in the convexity of heaven, cannot be devoid of inhabitants. But even if the earth is a sphere, it does not follow that that part is above water, or, even if this be the case, that it is inhabited; and it is too absurd to imagine that people from our parts could have navigated over the immense ocean to the other side, or that people over there could have sprung from Adam. With regard to the heavens, Augustine was, like his predecessors, bound hand and foot by the unfortunate water above the firmament. He says that those who defend the existence of this water point to Saturn being the coolest planet, though we might expect it to be much hotter than the sun, because it travels every day through a much greater orbit; but it is kept cool by the water above it. The water may be in a state of vapor, but in any case we must not doubt that it is there, for the authority of Scripture is greater than the capacity of the human mind. He devotes a special chapter to the figure of the heaven, but does not commit himself in any way though he seems to think that the allusions in Scripture to the heaven above us cannot be explained away by those who believe the world to be spherical. But anyhow Augustine did not, like Lactantius, treat Greek science with ignorant contempt; he appears to have had a wish to yield to it whenever Scripture did not pull him the other way, and in times of bigotry and ignorance this is deserving of credit. — *Dreyer.*

Arguing elsewhere that the soul perceives what the bodily eye cannot, Augustine avails himself of the geometrical analogy of the ideal straight line which shall have length without breadth or thickness, but he lapses into mysticism when he passes to the circle.

The biographer of St. Eligius (writing in 760 under Pepin) says 'What do we want with the so-called philosophies of Pythagoras, Socrates, Plato and Aristotle, or with the rubbish and nonsense of such shameless poets as Homer, Virgil and Menander? What service can be rendered to the servants of God by the writings of the heathen Sallust, Herodotus, Livy, Demosthenes or Cicero?' Fredegar . . . complains (about 600) that 'The world is in its decrepitude, intellectual activity is dead, and the ancient writers have no successors.' . . . — *G. H. Putnam,* Books of the Middle Ages.

The following is a broad survey of the whole period: —

The soft autumnal calm . . . which lingered up to the Antonines over that wide expanse of empire from the Persian Gulf to the Pillars of Hercules and from the Nile to the Clyde . . . was only a misleading transition to that bitter winter which filled the half of the second and the whole of the third century, to be soon followed by the abiding dark and cold of the Middle Ages. The Empire was moribund when Christianity arose. Rome had practically slain the ancient world before the Empire replaced the Republic. The barbarous Roman soldier who killed Archimedes absorbed in a problem, is but an instance and a type of what Rome had done always and everywhere by Greek art, civilization and science. The Empire lived upon and consumed the capital of preceding ages, which it did not replace. Population, production, knowledge, all declined and slowly died. . . .

The sun of ancient science, which had risen in such splendour from Thales to Hipparchus, was now sinking rapidly to the horizon; and when it at last disappeared, say, in the fifth century, the long night of the Middle Ages began. . . . The pursuit of knowledge for knowledge's sake was out of place. . . . All the outlets through which modern energy is chiefly expended were then closed; a man could not serve the state as a citizen, he could not serve knowledge

> as a man of science. . . . There was only one thing left for him to do, — to serve God. — *J. C. Morison,* The Service of Man.

The Eastern Empire. Edict of Justinian. — Only half a century after the fall of Rome the Greek schools in Athens were closed, in 529 A.D., by order of the emperor Justinian, and intellectual darkness settled down over Eastern Europe. Theology became more than ever the chief pursuit of the educated, and Greek learning more than ever neglected. Many Greek manuscripts, however, were hidden away, and many Greek scholars, though scattered, kept alive the feeble spark of Greek learning.

The Dark Ages. — After the mighty Roman Empire of the West had come to its end, the peoples of Christian Europe and of

> the Graeco-Roman world descended into the great hollow which is roughly called the Middle Ages, extending from the fifth to the fifteenth century, a hollow in which many great and beautiful and heroic things were done and created, but in which knowledge, as we understand it and as Aristotle understood it, had no place. The revival of learning and the Renaissance are memorable as the first sturdy breasting by humanity of the hither slope of that great hollow which lies between us and the ancient world. The modern man, reformed and regenerated by knowledge, looks across it and recognizes on the opposite ridge, in the far-shining cities and stately porticoes, in the art, politics and science of antiquity, many more ties of kinship and sympathy than in the mighty concave between, wherein dwell his Christian ancestry in the dim light of scholasticism and theology. — *Morison.*

The "great hollow" here so graphically portrayed may be described as the Middle or Medieval Age (c. 450–1450 A.D.) and of these ten centuries the first three, or thereabouts, are often called the Dark — as they certainly were the darkest — Ages.

> The darkest time in the Dark Ages was from the end of the sixth century to the revival of learning under Charles the Great (Charlemagne). Bad grammar was openly circulated and sometimes commended. St. Gregory the Great quoted the Bible in depreciation of the Humanities. (Ps. lxx. 15. 16.) The study of heathen authors was discouraged more and more. "Will the Latin grammar save an immortal soul?" "What profit is there in the record of pagan sages,

the labors of Hercules or of Socrates?" Books came to be scarce.... But the decline of education was not universal. If studies failed in Gaul or Italy, they flourished in Ireland and afterward in Britain, and returned later from these outer borders to the old central lands of the Empire. Further, in spite of depression and discouragement, there was a continuity of learning even in the darkest ages and countries. Certain school books hold their ground . . . Capella . . . Boethius . . . Cassiodorus . . . And later Isodorus of Seville with a number of other authors are found in the ages of distress and anarchy more or less calmly giving their lectures and preserving the standards of a liberal education. Much of this work was humble enough, but it was of great importance for the times that came after. . . . The darkest ages, with all their negligence, kept alive the life of the ancient world.

Boethius [in the sixth century A.D., see p. 148] is the interpreter of the ancient world and its wisdom, accepted by all the tribes of Europe from one age to another, and never disqualified in his office of teacher even by the most subtle and elaborate theories of the later schools. . . . Cassiodorus (490–585) is wanting in the graces of Boethius, and he is much sooner forgotten; but his enormous industry, his organization of literary production, his educational zeal have all left their effects indelibly in modern civilization. By his definition of the seven Liberal Arts, and by his examples of methods in teaching them, he is the spiritual author of the universities, the patron of all the available learning in the world. — *Ker,* Dark Ages.

THE ESTABLISHMENT OF SCHOOLS BY CHARLEMAGNE. — We have seen above how the schools of Athens were closed by Justinian in 529. Such schools as existed after that time were chiefly ecclesiastical and their teachings opposed to pagan or heathen (*i.e.* Greek) learning. At length, however, in 787 Charlemagne, moved it is said by the troublesome variety of writing as well as the general illiteracy of his people, ordered the establishment of schools in connection with every abbey of his realm, and summoned to take charge of them Peter of Pisa and Alcuin of York (735–804) (called by Guizot "the intellectual prime minister of Charlemagne"), whose names stand among the highest in a revival of learning thus begun in western Europe.

> In the later part of the eighth century begins the great age of medieval learning, the educational work of Charles the Great. . . . There was some leisure and freedom and much literary ambition. The Latin poets of the court of Charlemagne have an enthusiasm and delight in classical poetry. . . . In prose there was no less activity. Besides the scientific treatises and the commentaries, the edifying works of Alcuin and others, there were histories. . . . The scholarly spirit of the ninth century . . . is not limited to the orthodox routine. One of the chief scholars, with more Greek than most others, Erigena, is famous for more than his learning, as a philosopher, who, whatever his respect for the Church, acknowledged no authority higher than reason. — *Ker.*

Alcuin himself taught rhetoric, logic, mathematics and divinity, becoming master of the great school at St. Martin's of Tours. Of his arithmetic the following problem is an illustration: —

> If 100 bushels of corn are distributed among 100 people in such a manner that each man receives 3 bushels, each woman 2, and each child half a bushel; how many men, women and children are there?

Of six possible solutions Alcuin gives but one.

The mathematics taught in Charlemagne's schools would naturally include the use of the abacus, the multiplication table, and the geometry of Boethius. Beyond this, a little Latin with reading and writing sufficed for the needs of the church and her servants, and was supplemented by music and theology for her higher officers. The recognized intellectual needs of the world were indeed but slight. The civilization of Rome had been gradually submerged by successive waves of barbaric invasion from the north, as a similar fate was soon to be met by the still higher culture of Alexandria. The best intellect of the times was perforce drawn into other forms of activity, while such scholars as remained found no favorable environment for fruitful study. The Benedictine monasteries, indeed, sheltered a few studious monks whose scientific interest scarcely extended beyond the mathematics necessary for their simple accounts, and the computation connected with the determination of the date of Easter.

Near the close of the tenth century Gerbert of Aquitaine (940–1003), afterwards Pope Sylvester II, devoted his versatile genius in part to mathematical science. He constructed not only abaci, but terrestrial and celestial globes, and collected a valuable library. To him were also attributed a clock, and an organ worked by steam. He wrote works on the use of the abacus, on the division of numbers and on geometry. The last named contains a solution of the relatively difficult problem to find the sides of a right triangle whose hypotenuse and area are given. Unfortunately the latter part of his life was absorbed in political intrigue and his death in 1003 cut short his plans for attempting the recovery of the Holy Land.

Out of the schools of Charlemagne gradually grew up that subtle, minute and over-refined learning of the later Middle Ages which has come to be known as Scholasticism. Based as it was upon authority instead of experiment, and magnifying, as it did, details more than principles, it sharpened rather than broadened the intellect, and was indifferent if not unfavorable to science.

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CHAPTER VIII

HINDU AND ARABIAN SCIENCE. THE MOORS IN SPAIN

The grandest achievement of the Hindus and the one which, of all mathematical investigations, has contributed most to the general progress of intelligence, is the invention of the principle of position in writing numbers. — *Cajori.*

Indeed, if one understands by algebra the application of arithmetical operations to composite magnitudes of all kinds, whether they be rational or irrational number or space magnitudes, then the learned Brahmins of Hindustan are the true inventors of algebra. — *Hankel.*

In the ninth century the School of Bagdad began to flourish, just when the Schools of Christendom were falling into decay in the West and into decrepitude in the East. The newly-awakened Moslem intellect busied itself at first chiefly with Mathematics and Medical Science; afterwards Aristotle threw his spell upon it, and an immense system of orientalized Aristotelianism was the result. From the East, Moslem learning was carried to Spain; and from Spain Aristotle reëntered Northern Europe once more, and revolutionized the intellectual life of Christendom far more completely than he had revolutionized the intellectual life of Islam. — *Rashdall.*

ALEXANDRIA fell to the Arabs in 641 A.D. As a matter of historical perspective it is noteworthy that the interval between its foundation by Alexander the Great and its capture by the Mohammedans, — during most of which period it was the intellectual centre of the world, — is almost equal to that between Charlemagne's time and our own.

The preservation and transmission of portions of Greek science through the Dark Ages to the dawn of science in western Europe about 1200 A.D. was mainly effected through three distinct, though not quite independent, channels. First, there was to a limited extent a direct inheritance of ancient learning within the Italian peninsula, through all its political and military turmoil. Second, a substantial legacy was received indirectly through the Moors in Spain; while, third, additions of great importance came later through Italy from Constantinople. Before following the direct Latin-Italian line a brief sketch of Hindu and Arabic science is desirable.

HINDU MATHEMATICS. — The far-reaching conquests of Alexander the Great (330 B.C.) immensely stimulated communication of ideas between the Mediterranean world and Asia, and the East was able to make certain great contributions to mathematical science just where the Greeks were relatively weakest, namely in arithmetic and the rudiments of algebra and trigonometry. Several centuries before our era the Pythagorean theorem and an excellent approximation for $\sqrt{2}$ were known in India in connection with the rules for the construction of altars. The mathematicians however from whom we trace the later development of mathematics date from the sixth and following centuries.

About 530 A.D. Arya-bhata wrote a book in four parts dealing with astronomy and the elements of spherical trigonometry, and enunciating numerous rules of arithmetic, algebra and plane trigonometry. He gives the sums of the series

$$\begin{aligned} &1 + 2 + \dots + n \\ &1^2 + 2^2 + \dots + n^2 \\ &1^3 + 2^3 + \dots + n^3, \end{aligned}$$

solves quadratic equations, gives a table of sines of successive multiples of $3\frac{3}{4}°$ — *i.e.* twenty-fourths of a right angle, — and even uses the value $\pi = 3.1416$, correct to five places. His geometry is in general inferior.

Some years later, Brahmagupta composed a system of astronomy in verse, with two chapters on mathematics. In this he discusses arithmetical progression, quadratic equations, areas of triangles, quadrilaterals and circles, volume and surface of pyramids and cones. His value of π is $\sqrt{10} = 3.16+$. Typical problems and discussions are the following: —

Two apes lived at the top of a cliff of height 100, whose base was distant 200 from a neighboring village. One descended the cliff, and walked to the village, the other flew up a height x and then flew in a straight line to the village. The distance traversed by each was the same. Find x.

Beautiful and dear Lilavati, whose eyes are like a fawn's! tell me what are the numbers resulting from one hundred and thirty-five, taken into twelve? if thou be skilled in multiplication by whole or by parts, whether by subdivision or form or separation of digits. Tell me, auspicious woman, what is the quotient of the product divided by the same multiplier?

The son of Pritha exasperated in combat, shot a quiver of arrows to slay Carna. With half his arrows, he parried those of his antagonist; with four times the square-root of the quiver-full, he killed his horse; with six arrows, he slew Salya; with three he demolished the umbrella, standard and bow; and with one, he cut off the head of the foe. How many were the arrows, which Arjuna let fly?

For the volume contains a thousand lines including precept and example. Sometimes exemplified to explain the sense and bearing of a rule; sometimes to illustrate its scope and adaptation; one while to show variety of inferences; another while to manifest the principle. For there is no end of instances; and therefore a few only are exhibited. Since the wide ocean of science is difficultly traversed by men of little understanding; and, on the other hand, the intelligent have no occasion for copious instruction. A particle of tuition conveys science to a comprehensive mind; and having reached it, expands of its own impulse. As oil poured upon water, as a secret entrusted to the vile, as alms bestowed upon the worthy, however little, so does science infused into a wise mind spread by intrinsic force.

It is apparent to men of clear understanding, that the rule of three terms constitutes arithmetic; and sagacity, algebra. Accordingly I have said in the chapter of Spherics:

'The rule of three terms is arithmetic; spotless understanding is algebra. What is there unknown to the intelligent? Therefore, for the dull alone, it is set forth.'

Five centuries later Bhaskara also wrote an astronomy containing mathematical chapters, and the contents of this work soon became known through the Arabs to western Europe. While the preceding writers had no algebraic symbolism, but depended laboriously on words and sentences, Bhaskara made considerable progress in abbreviated notation. A partial list of subjects, treated in his first book, includes weights and measures, decimal numeration, fundamental operations, addition etc., square and cube root, fractions, equations of the first and second degrees, rule of three, progressions, approximate value of π, volumes. Applications are made to interest, discount, partnership, and the time of filling a cistern by several fountains. While there is reason to believe that the decimal system was known as early as the time of Brahmagupta, this work contains the first systematic discussion of it, including the so-called Arabic numerals and zero.

As an intermediate stage between the earlier use of entire words and our modern employment of single letters, he employs abbreviations, but multiplication, equality and inequality have still to be written out. The divisor is written under the dividend without a line, one member of an equation under the other with verbal context to insure clearness. Polynomials are arranged in powers, though without our exponents, coefficients follow the unknown quantities. In his "rules of cipher" he even gives the equivalent of $a \pm 0 = a$, $0^2 = 0$, $\sqrt{0} = 0$, $a \div 0 = \infty$.

In comparison with Greek mathematics, power and freedom are gained at the cost of some sacrifice of logical rigor. Among the Greeks, only the greatest appreciated the possibility and the importance of an unending series of numbers; but the Hindu imagination tended naturally in this direction. A notable achievement of the Hindus was the introduction of the idea of negative numbers and the illustration of positive and negative by assets and debts, etc.

On the whole, the Hindus, having received a part of their mathematics originally from the Greeks, made great contributions on the arithmetical and algebraic side, their influence on European science with which they had little or no direct contact being exerted mainly through the Arabs.

The Hindu mathematicians had no interest in what is termed mathematical method. They gave no definitions; preserved little logical order; they did not care whether the rules they used were properly established or not and were generally indifferent to fundamental principles. They never exalted mathematics as a subject of study and indeed their attitude to learning may be described as decidedly unmathematical. — *G. R. Kaye.*

HINDU ASTRONOMY. — In astronomy a parallel development took place. It seems probable that Greek planetary theory was introduced into India between the times of Hipparchus and Ptolemy, but Hindu astronomy is characterized as "a curious mixture of old fantastic ideas and sober geometrical methods of calculation." Aryabhata says indeed "The sphere of the stars is stationary, and the earth, making a revolution, produces the daily rising and setting of stars and planets," an opinion rejected by the later Brahmagupta.

MOHAMMED AND THE HEGIRA. — During the sixth and following centuries great events were happening in Arabia, an anciently settled country, but up to that time a blank in the history of civilization and of science. In 569 A.D. or thereabouts was born, probably in Mecca, — an insignificant commercial town 45 miles from the middle eastern shore of the Red Sea, — that extraordinary man Mohammed, whom millions of his fellow men still regard as the Prophet of the Almighty (Allah). In 622 Mohammed fled with a small company of his disciples to Medina, an agricultural town 250 miles to the north of Mecca, where he prosecuted his propaganda, and completed his Koran, — the Mohammedan Bible. Here also he died in 632 A.D.

In Mecca, Mohammed was "the despised preacher of a small congregation," but after his flight (*hejira*) to Medina, he became the leader of a powerful party and ultimately the autocratic ruler of Arabia. Even before his death his followers numbered thousands, while the religious zeal with which they were fired has never been surpassed. Taught by Mohammed to convert or kill, they threw themselves upon their neighbors with a fanatical fury which overcame all obstacles, so that within one short century their religion and its adherents swept like a tidal wave from the barren valleys of western Arabia northward and eastward through Syria over Asia Minor and Mesopotamia, and northwestward along the African shores of the Mediterranean to the straits of Gibraltar. Egypt, Alexandria, and Carthage fell before the Mohammedans, and the Arabian or Moslem empire soon rivalled in extent its great predecessor, the Roman. In 711 Moslems crossed the straits of Gibraltar and entered Spain, soon pushing northward into western France as far as Poitiers, where their great western and northern movement was finally checked by Charles Martel, in 732. This extraordinary onrush, occurring almost within a single century, naturally left the Moslems little time for the development of learning or for the arts and sciences. But after it was over, the Mohammedan invaders settled down in their various conquered countries and in some of them cultivated the arts of peace. The successive Arabian rulers (beginning with Al-Mansur, in 754) patronized learning, and to this end collected Greek manuscripts, which, after the closing of the Greek schools by Justinian in 529, had become scattered abroad. In particular, certain Nestorian Jews were brought to Bagdad and by them translations into Arabic were made of some of the works of Aristotle, Euclid, Ptolemy, and other Greek authors. The learning of India was also drawn upon, especially for the so-called Arabic numerals. Thus began a kind of Arabian science, chiefly imported at the outset, but destined within the next three centuries to take on characteristics of its own. It was, however, under the Caliph Al-Mamun (813–833), who has been called, as regards schools and learning, the Charlemagne of his people, that Aristotle was first translated into Arabic. Al-Mamun caused works on mathematics, astronomy, medicine, and philosophy to be translated from the Greek, and founded in Bagdad a kind of academy called the "House of Science," with a library and an observatory.

Arabian Mathematical Science. — While the Arabs themselves were not in general much addicted to scientific pursuits, their relations to the Greeks and Hindus, and subsequently to the nations of western Europe are of very great importance in the history of science. Even if we accept as typical the traditional dictum attributed to the Caliph Omar, that whatever in the library of Alexandria agreed with the Koran was superfluous, whatever disagreed was worse, and all should therefore be destroyed, it was inevitable that individuals in this active-minded race should fall under the spell of Greek mathematical science. Their religion was in fact more tolerant towards science than was contemporary Christianity.

It would appear that by 900 A.D. the Arabs were familiar on the one hand with Brahmagupta's arithmetic and algebra, including the decimal system, and on the other hand with the chief works of the great Greek mathematicians, some of which have come down to us only through Arabic translations.

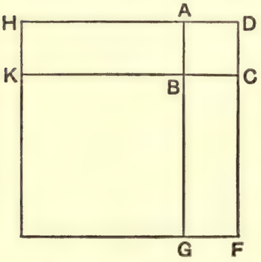
The Algebra of Alkarismi written about 830 was based on the work of Brahmagupta, and served in turn as the foundation for many later treatises. From its title is derived our word "algebra," from the author's name our "algorism." The book begins:—

> The love of the sciences with which God has distinguished Al-Mamun, ruler of the faithful, and his benevolence to scholars, have encouraged me to write a short work on computation by completion and reduction. Herein I limited myself to the simplest matters, and those which are most needed in problems of distribution, inheritance, partnership, land measurement, etc.

The first book contains a discussion of five types of quadratic equations:

$$ax^2 = bx,\ ax^2 = c,\ ax^2 + bx = c,\ ax^2 + c = bx,\ ax^2 = bx + c;$$

only real positive roots are accepted, but, unlike the Greeks, he recognizes the existence of two roots. He gives a geometrical solution of the quadratic equation analogous to those of Euclid. Suppose $x^2 + 10x = 39$ and let $AB = BC = x$, $AH = CF = 5$; then the areas are $AC = x^2$, $AK = BF = 5x$.



The sum of these is $x^2 + 10x$. Complete the square HF by adding $KE = 25$. $HF = (x + 5)^2 = 64$, whence $x = 3$.

The series $1^n + 2^n + 3^n + \ldots + m^n$ was summed for $n = 1,2,3,4$, and about 1000 A.D. Alkayami is said to have asserted the impossibility of finding two cubes whose sum should be a cube. Even a cubic equation was solved by the aid of intersecting conic sections. There is no definite separation of algebra and arithmetic, and the former, in spite of relatively rapid development, remains entirely rhetorical. The division line of fractions is introduced and the check of computation by "casting out nines."

In Physics, Al-Hazen (965? — 1038) wrote a work on optics enunciating the law of reflection and making a study of spherical and parabolic mirrors. He also devised an apparatus for studying refraction, being probably the first physicist to note the magnifying power of spherical segments of glass — *i.e.* lenses. He gave a detailed account of the human eye, and attempted to explain the change of apparent shape of the sun and the moon when approaching the horizon. The Arabs employed the pendulum for time measurement, and tabulated specific gravities of metals, etc. In the words of a modern physicist: —

> The Arabs have always reproduced what came down to them from the Greeks in thoroughly intelligible form, and applied it to new problems, and thus built up the theorems, at first only obtained for particular cases, into a greater system, adding many of their own. They have thus rendered an extraordinarily great service, such as would correspond in modern times to the investigations which have grown out of the pioneer work of such men as Newton, Faraday and Röntgen.
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> — *Wiedemann.*

ARABIAN ASTRONOMY. — In connection with astronomy the Arabs, following Greek precedents, developed trigonometry, introducing sines and other functions since current. They used masonry quadrants of large size, and even a combination of a horizontal circle with two revolving quadrants mounted upon it, foreshadowing the modern theodolite. Better and more complete observational data facilitated, and at the same time demanded, improved mathematical methods, while the necessary computations were accomplished much more economically, through the use of the decimal number system. Haroun Al-Raschid sent to Charlemagne an ingenious water clock, while under his successor, Al-Mamun, two learned mathematicians were commissioned to measure a degree of the earth's circumference.

> 'Choose a place in a level desert and determine its latitude. Then draw the meridian line and travel along it towards the pole-star. Measure the distance in yards. Then measure the latitude of the second place. Subtract the latitude of the first and divide the difference into the distance of the places in parasangs. The result multiplied by 360 gives the circumference of the earth in parasangs.'
>
> — *Wiedemann.*

The writer just quoted describes a second method involving the measurement of the angle of depression of the horizon as seen from the top of a high mountain.

It is not improbable that western Europe acquired from eastern Asia, through Arab channels, the mariner's compass and gunpowder.

> 'When the night is so dark that the captains can perceive no star to orient themselves, they fill a vessel with water and place it in the interior of the ship, protected from wind; then they take a needle and stick it into a straw, forming a cross. They throw this upon the water in the vessel mentioned and let it swim on the surface. Hereupon they take a magnet, put it near the surface of the water, and turn their hands. The needle turns upon the water; then they draw their hands suddenly and rapidly back, whereupon the needle points in two directions, namely north and south.' 1232 A. D.
>
> — *Wiedemann.*

The astronomical theory of the Arabs was merely that of Ptolemy. But they "were not content to consider the Ptolemaic system merely as a geometrical aid to computation; they required a real and physically true system of the world, and had therefore to assume solid crystal spheres after the manner of Aristotle." The various attempts to devise a better system all miscarried, their authors having no new guiding principle, nor superior mathematical power, and being more or less hampered by Aristotelian traditions, though Greek theories of the rotation of the earth seem not to have been unknown.

ASIATIC OBSERVATORIES. — Besides the work of the Arabian astronomers themselves, it is an interesting fact that their barbarian Mongol conquerors in the East acquired a temporarily active scientific interest, founding a fine observatory at Meraga near the northwest frontier of modern Persia. The instruments used here are said to have been superior to any used in Europe until the time of Tycho Brahe in the sixteenth century. The principal achievement of this observatory was the issue of a revised set of astronomical tables for computing the motions of the planets, together with a new star catalogue. The excellence of their work may be inferred from a determination of the precession of the equinoxes within 1″. This development lasted only a few years in the latter half of the thirteenth century. A similar brief outburst of astronomical activity occurred among the Tartars at Samarcand (Russian Turkestan) nearly 200 years later, that is, a little before the time of Copernicus, and here the first new star catalogue since that of Ptolemy was compiled. It is noteworthy that there was no hostility between science and the Mohammedan church. One of the uses of astronomy indeed was to determine the direction of Mecca.

No great original idea can be attributed to any of the Arab and other astronomers here discussed. They had, however, a remarkable aptitude for absorbing foreign ideas, and carrying them slightly further. They were patient and accurate observers, and skilful calculators. We owe to them a long series of observations, and the invention or introduction of several important improvements in mathematical methods. . . . More important than the actual contributions of the Arabs to astronomy was the service that they performed in keeping alive interest in the science and preserving the discoveries of their Greek predecessors. — *Berry.*

THE MOORS IN SPAIN. — We have already touched above upon the rapid spread of Mohammedanism westward from its home in Arabia, and the remarkable conquests of its followers in Spain and western France. These western Mohammedans included not only some of pure Arabian stock, but more of mixed descent, especially from that part of northern Africa once known as Mauretania, — whence the term Moors, generally applied to the conquering Mohammedans of the west. The Moors entered Spain early in the eighth century, bringing after them the learning of the Arabs, so that Hindu and Arabian science, and to some extent Greek science, were making their way into southwestern Europe even before the schools of Charlemagne were established toward the end of the same century in central (Christian) Europe. In the ninth and tenth centuries a remarkable civilization arose in Spain, — the highest that the Arabian race has ever reached. The developments of science in Mohammedan Spain are more or less typical of what occurred throughout the whole Arabian empire, and in such cities as Cordova, Toledo, and Seville a type of civilization and a stage of learning were reached higher in many respects than existed at the same time and even for centuries afterward anywhere in Christian Europe.

> Scarcely had the Arabs become firmly settled in Spain when they commenced a brilliant career. Adopting what had now become the established policy of the Commanders of the Faithful in Asia, the Emirs of Cordova distinguished themselves as patrons of learning, and set an example of refinement strongly contrasting with the condition of the native European princes. Cordova, under their administration, at its highest point of prosperity, boasted of more than two hundred thousand houses, and more than a million of inhabitants. After sunset, a man might walk in a straight line for ten miles by the light of the public lamps. Seven hundred years after this time there was not so much as one public lamp in London. Its streets were solidly paved. In Paris, centuries subsequently, whoever stepped over his threshold on a rainy day stepped up to his ankles in mud.
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> — *Draper.*

The Mohammedans made some additions to medical science, and yet their medicine hardly goes beyond that of Galen, whom they specially revered. In alchemy they are notable, though more by their attempts than their achievements. Too often it was simply a search for "potable gold" or other "elixirs of life," "the philosopher's stone," and the like. In the arts and industries, however, the Moors deserve special mention. Cordovan and Morocco leather are well known. Toledo and Damascus blades (swords) were long famous. Arabian horses furnished Europe with one of the most serviceable strains of that useful animal, and many Arabian words have been adopted into our language, *e.g. alcohol, elixir, algebra, alembic, zenith, nadir*, etc.

". . . Under the caliphs, Moslem Spain became the richest, most populous, and most enlightened country in Europe. The palaces, the mosques, bridges, aqueducts, and private dwellings reached a luxury and beauty of which a shadow still remains in the great mosque of Cordova. New industries, particularly silk weaving, flourished exceedingly, 13,000 looms existing in Cordova alone. Agriculture, aided by perfect systems of irrigation for the first time in Europe, was carried to a high degree of perfection, many fruits, trees and vegetables hitherto unknown being introduced from the East. Mining and metallurgy, glass making, enamelling, and damascening kept whole populations busy and prosperous. From Malaga, Seville, and Almeria went ships to all parts of the Mediterranean loaded with the rich produce of Spanish Moslem taste and industry, and of the natural and cultivated wealth of the land. Caravans bore to farthest India and darkest Africa the precious tissues, the marvels of metal work, the enamels, and precious stones of Spain. All the luxury, culture, and beauty that the Orient could provide in return, found its way to the Moslem cities of the Peninsula. The schools and libraries of Spain were famous throughout the world; science and learning were cultivated and taught as they never had been before. Jew and Moslem, in the friendly rivalry of letters, made their country illustrious for all time by the productions of their study. . . . The schools of Cordova, Toledo, Seville, and Saragossa attained a celebrity which subsequently attracted to them students from all parts of the world. At first the principal subjects of study were literary, such as rhetoric, poetry, history, philosophy, and the like, for the fatalism of the faith of Islam to some extent retarded the adoption of scientific studies. To these, however, the Spanish Jews opened the way, and when the barriers were broken down, the Arabs themselves entered with avidity into the domain of science. Cordova then became the centre of scientific investigation. Medicine and surgery especially were pursued with intense diligence and success, and veterinary surgery may be said to have there first crystallized into a science. Botany and pharmacy also had their famous professors, and astronomy was studied and taught as it had never been before; algebra and arithmetic were applied to practical uses, the mariner's compass was invented, and science as applied to the arts and manufactures made the products of Moslem Spain — the fine leather, the arms, the fabrics, and the metal work — esteemed throughout the world. . . . Canals and water wheels for irrigation carried marvellous fertility throughout the south of Spain, where the one thing previously wanting to make the land a paradise was water. Rice, sugar, cotton, and the silkworm were all introduced and cultivated with prodigious success; the silks, brocades, velvets, and pottery of Valencia, the beautiful damascened steel of Seville, Toledo, Murcia, and Granada, the stamped embossed leather of Cordova, and the fine cloths of Seville brought prosperity to Moslem and Mozarab alike under the rule of the Omeyyad caliphs, while the systematic working of the silver mines of Jaen, the corals on the Andalusian coasts, and the pearls of Catalonia supplied the material for the lavish splendor which the rich Arabs affected in their attire and adornment.

The Moors of Andalusia and Valencia acclimatized and cultivated a large number of semitropical fruits and plants hitherto little known in Europe, and studied arboriculture and horticulture not only practically but scientifically. The famous work on the subject by Abu Zacaria Al-Awan was the foundation of such books, and of the application of science to gardening. It was mainly derived from Chaldean, Greek, and Carthaginian manuscripts now lost. Curiously, Spain had produced under the Romans a famous book on agriculture by Columella; but for scientific knowledge it cannot be compared to the Treatise on Agriculture by Abu Zacaria. . . . From the earliest times the wool of Spain had been the finest in the world. . . . Vast herds of stunted, ill-looking, but splendidly fleeced sheep belonged to the nobles and ecclesiastical lords, and quite early in the period of reconquest, when these classes were all-powerful, a confederacy of sheep owners was formed, which by the fourteenth and fifteenth centuries had developed into a corporation of immense wealth. This was called the Mesta. . . . The fleeces were extremely fine, often weighing 12 pounds per animal, and the wool was sought after throughout the world, especially by Flemish and French cloth workers. Even in the ninth century Spanish wool was famous in Persia and in the East; and as early as the time of the Phœnicians it was considered the finest in the world. — *Hume.*

The tenth century was the golden age of Moorish science in Spain. Another hundred years and it had gone down forever. Its permanent importance, even in conserving the work of the ancients, has been questioned, and a recent writer cleverly compares the whole western movement of the Arabians to the sands of their deserts, — now fierce and pitiless when driven by some force such as the wind, — now sinking into inert, helpless, infertile heaps when left to themselves.

An Arab renaissance as early as the eighth century had revived something of classic knowledge. The poetry and philosophy of Greece were studied, and the taste for learning was cultivated with the enthusiasm which the Arabs infused into all their undertakings. Every one knows the fascinating account of these things in the pages of Gibbon. How the tide of progress flowed from Samarcand and Bokhara to Fez and Cordova. How a vizir consecrated 200,000 pieces of gold to the foundation of a college. How the transport of a doctor's books required four hundred camels. How a single library in Spain contained 600,000 volumes, while seventy public libraries were opened in Andalusia alone. How the Arabian schools of Spain and Italy were resorted to by scholars from every country in Europe. . . .

Here was a state of luxury and learning which contrasted strongly enough with the barbarism of the age. But under all this show where was the substantial basis? How much of all this was real? Arab architecture, in so far as it was Arab, and not built for them by the Greeks, was a concoction of whim and fantasy. In those nervous hands every strong and simple feature was distorted into endless complications, and, as always happens, lost in stability what it gained in eccentricity. Their learning was of the same character. Though they disputed interminably on the rival merits of the Greek philosophers, they were content to receive all their knowledge of them through indifferent translations. When the real revival of learning came, and a genuine Renaissance set in, the six or seven centuries of Arab civilization were simply ignored and passed over. . . .

The Arab mind seems to turn by a sort of instinct to the occult, the mystical, the fantastic. It is always sighing for new worlds to conquer before it has made good the ground it stands on. It has the curious gift of turning everything it touches from substance to shadow.

> Astronomy changes into astrology, and the main business of the science becomes the casting of horoscopes. The study of medicine changes into the composition of philtres and talismans and the reciting of incantations. Chemistry changes into a search for the secret of the transmutation of metals and the elixir of immortal health. In short, the tendency always was to shift the appeal from the intellect and reason to the fancy and imagination; and their zeal, instead of being devoted to laying firm foundations, evaporated in vague aspirations after the unintelligible or the unobtainable. . . .
>
> And the consequence is that not only has the Arab left us little or nothing, but his whole history seems already more legendary than real. Other civilizations abide our question. Not the Greek and Roman only, but the remote Assyrian and Egyptian, are definite and real in comparison with the Arabian. This seems of another texture. It is such stuff as dreams are made of.
>
> Those so-called conquests of his [the Arab's] were really the taking advantage of a unique opportunity for destroying and pulling down. The collapse of the Western Empire, and weakness and paralysis of the Eastern, afforded the Arab a fine field for the display of his peculiar prowess. He took to the lumber and *dêbris* of these crumbling empires as fire takes to rotten wood. But if in the void that separates ancient civilisation from modern the Arab appears to advantage, there no sooner entered on the scene nations of solid character and creative genius than he retired before them, and yielded to their advance.
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> — *March Phillips.*

The Golden Age of Moorish learning in the tenth century came and went, leaving behind it singularly few permanent results. Owing to the racial and religious hatreds of the time the Christian conquerors of the Moslems, like their Roman prototypes in the first few centuries after Christ, had small respect for Greek — and less for Mohammedan — learning. Hence, doubtless, it came about that to-day in Cordova, for example, almost no traces remain of that Arabian learning of which it was once the celebrated seat. Even the site of its illustrious university has faded from memory and only its great mosque (of which the heart is occupied by a Christian church) remains to bear visible witness to Mohammedan Cordova. The same is true of other once famous centres of Spanish Mohammedan-Greek learning. Toledo still possesses some of its Arabian walls and gateways, and Seville its lovely Giralda — "the first astronomical observatory in Europe" — and its Tower of Gold; but it is only in the Alhambra of Granada that any adequate vision can be had of Mohammedan life and influence in Spain. Here the quiet, the seclusion, the rich ornamentation, and the music of abundant running waters, still communicate an impression of wealth, taste, and power, and suggest possibilities of uninterrupted study and an intellectual life. Elsewhere, evidences of the Mohammedan love of inquiry, of libraries, of decoration, and even of fruits and gardens, have been almost wholly blotted out.

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HUME, M. *History of the Spanish People.*
MARCH PHILLIPS. *In the Desert.*
GIBBON. *Decline and Fall.*

CHAPTER IX

PROGRESS OF SCIENCE TO 1450 A.D.

It cannot be too emphatically stated that there is no historical evidence for the theory which connects the new birth of Europe with the passing away of the fateful millennial year and with it, the awful dread of a coming end of all things. Yet, although there was no breach of historical continuity at the year 1000, the date will serve as well as any other that could be assigned to represent the turning-point of European history, separating an age of religious terror and theological pessimism from an age of hope and vigor and active religious enthusiasm. . . . The change which began to pass over the schools of France in the eleventh century and culminated in the great intellectual Renaissance of the following age, was but one effect of that general revivification of the human spirit which should be recognized as constituting an epoch in the history of European civilization not less momentous than the Reformation or the French Revolution. . . . The schools of Christendom became thronged as they were never thronged before. A passion for inquiry took the place of the old routine. The Crusades brought different parts of Europe into contact with one another and into contact with the new world of the East, — with a new Religion and a new Philosophy, with the Arabic Aristotle, with the Arabic commentators on Aristotle, and eventually even with Aristotle in the original Greek. . . . Whatever the causes of the change, the beginning of the eleventh century represents, as nearly as it is possible to fix it, the turning-point in the intellectual history of Europe. — *Rashdall.*

THE CRUSADES. — From the time of Mohammed's hegira from Mecca to Medina in 622 A.D. to the siege of Vienna by his followers in 1683 — a period of more than 1000 years — Europe stood in constant dread of Mohammedan conquest. Fifteen years after the hegira, Jerusalem was captured by Omar, and remained under Mohammedan control till the end of the first Crusade, since which time it has been sometimes in Christian, sometimes in Mohammedan, possession. Toleration of Christians in the Holy

Land was, however, the rule until the eleventh century, and between 700 and 1000 A.D. pilgrimages to Jerusalem were frequently undertaken by Christians in the West. But after 1010 such pilgrimages began to be seriously interfered with, and matters steadily grew worse, until in 1071 Seljukian Turks displaced Arabian Mohammedans as rulers of Jerusalem. These Turks, though more rough than intolerant, eventually interfered with both trade and pilgrimages, until for this and other reasons the conquest of the Holy Land became a passion with the Christian nations.

In the spring of 1097, after several years of widespread preparation, a great host of western Christians, variously estimated at 150,000 to 600,000, gathered at Constantinople charged with warlike and religious zeal and bent on wresting Jerusalem and the Holy Land from the possession of the Mohammedan "Infidel." This was the beginning of those expeditions under the banner of the Cross, — hence known as the Crusades, — which may be regarded as intermittent reactions of the Christian West against the pressure of the Mohammedan East. The spirit of Christian Europe in the Middle Ages being essentially religious and ecclesiastical, it was natural that its more bold and adventurous youth should regard with jealousy and indignation the wide extent of the Mohammedan empire and especially its possession of Jerusalem and other holy places. In all, eight such Crusades are recognized by historians, and of these the influence upon Christian Europe must have been immense. In the first place, the expansion of the intellectual outlook due to the mere experiences of travel, for men born and bred under the parochial limitations of feudalism and monasticism, must have been great. Then, too, the arts and appliances observed abroad, the different standards of all sorts, the wealth and luxury of the distant East, doubtless had a powerful effect upon Europe when reported or introduced by the Crusaders upon their return. When we reflect upon the ages of darkness which had rested upon Christian Europe from the fall of Rome into the hands of the barbarians to the fall of Jerusalem into the hands of the Turks — a period of almost exactly six hundred years — we may agree with those who are disposed to look upon the Crusades as an age of discovery comparable with that of the new world by Columbus and his followers, — but a discovery of the East instead of the West.

The period of the Crusades extends over about two centuries, viz: — from 1090 to 1290, and thus immediately precedes the Renaissance, of which it was apparently one of the most important factors.

TRIVIUM: QUADRIVIUM. SCHOLASTICISM. — Meantime, following the mandate of Charlemagne establishing schools in connection with all the abbeys and monasteries of his vast domain of central Europe, a characteristic technical and essentially verbal scholarship gradually arose which, although chiefly ecclesiastical in substance, and so narrow in its range as almost completely to neglect natural science, was often thorough and sometimes profound. This learning in its later development is known as "Scholasticism," of which the foundation and essence was the famous curriculum of "the seven liberal arts," founded upon the educational doctrines of Plato, but adapted to the fashion of the Middle Ages. These consisted of a *quadrivium* — geometry, astronomy, music and arithmetic — and a *trivium* — grammar, logic, and rhetoric. (p. 148.)

In the introduction to the Logic of Aristotle which was in the hands of every student even in the Dark Ages, the *Isagoge* of Porphyry, the question was explicitly raised in a very distinct and emphatic manner. The words in which this writer states, without resolving, the problem of the Scholastic Philosophy, have played perhaps a more momentous part in the history of thought, than any other passage of equal length in all literature outside the canonical Scriptures. They are worth quoting at length:

'Next, concerning genera and species, the question indeed whether they have a substantial existence, or whether they consist in bare intellectual concepts only, or whether, if they have a substantial existence, they are corporeal or incorporeal, and whether they are separable from the sensible properties of the things (or particulars of sense), or are only in those properties and subsisting about them, I shall forbear to determine. For a question of this kind is a very deep one and one that requires a longer investigation.' — *Rashdall.*

To show the low state of natural history it suffices to refer to an extraordinary work, the so-called *Physiologus* or Bestiary, a kind of scriptural allegory of animal life, originally Alexandrian, but surviving in mutilated forms and widely used in medieval times. The childish and grotesque character of this curious compendium shows how ill-adapted were the centuries of crusading to the calm pursuits of science; they were indeed almost barren in this direction.

Scholasticism, nevertheless, lingered long after the Crusades were ended, and abundant survivals of it exist even today.

MEDIEVAL UNIVERSITIES. — The origin of the universities which play so great a part in the cultivation and dissemination of learning in the later middle ages is involved in obscurity. The medical school at Salerno in southern Italy seems to have become known in the ninth century, so that the University of Salerno is sometimes called the oldest in Europe. It was still famous in the thirteenth century. The law school at Bologna, in northern Italy, became well known about 1000 A.D., though the date of the University of Bologna is usually given as near the end of the twelfth century. The University of Paris is often dated from the early part of the same century. None of these early universities was much more than an association or gild of masters and pupils. Laboratories for instruction were of course unknown.

In the eleventh and twelfth centuries there was a gradual development from the previous monastic schools to the beginnings of modern universities at Paris, Bologna, Salerno, Oxford, and Cambridge, the schools themselves however continuing along their previous lines; and from that time onward to our own, the universities have played the chief part in the advancement of learning in general and of science in particular. In their development theological influences were naturally dominant, and it is interesting to observe that the use of Aristotle's Natural Philosophy, which became later the stronghold of orthodox conservatism, was prohibited in the thirteenth century.

Medieval academic standards were naturally low. The university was a voluntary and privileged society of scholars. Not until

1426 is there a record of the refusal of a degree for poor scholarship, and the victim then sought redress by legal proceedings, though in vain. In most of the early universities logic, philosophy, and theology were cultivated rather than even mathematical science.

TRANSMISSION OF SCIENCE THROUGH MOORISH SPAIN. — The meagre rivulet of classical science derived directly from Greek and Roman sources is now mingled with the current which found its way through northern Africa and Spain under the Moors. Boethius' rudimentary work was supplanted, and before 1400, the first five books of Euclid were taught at many universities. Ptolemy's Almagest was also translated from the Arabic into Latin early in the twelfth century, probably with the use of Arabic numerals. Near the close of the Moorish domination of Spain, King Alfonso X of Castile (1223–1284) collected at Toledo a body of Christian and Jewish scholars who under his direction prepared the celebrated Alfonsine Tables, using the new Arabic numerals. These enjoyed a high reputation for three centuries, though first printed in 1483.

While we thus owe to the Arabs a considerable debt for preserving for the use of later ages the precious heritage of Greek learning, the revival of learning in the fourteenth century came chiefly from other quarters and would probably have come in due time even if Arabic influences had not been at work. Yet it is noteworthy that early in the twelfth century re-translations of the Greek classics began to be made from the Arabic, and these may well have supplied the very limited demand for them tolerated by the church for the next hundred years. In spite of jealous exclusiveness the learning of the great schools of Granada, Cordova, and Seville gradually found its way to Paris, Oxford, and Cambridge.

> During the course of the twelfth century a struggle had been going on in the bosom of Islam between the Philosophers and the Theologians. It was just at the moment when, through the favor of the Caliph Al-Mansur, the Theologians had succeeded in crushing the Philosophers, that the torch of Aristotelian thought was handed on to Christendom. . . .

It was from this time and from this time only (though the change had been prepared in the region of pure Theology by Peter the Lombard) that the Scholastic Philosophy became distinguished by that servile deference to authority with which it has been in modern times too indiscriminately reproached. And the discovery of the new Aristotle was by itself calculated to check the originality and speculative freedom which, in the paucity of books, had characterized the active minds of the twelfth century. The tendency of the sceptics was to transfer to Aristotle or Averroës the authority which the orthodox had attributed to the Bible and the Fathers of the Church.

— *Rashdall.*

DAWN OF THE RENAISSANCE. — In the thirteenth century it becomes plain that a new spirit is arising in Europe. We cannot fail to detect at this time the existence, even at places as far apart as Oxford and Bologna — infinitely further apart then than now, — of a widespread desire for knowledge and a zeal for learning such as had not been known for centuries. Arabic mathematical science is introduced from northern Africa by Leonardo Pisano. A fresh and notable philosopher — Albertus Magnus — appears. Thomas Aquinas writes his famous *Imitatio Christi.* Great Gothic cathedrals arise, more universities are founded, and, most noteworthy of all for the history of science, an original student of nature appears, in Roger Bacon.

By the beginning of the thirteenth century, in consequence of the opening up of communications with the East — through intercourse with the Moors in Spain, through the conquest of Constantinople, through the Crusades, through the travels of enterprising scholars — the whole of the works of Aristotle were gradually making their way into the Western world. Some became known in translations direct from the Greek; more in Latin versions of older Syriac or Arabic translations. And now the authority which Aristotle had long enjoyed as a logician — nay, it may almost be said the authority of logic itself — communicated itself in a manner to all that he wrote. Aristotle was accepted as a well-nigh final authority upon Metaphysics, upon Moral Philosophy, and with far more disastrous results upon Natural Science. The awakened intellect of Europe busied itself with expounding, analysing and debating the new treasures unfolded before its eyes. . . .

And of the scientific side of this revival Italy was the centre. This branch of the movement began, indeed, before the twelfth century. It was in Italy that the Latin world first came into contact with the half-forgotten treasures of Greek wisdom, with the wisdom which the Arabs had borrowed from the Greeks and with original products of the remoter East. Of the Medical School of Salerno we have already spoken. It was probably in Italy and through the Arabic that the Englishman Adelard of Bath translated Euclid into Latin during the first half of the eleventh century. At about the same time modern musical notation originated with the discoveries of the Camaldulensian monk, Guido of Arezzo. In the first years of the following century the Algebra and the Arithmetic which the Arabs had borrowed from the Hindus were introduced into Italy by the Pisan merchant, Leonardo Fibonacci. . . . It was to this Arabo-Greek influence that Bologna owed its very important School of Medicine and Mathematics — two subjects more closely connected then than now through their common relationship to Astrology.

— *Rashdall.*

MATHEMATICAL SCIENCE IN THE THIRTEENTH CENTURY. — Increasing activity in mathematical science was due largely to Leonardo Pisano of Italy, Jordanus Nemorarius of Saxony, and Roger Bacon of England.

Leonardo Pisano or Fibonacci (born 1175) was educated in Barbary, where his father was in charge of the custom-house, and thus became familiar with Alkarismi's algebra, and the Arabic decimal system. He appreciated their advantages and on his return to Italy published in his *Liber Abaci* an account which gave them currency in Europe "in order that the Latin race might no longer be deficient in that knowledge." As the mathematical masterpiece of the Middle Ages, it remained a standard for more than two centuries. His algebra is rhetorical, but gains by the employment of geometrical methods. He discusses the fundamental operations with whole numbers and fractions, using the present line for division. Fractions are decomposed into parts with unit numerators as in early Egypt. Through the Arabs

Leonardo inherits Egyptian as well as Greek traditions, for example, the type of fraction just mentioned, square and cube root, progressions, the method of false assumption. It would appear that when the Arabs conquered Alexandria some of the old Egyptian culture was preserved. The rule of three, partnership, powers and roots, and the solution of equations are also included.

In 1225 the emperor, impressed by the accounts of Pisano's mathematical power, arranged a mathematical tournament of which the challenge questions are preserved:

> 'To find a number of which the square, when either increased or diminished by 5, would remain a square.
>
> 'To find by the methods used in the tenth book of Euclid a line whose length x should satisfy the equation $x^3 + 2x^2 + 10x = 20$.
>
> 'Three men, A, B, C, possess a sum of money u, their shares being in the ratio 3 : 2 : 1. A takes away x, keeps half of it, and deposits the remainder with D; B takes away y, keeps $\frac{2}{3}$ of it, and deposits the remainder with D; C takes away all that is left, namely z, keeps $\frac{5}{6}$ of it, and deposits the remainder with D. This deposit is found to belong to A, B, and C in equal proportions. Find u, x, y and z.'
>
> Leonardo gave a correct solution of the first and third, also a root of the cubic equation correct to nine decimals. — *Ball.*

Jordanus Nemorarius wrote important Latin works on arithmetic, geometry, and astronomy. His *De Triangulis* — the most important of these — consists of four books dealing not only with triangles, but with polygons and circles. He generally uses Arabic numerals, and denotes quantities known or unknown by letters. He solves the problem of finding two numbers having a given sum and product, by a method equivalent to our elementary algebra. This is practically the first European syncopated algebra, but seems to have become too little known to have far-reaching results in a time not yet ripe for this invention. A book on Weights contains elements of mechanics.

Albertus Magnus, born near the end of the twelfth century, became an ardent champion of the newly discovered but proscribed works of Aristotle. In particular he interpreted the Milky

Way as an accumulation of small stars, and ridiculed the current objections to antipodes, striving, however, always to harmonize the ancient science with the theology of his church.

Two Oxford scholars, John of Holywood (*Sacrobosco*) and Roger Bacon, have next to be mentioned. Sacrobosco lectured at Paris on arithmetic and algebra, and wrote standard books on the former with rules but no proofs, and an astronomy of which more than sixty editions were afterwards printed.

ROGER BACON (1214–1294?). — In the history of natural science one thirteenth century name stands out before all others, viz.: that of Roger or "Friar" Bacon, a member of the Franciscan order, born at Ilchester, England, in 1214. He was a pupil of Robert Grosseteste "who had especially devoted himself to mathematics and experimental science," and had studied the works of the Arabian authors. Bacon also travelled abroad and studied at the University of Paris, — at that time the centre of European learning. Here he took the degree of Doctor of Theology and probably also here became a Franciscan friar. He taught at Oxford, where he had a kind of laboratory for alchemical experiments. Doubtless it was for this that he became reputed as a worker in "magic" and the "black arts," for in 1257 he was forbidden by the head of his order to teach, and was sent to Paris, where he underwent great privations. In 1266 he was invited by Pope Clement IV to prepare and send to him a treatise on the sciences, and within 18 months he had written and sent three important works — his *Opus Majus, Opus Minus,* and *Opus Tertium.* In 1268 he returned to Oxford and there composed several more works, but under a later Pope his books were condemned and he was thrown into prison where he remained until about a year before his death.

In Paris, Bacon devoted himself particularly to physical science and mathematics. His *Opus Majus* (1267) contains both a summary of ancient and current physical science, and a philosophy of learning based on Greek, Roman, and Arabic authorities. He insisted that natural science must have an experimental basis, and that astronomy and the physical sciences must be founded on mathematics, "the alphabet of all philosophy." On the other hand he says: —

> We must consider that words exercise the greatest influence. Almost all wonders are accomplished through speech. In words the highest enthusiasm expresses itself. Therefore words, deeply thought . . . keenly realized, well calculated, and spoken with emphasis, have notable power.

Bacon enunciated the essential principles of calendar reform, recognizing that the current plan of 365¼ days led to an error of one day in 130 years. He made an acute criticism of the arbitrary assumptions and the artificial complexity of the Ptolemaic astronomy; he discussed reflection and refraction, spherical aberration, rainbows, magnifying glasses, and shooting stars; he attributed the tides to the action of the lunar rays. In a chapter on geography he "comes to the conclusion that the ocean between the east coast of Asia and Europe is not very broad. This . . . was quoted by Columbus in 1498. . . . It is pleasant to think that the persecuted English monk, then two hundred years in his grave, was able to lend a powerful hand in widening the horizon of mankind." (See Appendix.)

Most of this remarkable work — not printed for nearly 500 years — was so far in advance of the age that it not only failed of appreciation, but exposed the author to accusations of magic, and even to imprisonment. In spite of his many attainments he believed in astrology, in the doctrine of "signatures" and in the "philosopher's stone," and "knew" that the circle had been squared. He prophesied ships propelled swiftly by mechanical means and carriages without horses. He repudiated belief in witchcraft,[1] and paid the penalty for his courage by many years in prison.

Dante Alighieri (1265–1321). — Another notable scholar of the thirteenth century is Dante, the greatest poetical genius of the Middle Ages, who requires our notice not only because of his influence in awakening and stimulating the minds of his own and later times, but also as the author of a treatise On Water and the Earth (*De Aqua et Terra*) which, according to himself, was delivered at Mantua in 1320 as a contribution to the question, then much discussed, "whether on any part of the earth's surface water is higher than the earth." In his cosmology, Dante seems to derive from Aristotle and Pliny, without having attained familiarity with the Ptolemaic system.

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[1] Not merely astrology and alchemy but even magic and necromancy were at this time the subjects of university lecture courses.

COMPUTATION IN THE MIDDLE AGES. — During the fourteenth century there was continued activity in the gradual dissemination of Arabic learning, largely through the medium of almanacs and calendars, so that Arabic computations, Euclidean geometry, and Ptolemaic astronomy became widely known. Some of these calendars emphasized the religious side and gave dates of church festivals for a series of years, others specialized in astrology, medicine, or astronomy. For ecclesiastical purposes Roman numerals were preferred, but at least an explanation of the new Arabic characters and their use was generally given.

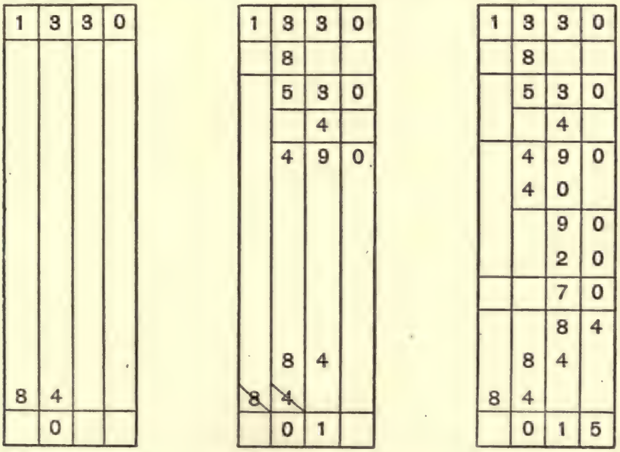
The arithmetic of Boethius, based on Roman numerals, retained its vogue in northern Europe as late as about 1600. Arabic arithmetic, or algorism, based on the *Liber Abaci* of Leonardo Pisano, employing the decimal scale and including the elements of algebra, came into general use among the Italian merchants in the thirteenth and fourteenth centuries, though not without meeting serious opposition. Outside of Italy, however, accounts were kept in Roman numerals till about 1550, and in the more conservative religious and educational institutions, for a hundred years longer. The Florentines at the same time considerably simplified the classification of arithmetical operations, in accordance with our modern list: — numeration, addition, subtraction, multiplication, division, involution and evolution.

Addition and subtraction were begun at the left. The multiplication table, at first little known, ended with 5 × 5. For further products up to 10 × 10, a system of finger reckoning was widely used, the rule running: —

> Let the number five be represented by the open hand; the number six by the hand with one finger closed; the number seven by the hand with two fingers closed; the number eight by the hand with three

fingers closed; and the number nine by the hand with four fingers closed. To multiply one number by another let the multiplier be represented by one hand, and the number multiplied by the other, according to the above convention. Then the required answer is the product of the number of fingers (counting the thumb as a finger) open in the one hand by the number of fingers open in the other together with ten times the total number of fingers closed.[1]

Long division naturally required the skill of a mathematical expert. For example, if it is necessary to divide 1330 by 84 (*Ball*, p. 191) the Arabic or Persian method may be represented as follows, the right hand figure summing up the whole process:



The galley or "scratch" method generally employed in Italy would take for the same problem, the successive forms (*Ball*, p. 192):

$$\begin{array}{ccccc}
 & & & & 7 \\
 & 4 & 4 & \cancel{4}9 & \cancel{4}\cancel{9} \\
5 & \cancel{5}9 & \cancel{5}9 & \cancel{5}\cancel{9} & \cancel{5}\cancel{9} \\
\cancel{1}\cancel{3}30(1 & \cancel{1}\cancel{3}\cancel{3}0(1 & \cancel{1}\cancel{3}\cancel{3}0(15 & \cancel{1}\cancel{3}\cancel{3}0(15 & \cancel{1}\cancel{3}\cancel{3}0(15 \\
\cancel{8}4 & \cancel{8}\cancel{4} & \cancel{8}\cancel{4}4 & \cancel{8}\cancel{4}4 & \cancel{8}\cancel{4}\cancel{4} \\
 & & 8 & \cancel{8} & \cancel{8}
\end{array}$$

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[1] In modern notation: If x is the number of fingers closed in one hand, y the number closed in the other, then

$$(5 + x)\ (5 + y) = (5 - x)\ (5 - y) + 10\ (x + y).$$

This method was considered simpler than our modern long division, and remained in use till the seventeenth century.

The signs +, −, ÷, and the use of decimal fractions belong to a somewhat later period.

The characteristics of the algoristic arithmetic are: (1) the use of the Hindu-Arabic system of notation; (2) the system of local value; (3) the use of the zero; (4) the entire discarding of the abacus; (5) the combined use of symbols and numbers (in reality a combination of algebra and arithmetic, as these terms are understood to-day); and (6) the introduction into Western Europe of a vast amount of arithmetical material from the East by means of Latin translations from Arabian sources. While the general tendency of this period was to approach the study of arithmetic from its practical and scientific sides, the mystical aspects of the subject — so popular in the earlier periods — are by no means neglected. The fantastic treatment of the properties of numbers is still common in this age. . . .

Thus the beginning of the thirteenth century marks the introduction of the Arabian system of notation and its adoption in place of both the Roman notation and the abacus. This fundamental revolution was brought about only gradually, and that of the algorism can be traced in the translated literature of the Hindu-Arabian arithmetic. — *Abelson.*

MATHEMATICS IN THE MEDIEVAL UNIVERSITIES. — The state of mathematics in the universities toward the close of the fourteenth century may be inferred from the requirements for the master's degree at Prague (1384) and Vienna (1389). The former included Sacrobosco's Sphere, Euclid Books I–VI, optics, hydrostatics, theory of the lever, and astronomy. Lectures were given on arithmetic, finger-reckoning, almanacs, and Ptolemy's Almagest. At Vienna, Euclid I–V, perspective, proportional parts, mensuration, and a recent version of Ptolemy were required. In Leipsic, however, in 1437 and 1438 mathematical (?) lectures were confined to astrology, and conditions seem to have been much the same at the Italian universities, while Oxford and Paris probably occupied an intermediate level.

There can be no doubt that at all times medieval schools taught all that their respective generations knew of arithmetic; that the teachers of arithmetic in the schools were often the famous mathematicians of their day; that this teaching, since it kept pace with the increase in the knowledge of the subject, was progressive in character, and that at no time, not even in the barren generations at the close of the Middle Ages, when the scholastic education had outlived its usefulness, did arithmetic cease to be a subject of study in the arts faculties of the medieval universities. — *Abelson.*

THE RENAISSANCE. — With the fourteenth century we enter upon one of the most interesting and noteworthy periods of human history; viz. the Renaissance. Neither the term nor the period is, however, sharply defined, the former signifying an awakening or "new birth," the latter covering loosely the fourteenth to the sixteenth centuries. It is only necessary to recapitulate briefly some of the phenomena touched upon in the present chapter, to realize that the civilization of the later Middle Ages has been undergoing great changes. The Crusades marked the first and perhaps most important of these, while the rediscovery or recovery of the classics from Arabian and other sources in the eleventh to the thirteenth centuries, followed by the revival of (classical) learning in the fourteenth must have been powerful ferments of the medieval scholastic mind, expanded and uplifted as it was by the poetical philosophy of Dante and challenged by the naturalism and rationalism of Roger Bacon.

The great events of the fourteenth century were in part new, and in part the natural extension and development of those of the thirteenth. A strange and appalling natural phenomenon was the famous epidemic known as the "black death," a quickly fatal disease which carried off from one quarter to one half of all the inhabitants of Europe, producing social changes — such as the rise of wages — which are still felt.

HUMANISM. — The development of better education begun in the thirteenth century was marked in the fourteenth by the founding of many now famous universities and colleges and by that revival of ancient learning which is associated especially with the name of Petrarch (1304–1374). This revival, while at first chiefly literary and philosophical, brought with it translations into Latin — the current language of scholars at that time — of Aristotle and other classical writers of scientific importance, and thus aided in bringing on a new birth or renaissance in science as well as in other branches.

Precisely as there is one great name in thirteenth century literature, viz. that of Dante, which must be regarded with attention by all students of history, so in the fourteenth the name and work of Petrarch require careful consideration. Francesco Petrarca, commonly called Petrarch, a gifted Italian poet and scholar, greatly promoted the revival of ancient learning by insisting on the importance and merits of the Greek and Roman authors.

> Petrarch was less eminent as an Italian poet than as the founder of Humanism, the inaugurator of the Renaissance in Italy. . . . Standing within the kingdom of the Middle Ages, he surveyed the kingdom of the modern spirit and, by his own inexhaustible industry in the field of scholarship and study, he determined what we call the revival of learning. By bringing the men of his own generation into sympathetic contact with antiquity, he gave a decisive impulse to that European movement which restored freedom, self-consciousness and the faculty of progress to the human intellect. . . . He was the first man to collect libraries, to accumulate coins, to advocate the preservation of antique monuments, and to collate manuscripts. Though he knew no Greek, he was the first to appreciate its vast importance; and through his influence, Boccaccio laid the earliest foundations of its study. . . . For him the authors of the Greek and Latin world were living men, — more real in fact than those with whom he corresponded; and the rhetorical epistles he addressed to Cicero, Seneca and Varro prove that he dwelt with them on terms of sympathetic intimacy. — *Symonds.*

Rich as the fourteenth and fifteenth centuries are in mathematical science and geographical discovery, and in art and invention, they are almost destitute of positive achievement in natural science. Doubtless the scientific spirit of curiosity and inquiry was alive and active, but thus far it had taken other directions.

ALCHEMY. — What astrology was to astronomy, alchemy was to chemistry; viz. the crude and often magic-working predecessor. The search for such will o' the wisps as the "philosopher's stone," the "elixir of life," "potable gold" and the "transmutation of elements," is probably as old as human history. The ancients seem to have dabbled in it, the Arabs to have been devoted to it, and the men of the Middle Ages, and even of the fourteenth and fifteenth centuries, to have spent much time upon it. Alembics and receivers, "Moors' Heads" and "Moors' Noses," calcification, distillation, and the like typify interesting and by no means fruitless gropings after the real composition of things. The names of Albertus Magnus, Bernard of Treviso, Eck of Salzburg, and Basil Valentine are some which have come down to us as most important at this time, and as we read of the preparation of the "spirits of salt" (hydrochloric acid), the calcification (oxidation) of mercury, etc., we realize that their labors, though often misdirected, were the prelude to better things.

THE MARINER'S COMPASS. — The loadstone was certainly known to antiquity as a stone having the power of attracting and carrying a load of iron, but its directive property seems to have been first recognized and used for guidance on land or sea by the Chinese, since according to Humboldt, Chinese ships navigated the Indian Ocean with the magnetic needle in the third century of our era. The Arabs are also credited with its invention and use, as stated in the preceding chapter. The first reference to it in Christian Europe is said to be in a poem by Guyot of Provence, dated 1190, while references are also made to the compass in works of the thirteenth century. One of these runs: —

> No master mariner dares to use it lest he should be suspected of being a magician; nor would the sailors venture to go to sea under the command of a man using an instrument which so much appeared to be under the influence of the powers below.

It is probable, however, that the compass was first made commonly useful to western Europe early in the fourteenth century, by Flavio Gioja, a native of Amalfi, a small port near Naples in Italy, who first poised the needle on a pivot instead of a card floating on water, as had been the custom before his time. (See page 164.)

CLOCKS. — Clocks with wheels seem to have come into occasional use from the twelfth to the fourteenth centuries, and one of the first is said to have been sent by the Sultan of Egypt in 1232 to the Emperor Frederick II.

> It resembled a celestial globe, in which the sun, moon and planets moved, being impelled by weights and wheels so that they pointed out the hour, day and night, with certainty.

Another is mentioned as in Canterbury cathedral, while still another at St. Albans, made by R. Wallingford who was abbot there in 1326, is said to have been so notable "that all Europe could not produce such another." It remained for Huygens in the seventeenth century to apply pendulums to clocks.

WOOL AND SILK. TEXTILES IN THE MIDDLE AGES. — As an example of the industrial history of the times the following account of conditions in Spain is given: —

> The cloth manufactures in Spain continued to be of the coarsest character until after the marriage of Catharine of Lancaster to the heir of Castile (1388) when finer cloths were manufactured and improved methods adopted. Up to that time the cloths used by people of the higher class came from Bruges, from London, and from Montpellier. James II of Aragon — the sovereign of Barcelona, where there were at the time hundreds of looms at work making a coarse woolen — wished to send a present to the Sultan of Egypt (1314 and 1322), and chose green cloths from Chalons and red cloths from Rheims and Douai, but sent no Spanish stuff; while the steward's accounts of Fernando V show that all his household were dressed in garments of imported stuffs. The great centre for the sale of wool was at Medina del Campo, and the cloth factories of Segovia and Toledo were the most active and celebrated in Castile, while those of Barcelona were the principal in the east of Spain. It is asserted that the improvement in the qualities of the Spanish cloth after the coming of the Plantagenet princess to Spain was partly owing to the fact that some herds of English sheep formed part of her

dowry, and the blending of staples enabled a better cloth to be made. The Flemish weavers mixed Spanish with English wool for their best textures.

During the Arab domination of the south, Jaen, Granada, Valencia, and Seville had been great centres of silk culture and manufacture. Edrisi says that in the kingdom of Jaen in the thirteenth century there were 3000 villages where the cultivation of the silkworm was carried on, while in Seville there were 6000 silk looms, and Almeria had 800 looms for the manufacture of fancy brocades, etc. We are also told that a minister of Pedro the Cruel owned 125 chests of silk and gold tissue. In the twelfth century, a very flourishing trade in silks, velvets, and brocades was carried on with Constantinople and the East generally. Even in the fourteenth and early fifteenth centuries, the silks of Valencia and the bullion embroideries and gold and silver tissues of Cordova and Toledo were unsurpassed in Christendom, though heavily handicapped by the growing burdens placed upon craftsmen by labor laws and racial prejudice, and the discouragement of luxury by sumptuary regulations. — *Hume.*

THE INVENTION OF PRINTING. — Before the middle of the fifteenth century, printing was done chiefly from fixed blocks of wood, metal, or stone, as is the case to-day in the printing of engravings, wood cuts and the like. The introduction of movable types, capable of an almost infinite variety of combination was therefore a forward step of fundamental importance, since the same letter or picture could be used over and over in new combinations where previously it could be used but once. Until quite recently, it was generally held that the invention of the art of printing from movable types was the work of Johann Gutenberg (1397–1468) of Mainz on the Rhine, aided by Johann Faust or Fust, a rich citizen of Mainz. Of late, however, the claim of Gutenberg has been much disputed.

The controversy about the person and nationality of the inventor [of the art of printing] and the place of invention resembles the rival claims of seven cities to be the birthplace of Homer. . . . The best authorities agree on Gutenberg. Jacob Wimpheling wrote in 1507 . . . 'Of no art can we Germans be more proud than of the art of printing, which made us the intellectual bearers of the doctrines of Christianity, of all divine and earthly sciences, and thus benefactors of the whole race.' — *Schaff*.

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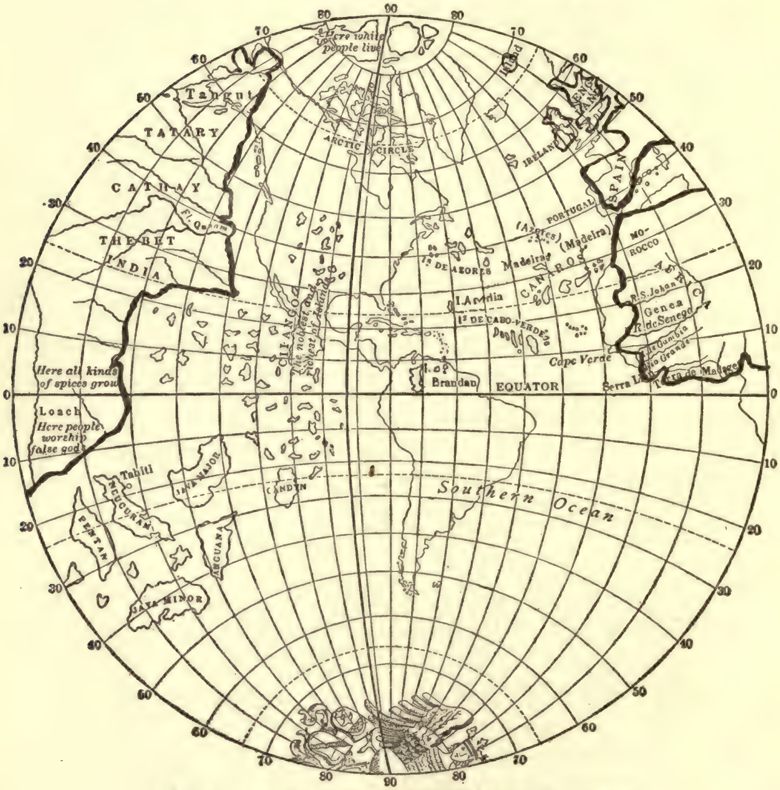
Here white people live
Tangut
TATARY
CATHAY
Fl. Quinsai
THEBET
INDIA
Here all kinds of spices grow
Loach
Here people worship false gods
Tabili
NUCURAN
PENTAN
IAVA MAJOR
ANGUANA
JAVA MINOR
CANDYN
CIPANGO
The noblest and richest of islands
ARCTIC CIRCLE
Island
ENGLAND
IRELAND
SPAIN
PORTUGAL
(Azores)
(Madeira)
MOROCCO
Is DE AZORES
Madeira
CANAROS
I. Antilia
Is DE CABO-VERDE
Cape Verde
AFRICA
R. S. Johan
Genea
R. de Senega
R. de Gambia
Rio Grande
Serra Lio
Terra de Malagete
S. Brandan
EQUATOR
Southern Ocean
ARCTIC CIRCLE
90
80
70
60
50
40
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20
10
0

A MAP OF THE GLOBE IN THE TIME OF COLUMBUS
(After J. H. Robinson. Courtesy of Messrs. Ginn & Co.)

In 1492 a German mariner, Behaim, made a globe which is still preserved in Nuremberg. He did not know of the existence of the American continents or of the vast Pacific Ocean. . . . He places Japan (Cipango) where Mexico lies. In the reproduction many names are omitted, and the outlines of North and South America are sketched in.
— J. H. ROBINSON, *Mediæval and Modern Times.*

CHAPTER X

A NEW ASTRONOMY AND THE BEGINNINGS OF MODERN NATURAL SCIENCE

The breeze from the shores of Hellas cleared the heavy scholastic atmosphere. Scholasticism was succeeded by Humanism, by the acceptance of this world as a fair and goodly place given to man to enjoy and to make the best of. In Italy the reaction became so great that it seemed destined to put paganism once more in the place of Christianity; and though it produced lasting monuments in art and poetry, the earnestness was wanting which in Germany brought about the revival of science, and later on the rebellion against spiritual tyranny. . . . Astronomy profited more than any other science by this revival of learning, and about the middle of the fifteenth century the first of the long series of German astronomers arose who paved the way for Copernicus and Kepler, though not one of them deserves to be called a precursor of these heroes. — *Dreyer.*

The silent work of the great Regiomontanus in his chamber at Nuremberg computed the ephemerides which made possible the discovery of America by Columbus. — *Rudio.*

The extension of the geographical field of view over the whole earth and the release of thought and feeling from the restrictions of the Middle Ages mark a division of equal importance with the fall of the ancient world a thousand years earlier. — *Dannemann.*

Science begins to dawn, but only to dawn, when a Copernicus, and after him a Kepler or a Galileo, sets to work on these raw materials, and sifts from them their essence. She bursts into full daylight only when a Newton extracts the quintessence. There has been as yet but one Newton; there have not been very many Keplers. — *Tait.*

THE AGE OF DISCOVERY. — With the end of the fifteenth century and the beginning of the sixteenth opens one of the most marvellous chapters in all history; viz. *the Discovery of the New World.* At about the same time further explorations of the old world attained equal extent and interest. We have referred above (p. 174) to the *Discovery of the East* by the Crusaders, and now with Columbus, Magellan, and their successors, we have an even more pregnant *Discovery of the West.* Meanwhile, Diaz and da Gama pushed the explorations of Prince Henry of Portugal, "the Navigator," to the south, and in rounding the Cape of Good Hope completed the *Discovery of the South.* To the north, explorers had already advanced to regions of perpetual snow and ice, so that in all directions there were new problems of intense interest profoundly moving the imagination of mankind.

THE REFORMATION. — Another potent element was added to the already complex fermentation of medieval ideas when in 1517 a widespread insurrection began in the Christian Church, the most conservative and most powerful institution of the Middle Ages. This revolution, — for such it proved to be, — with which the name of Luther will always be chiefly associated, soon aroused a wave of determined opposition, naturally strongly conservative, known to-day as the "counter-reformation," of which the Inquisition was one instrument.

The increased importance of the art of navigation reacted powerfully on the underlying sciences of mathematics and astronomy, particularly through the demand for improved astronomical tables. The Church, even, had a strong, if restricted, interest in astronomy on account of the necessity of more accurate data for its calendar.

PIONEERS OF THE NEW ASTRONOMY. — Nicholas of Cusa (1401–1464), later Bishop of Brixen, wrote on Learned Ignorance, arguing that the universe, being infinite in extent, could have no centre, and that the earth has diurnal rotation. "It is now clear that the earth really moves, if we do not at once observe it, since we perceive motion only through comparison with something immovable." In mathematics he follows Euclid and Archimedes, coöperating in a translation of the latter from Greek into Latin, and dealing with the squaring of the circle.

He makes a map of the known world, using central projection. He is said to have determined areas of irregular boundary by the then novel method of cutting them out and weighing, and is one of the first to emphasize the importance of measurement in all investigations. He showed independence of thinking, but his astronomical theories were too little developed — and too speculative — to constitute real progress in an age not yet quite ripe for their reception.

Peurbach (1423–1461), who had as a youth met Nicholas of Cusa in Rome, became professor of astronomy and mathematics in Vienna and has been called "the founder of observational and mathematical astronomy in the West." Recognizing the imperfections of the Alfonsine tables he published a new edition of the Almagest with tables of natural sines — instead of chords — computed for every ten minutes. He depended mainly, however, on imperfect Arabic translations.

His more eminent pupil and successor, Johann Müller, of Königsberg, better known as Regiomontanus (1436–1476), was the most distinguished scientific man of his time. After the fall of Constantinople he was among the first to avail himself of the opportunities for more direct acquaintance with the works of Archimedes, Apollonius, and Diophantus. For the defective version of the Almagest which had come through Arabic channels he substituted the Greek original, while his tables, published in 1475, were important both for astronomy and for the voyages of discovery of Vasco da Gama, Vespucci, and Columbus. These tables covered the period 1473 to 1560, giving sines for each minute of arc, longitudes for sun and moon, latitude for the moon, and a list of predicted eclipses from 1475 to 1530. Another work on astrology includes a table of natural tangents for each degree. A wealthy merchant of Nuremberg erected an elaborately equipped observatory for Regiomontanus, and the printing-press recently established there became the most important in Germany. Accepting, however, a summons to Rome to reform the calendar, he was murdered at the age of 40.

His *De Triangulis* (1464) is the earliest modern trigonometry. Four of its five books are devoted to plane trigonometry, the other to spherical. He determines triangles from three given conditions, using sines and cosines, and employs quadratic equations successfully in some of his solutions. One of his problems is "to determine a triangle when the difference of two sides, the perpendicular on the base, and the difference between the segments into which the base is divided are given: *i.e.* $a - b$, $a \sin B$, $a \cos B - b \cos A$ are known; to find, a, b, c, A, B, C." Another is to construct from four given lines a quadrilateral which can be inscribed in a circle.

CONDITIONS NECESSARY FOR PROGRESS. — The genius of Hipparchus and Ptolemy had brought Greek astronomy to its culmination. Higher it could not rise until three conditions should be fulfilled, even though here and there the heliocentric hypothesis might be adopted through an unsupported inspiration of individuals. First, there must be better astronomical instruments and more accurate observations, extended over long periods. Second, there must be improved methods of mathematical computation for the reduction and interpretation of these observations. Third, there must be substantial progress towards clear thinking as to the fundamental facts and laws of motion. These conditions were met one after another during the sixteenth and seventeenth centuries by an extraordinary series of men of genius, among whom the chief were Copernicus, Tycho Brahe, Kepler, Galileo, and Newton. Their work constitutes a great part of the history of science during these two centuries — and one of the most wonderful chapters of all time.

Of these five, Copernicus and Kepler were predominantly interested on the mathematical and theoretical side, Tycho Brahe was a great observer, Galileo combined experimental and observational skill with a new appreciation of physical laws, while Newton, building on the foundation laid by all the others, made a magnificent synthesis of their results into a rational and consistent mathematical theory of the solar system. These five represent Poland, South Germany, Denmark, Italy, and England. Scientific progress is no longer localized or dependent on princely patronage. It has now become international.

NICOLAUS COPERNICUS (1473–1543) was born in the remote little city of Thorn on the Vistula, and having relatives in the Church, prepared himself for an ecclesiastical career. This led him, after medical study at Cracow, first to the university of Vienna, then to the chief Italian universities, Bologna, Padua, Ferrara, and Rome, where he found opportunity to cultivate his mathematical talents and to master what was then known of astronomy. He became canon at Frauenburg in his native land in 1497, and from 1512 until his death thirty years later, was settled there, rendering varied public services, and practising gratuitously, as needful, the medical art he had also learned. At the same time he found it possible to devote much attention to astronomical studies.

In his study of the classical writers he came upon a statement that certain Pythagorean philosophers explained the phenomena of the daily and yearly motions of the heavenly bodies by supposing the earth itself to rotate on its axis and to have also an orbital motion.

> 'Occasioned by this, I also began to think of a motion of the earth, and although the idea seemed absurd, still, as others before me had been permitted to assume certain circles in order to explain the motions of the stars, I believed it would readily be permitted me to try whether on the assumption of some motion of the earth better explanations of the revolutions of the heavenly spheres might not be found. And thus I have, assuming the motions which I in the following work attribute to the earth, after long and careful investigation, finally found that when the motions of the other planets are referred to the circulation of the earth and are computed for the revolution of each star, not only do the phenomena necessarily follow therefrom, but the order and magnitude of the stars and all their orbs and the heaven itself are so connected that in no part can anything be transposed without confusion to the rest and to the whole universe.' — *Dreyer.*
>
> 'I made every effort to read anew all the books of philosophers I could obtain, in order to ascertain if there were not some one of them of the opinion that other motions of the heavenly bodies existed than are assumed by those who teach mathematical sciences in the schools. So I found first in Cicero that Hicetas of Syracuse believed the earth moved. Afterwards I found also in Plutarch that others were likewise of this opinion. . . . Starting thence I began to reflect on the mobility of the earth.' — *Timerding.*

Copernicus was not a great observational astronomer. His instruments were poor, his eyesight not keen, his location unfavorable for clear skies. His recorded observations are few, chiefly of eclipses or oppositions of planets, and of no high degree of accuracy. His interest and genius lay rather in the direction of profound analysis and careful mathematical revision of the current geocentric theory, practically unchanged since its formulation by Ptolemy thirteen centuries earlier. Unfortunately the conditions of the time were adverse to the publication of so radical an innovation as a heliocentric theory of the solar system; nor was Copernicus ever greatly interested in any publication of his results, being both indifferent to reputation and averse to controversy.

> 'The scorn,' he says, 'which I had to fear in consequence of the novelty and seeming unreasonableness of my ideas, almost moved me to lay the completed work aside.'

Moreover, he realized the futility of publishing his revolutionary theories until he should have buttressed them with a planetary system so completely worked out that its superiority to the long-intrenched Ptolemaic system should be unquestionable — a herculean, if congenial labor. Nevertheless, he gradually formulated his astronomical system in manuscript, and about 1529 issued a *Commentariolus* giving an outline of his theory, which thus became gradually but vaguely known to scholars. Ten years later George Joachim — *Rheticus* — a young professor of mathematics from the Lutheran university of Wittenberg, visited Copernicus, eager to learn more of the new doctrine. The Lutheran church was not more hospitable than the Roman Catholic to scientific novelty and Luther himself called Copernicus a fool.

De Revolutionibus.—In 1540 appeared the *Prima Narratio* by Rheticus containing a considerable admixture of astrology, and in 1543 the immortal *De Revolutionibus Orbium Celestium,* a copy reaching Copernicus, it is said, on his death-bed. He begins with certain postulates: first, that the universe is spherical; second, that the earth is spherical; third, that the motions of the heavenly bodies are uniform circular motions or compounded of such motions. The slender basis for the first and third of these may be inferred from his statement in regard to certain hypothetical causes of want of uniformity: —

> Both of which things the intellect shrinks from with horror, it being unworthy to hold such a view about bodies which are constituted in the most perfect order.

He makes the relative character of the motions involved of fundamental importance. In his own words: —

> For all change in position which is seen is due to a motion either of the observer or of the thing looked at, or to changes in the position of both, provided that these are different. For when things are moved equally relatively to the same things, no motion is perceived, as between the object seen and the observer.

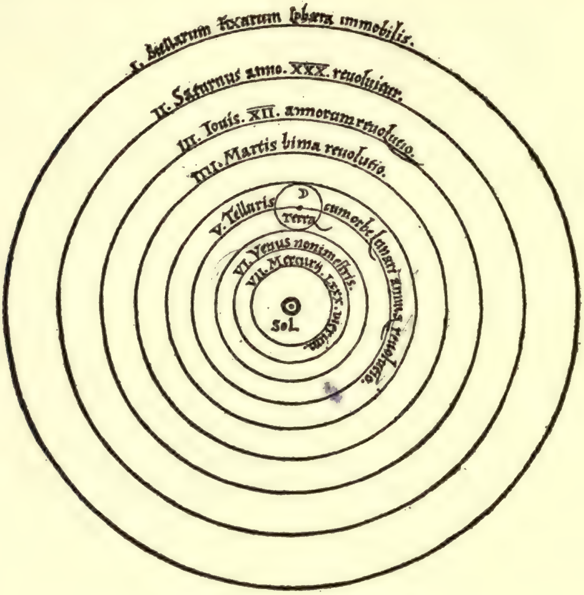
Thus the daily revolution of sun, moon, and stars about a stationary earth would have the same apparent effect as rotation of the earth in the opposite direction about its own axis, and the apparent yearly motion of the sun about the earth is equivalent to an orbital motion of the latter.

'It is,' he says, 'more probable that the earth turns about its axis than that the planets at their various distances, the comets sweeping through space, and the endless multitude of the fixed stars, describe the same regular daily motion about the earth.'

The apparent irregularities in the motions of the five known planets had been a perpetual stumbling-block to the ancient astronomers, requiring more and more complicated hypotheses for their explanations as accuracy of observations increased. The heliocentric theory of Copernicus, inaccurate as it was in some respects, afforded a simple explanation of the fact that Mercury and Venus seem merely to oscillate east and west of the sun, while Mars, Jupiter, and Saturn recede indefinitely from it, exhibiting also periodic reversals of the direction of their motion.

The new explanation obviously accounted also for the variations in the brightness of these planets.

'It is certain,' he says, 'that Saturn, Jupiter and Mars are always nearest the earth when they rise in the evening, that is when they are in opposition to the sun, as the earth is situated between them and the sun. On the contrary, Mars and Jupiter are farthest from the earth when they set in the evening, the sun lying between them and us. This proves sufficiently that the sun is the centre of their orbits, as of those of Venus and Mercury. Since thus all planets move about one centre it is necessary that the space which remains between the circles of Venus and Mars, contain the earth and its accompanying moon.'


I. Stellarum fixarum sphæra immobilis.
II. Saturnus anno. XXX. reuoluitur.
III. Iouis. XII. annorum reuolutio.
IIII. Martis bima reuolutio.
V. Telluris cum orbe lunari annua reuolutio.
Terra
VI. Venus nonimestris.
VII. Mercurij. LXXX. dierum.
Sol.

The Copernican System

He is, therefore, not afraid to maintain that the earth with the moon encircling it, traverses a great circle in its annual motion among the planets about the sun. The universe, however, is so vast, that the distances of the planets from the sun are insignificant in comparison with that of the sphere of the stars. He holds all this easier of comprehension, than if the mind is confused by an almost endless mass of circles, as is necessary for those who put the earth in the centre of the universe.

'So in fact the sun seated on the royal throne guides the family of planets encircling it. We find thus in this arrangement a harmonious connection not otherwise realized. For here one can see why the forward and backward motions of Jupiter seem greater than those of Saturn and smaller than those of Mars.'

His adherence to the Greek assumption of uniform circular motion leaves him still under the necessity of retaining an elaborate system of epicycles, but he rejects Ptolemy's equant.

> . . . He his fabric of the heavens
> Hath left to their disputes, perhaps to move
> His laughter at their quaint opinions wide;
> Hereafter when they come to model heaven
> And calculate the stars, how will they wield
> The mighty frame! how build, unbuild, contrive
> To save appearances! how gird the sphere
> With centric and eccentric scribbled o'er,
> Cycle in epicycle, orb in orb!
>
> — *Milton,* Paradise Lost, VIII.

The epicycles of Copernicus numbered however but 34,— sufficing "to explain the whole construction of the world and the whole dance of the planets" — against the 79 to which the Ptolemaic theory had gradually attained. The completeness of mathematical detail with which the whole theory is worked out can not here be adequately described. He includes so much trigonometry as his astronomical work requires, also a revision of Ptolemy's star catalogue. He computes a very accurate value of the equinoctial precession, and interprets this correctly as due to a slow conical motion of the earth's axis, like that of a top coming to rest.

Copernicus estimates the relative sizes of moon, earth and sun as 1 : 43 : 6937, and the distance from earth to sun — according to the method of Aristarchus — at about 1200 earth-radii, that is about $\frac{1}{20}$ of the actual.

Revolutionary as were the theories expounded by Copernicus they were not clothed in such popular form as to occasion immediate or general controversy. In dedicating his work to the Pope, Copernicus says in substance: —

> It seems to me that the church can derive some advantage from my labors. Under Leo X indeed the rectification of the calendar was not possible, since the length of the year and the motions of the sun and moon were not exactly determined. I have sought to determine these more closely. What I have accomplished, I leave to the judgment of your Holiness, and of the learned mathematicians. (See Appendix.)

Moreover criticism was in considerable measure disarmed by a fraudulent preface inserted by Osiander, a Lutheran theologian of Nuremberg, to whom the care of publication had been partially intrusted by Rheticus. In this preface, ostensibly by Copernicus himself, it is stated, —

> that though many will take offence at the doctrine of the earth's motion, it will be found on further consideration that the author does not deserve blame. For the object of an astronomer is to put together the history of the celestial motions from careful observations, and then to set forth their causes or hypotheses about them, if he cannot find the real causes, so that those motions can be computed on geometrical principles. But it is not necessary that his hypotheses should be true, they need not even be probable; it is sufficient if the calculations founded on them agree with the observations. Nobody would consider the epicycle of Venus probable, as the diameter of the planet in its perigee ought to be four times as great as in the apogee, which is contradicted by the experience of all times. Science simply does not know the cause of the apparently irregular motions, and an astronomer will prefer the hypothesis which is most easily understood. Let us therefore add the following new hypotheses to the old ones, as they are admirable and simple, but

nobody must expect certainty about astronomy, for it cannot give it; and whoever takes for truth what has been designed for a different purpose, will leave this science as a greater fool than he was when he approached it.

INFLUENCE OF COPERNICUS.—The publication of *De Revolutionibus* was naturally a powerful stimulus to astronomical and mathematical studies. Thus Rheticus, whose relations to Copernicus had been so fruitful, calculated a new and extensive set of mathematical tables, while Reinhold, who had hailed Copernicus as a new Ptolemy, published astronomical tables — the Prutenic or Prussian — on the basis of Copernicus' work, superior to the Alfonsine, previously current.

Before the new doctrine should be completely justified or the reverse, it was necessary that certain mechanical notions should be clarified, and that more accurate observational data should be systematically collected. Copernicus had based his imposing structure on a very slender foundation of actual fact, and had professed his complete satisfaction if his theoretical results should come within ten minutes of the observed positions of the planets, — a degree of accuracy which he did not, in fact, attain. On the other hand, he could indeed answer, but not rise entirely above, the traditional notions that the four elements of the ancients must have rectilinear, the heavenly bodies circular, motion; also, that if the earth rotated in twenty-four hours, loose bodies would long since have been thrown off, falling bodies would not fall, and clouds would always be left behind in the west.

As suggested by Dreyer: —

> It is interesting, though useless, to speculate on what would have been the chances of immediate success of the work of Copernicus if it had appeared fifty years earlier. Among the humanists there certainly was considerable freedom of thought, and they would not have been prejudiced against the new conception of the world because it upset the medieval notion of a set of planetary spheres inside the empyrean sphere, with places allotted for the hierarchy of angels. If one of the leaders of the Church (at least in Italy) at the beginning

of the sixteenth century had been asked whether the idea of the earth moving through space was not clearly heretical, he would probably merely have smiled at the innocence of the enquirer and have answered in the words of Pomponazzi that a thing might be true in philosophy and yet false in theology. But the times had changed. The sun of the Renaissance had set when, in 1527, the hordes of the Constable of Bourbon sacked and desecrated Rome; the Reformation had put an end to the religious and intellectual solidarity of the nations, and the contest between Rome and the Protestants absorbed the mental energy of Europe. During the second half of the sixteenth century science was therefore very little cultivated, and though astronomy and astrology attracted a fair number of students (among whom was one of the first rank), still theology was thought of first and last. And theology had come to mean the most literal acceptance of every word of Scripture; to the Protestants of necessity, since they denied the authority of Popes and Councils, to the Roman Catholics from a desire to define their doctrines more narrowly and to prove how unjustified had been the revolt against the Church of Rome. There was an end of all talk of Christian Renaissance and of all hope of reconciling faith and reason; a new spirit had arisen which claimed absolute control for Church authority. Neither side could therefore be expected to be very cordial to the new doctrine.

Robert Recorde, in his Pathway to Knowledge (1551), has his "Master" state to a "scholar":

'Eraclides Ponticus, a great philosopher, and two great clerkes of Pythagoras schole, Philolaus and Ecphantus, were of the contrary opinion, but also Nicias Syracusius and Aristarchus Samius seem with strong arguments to approve it.' After saying that the matter is too difficult and must be deferred till another time, the Master states that 'Copernicus, a man of great learning, of muche experience and of wondrefull diligence in obseruation, hathe renewed the opinion of Aristarchus Samius, and affirmeth that the earthe not only moueth circularlye about his own centre, but also may be, yea and is continually out of the precise centre 38 hundredth thousand miles; but bicause the vnderstanding of that controuersy dependeth of profounder knowledge than in this introduction may be vttered conueniently, I will let it passe tyll some other time.'

A little later Francis Bacon writes: —

'In the system of Copernicus there are many and grave difficulties; for the threefold motion with which he encumbers the earth is a serious inconvenience, and the separation of the sun from the planets, with which he has so many affections in common, is likewise a harsh step; and the introduction of so many immovable bodies into nature, as when he makes the sun and the stars immovable, the bodies which are peculiarly lucid and radiant, and his making the moon adhere to the earth in a sort of epicycle, and some other things which he assumes, are proceedings which mark a man who thinks nothing of introducing fictions of any kind into nature, provided his calculations turn out well.'

Bacon himself was very ignorant of all that had been done by mathematics; and, strange to say, he especially objected to astronomy being handed over to the mathematicians. Leverrier and Adams, calculating an unknown planet into a visible existence by enormous heaps of algebra, furnish the last comment of note on this specimen of the goodness of Bacon's view. . . . Mathematics was beginning to be the great instrument of exact inquiry; Bacon threw the science aside, from ignorance, just at the time when his enormous sagacity, applied to knowledge, would have made him see the part it was to play. If Newton had taken Bacon for his master, not he, but somebody else, would have been Newton. — *De Morgan.*

Copernicus cannot be said to have flooded with light the dark places of nature — in the way that one stupendous mind subsequently did — but still, as we look back through the long vista of the history of science, the dim Titanic figure of the old monk seems to rear itself out of the dull flats around it, pierces with its head the mists that overshadow them, and catches the first gleam of the rising sun, . . .

Like some iron peak, by the Creator
Fired with the red glow of the rushing morn.

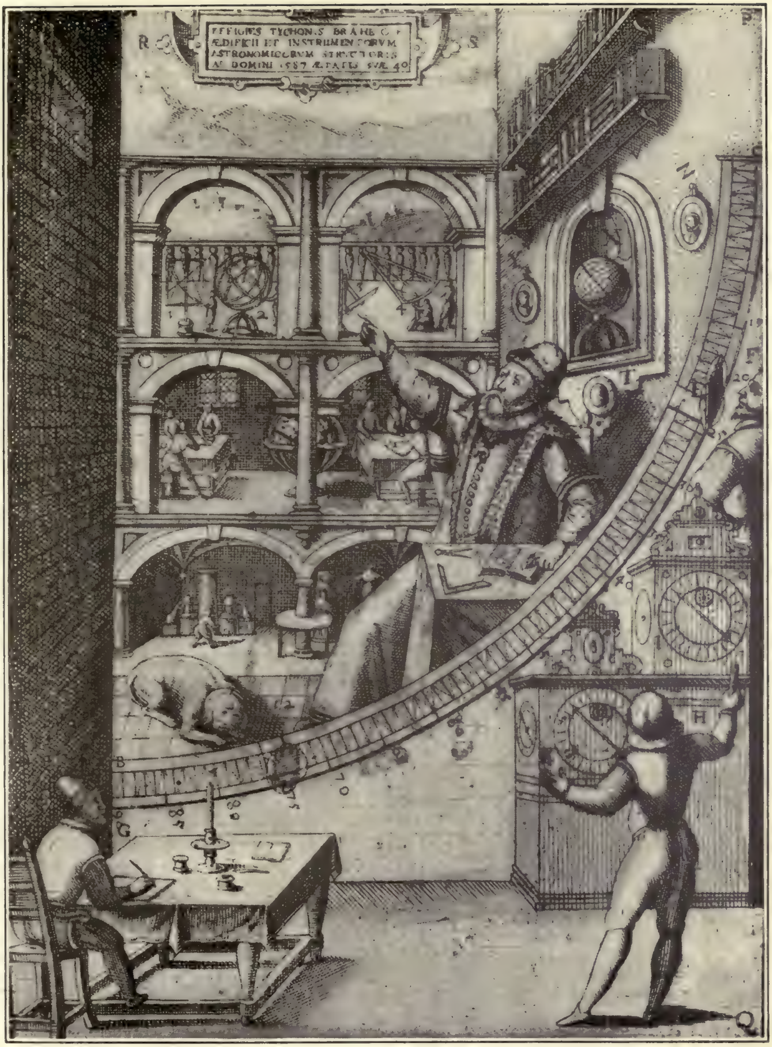
— *E. J. C. Morton.*

Tycho Brahe (1546–1601). — The first great need of the new Copernican astronomy — adequate and accurate data — was soon to be supplied by Tycho Brahe, born in 1546 of a noble Danish family. While a student at the University of Copenhagen his interest in astronomy was enlisted by an eclipse, and later, at Leipsic, he persisted in devoting to his new avocation the time and attention he was expected to give to subjects more highly esteemed for a man of birth and fortune.

From a lunar eclipse which took place while he was at Leipsic, Tycho foretold wet weather, which also turned out to be correct.

Here, too, he began his life work of procuring and improving the best instruments for astronomical observations, at the same time testing and correcting their errors. Returning to Denmark from travels in Germany, his predilection for astronomy was powerfully stimulated by the appearance in the constellation Cassiopeia, in November, 1572, of a brilliant new star, which remained visible for 16 months. The great importance attached to this occurrence by Tycho and his contemporaries was due to the evidence it afforded against the truth of the Aristotelian conviction that the heavens were immutable, since Tycho's careful observations showed that the star must certainly be more distant than the moon, and that it had no share in the planetary motions. He reluctantly published an account of the new star, expressing still his adherence to the current pre-Copernican notions of crystalline spheres for the different heavenly bodies and of atmospheric comets, all combined with astrological reflections and inferences, as illustrated by the following passages from Dreyer's biography: —

> The star was at first like Venus and Jupiter, and its effects will therefore first be pleasant; but as it then became like Mars, there will next come a period of wars, seditions, captivity, and death of princes and destruction of cities, together with dryness and fiery meteors in the air, pestilence, and venomous snakes. Lastly, the star became like Saturn, and there will therefore, finally, come a time of want, death, imprisonment, and all kinds of sad things.
>
> As the star seen by the wise men foretold the birth of Christ, the new one was generally supposed to announce His last coming and the end of the world.
>
> That an unusual celestial phenomenon occurring at that particular moment should have been considered as indicating troublous times, is extremely natural when we consider the state of Europe in 1573. The tremendous rebellion against the Papal supremacy, which for a long time had seemed destined to end in the complete overthrow of


EFFIGIES TYCHONIS BRAHE O. F.
ÆDIFICII ET INSTRUMENTORUM
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Tycho Brahe's Quadrant.

the latter, appeared now to have reached its limit, and many people thought that the tide had already commenced to turn.

Tycho considered that the new star was formed of 'celestial matter,' not differing from that of which the other stars are composed, except that it was not of such perfection or solid composition as in the stars of permanent duration. It was therefore gradually dissolved and dwindled away. It became visible to us because it was illuminated by the sun, and the matter of which it was formed was taken from the Milky Way, close to the edge of which the star was situated, and in which Tycho believed he could now see a gap or hole which had not been there before.

But the star had a truer mission than that of announcing the arrival of an impossible golden age. It roused to unwearied exertions a great astronomer, it caused him to renew astronomy in all its branches by showing the world how little it knew about the heavens; his work became the foundation on which Kepler and Newton built their glorious edifice, and the star of Cassiopeia started astronomical science on the brilliant career which it has pursued ever since, and swept away the mist that obscured the true system of the world. As Kepler truly said, 'If that star did nothing else, at least it announced and produced a great astronomer.'

At the same time the book bears witness to the soberness of mind which distinguishes him from most of the other writers on the subject of the star. His account of it is very short, but it says all there could be said about it — that it had no parallax, that it remained immovable in the same place, that it looked like an ordinary star — and it describes the star's place in the heavens accurately, and its variations in light and color. Even though Tycho made some remarks about the astrological significance of the star, he did so in a way which shows that he did not himself consider this the most valuable portion of his work. To appreciate his little book perfectly, it is desirable to glance at some of the other numerous books and pamphlets which were written about the star, and of most of which Tycho himself has in his later work given a very detailed analysis.

In 1575 Tycho obtained while travelling a copy of Copernicus' *Commentariolus,* and in the following year received from King Frederick II the island of Hveen, with funds for the maintenance of an observatory upon it. As to the former his opinion is that

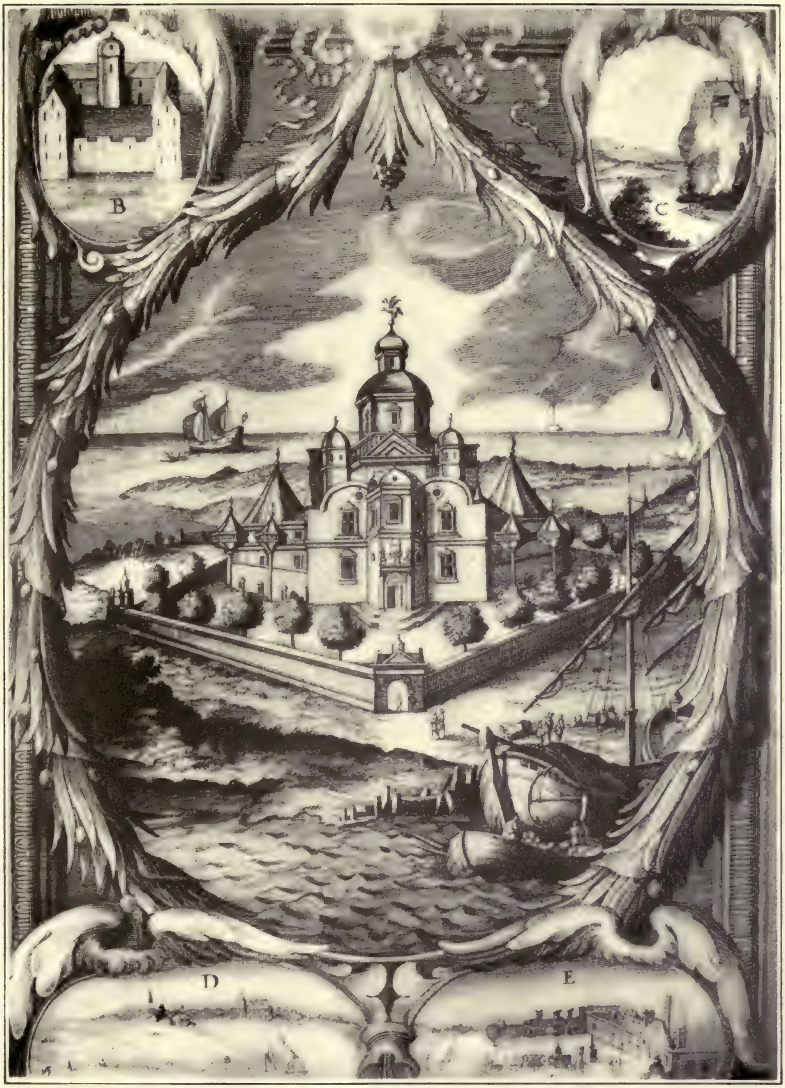
> 'The Ptolemean system was too complicated, and the new one which that great man Copernicus had proposed, following in the footsteps of Aristarchus of Samos, though there was nothing in it contrary to mathematical principles, was in opposition to those of physics, as the heavy and sluggish earth is unfit to move, and the system is even opposed to the authority of Scripture.'
>
> — *Dreyer,* Tycho Brahe.

URANIBORG. — The observatory of Uraniborg — the castle of the heavens — at Hveen was an extraordinary establishment.

In a large square inclosure oriented according to the points of the compass, were several observatories, a library, laboratory, living-rooms and, later, workshops, a paper-mill and printing-press, and even underground observatories. The whole establishment was administered with lavish extravagance, while Tycho was neither careful of his obligations nor free from arbitrary arrogance in his personal and administrative relations. In spite of these difficulties "a magnificent series of observations, far transcending in accuracy and extent anything that had been accomplished by his predecessors" was carried on for not less than 21 years. At the same time medicine and alchemy were also cultivated.

Concerned as he was to secure the greatest possible accuracy, Tycho constructed instruments of great size; for example, a wooden quadrant for outdoor use with a brass scale of some ten feet radius, permitting readings to fractions of a minute.

> The best artists in Augsburg, clockmakers, jewellers, smiths, and carpenters, were engaged to execute the work, and from the zeal which so noble an instrument inspired, the quadrant was completed in less than a month. Its size was so great that twenty men could with difficulty transport it to its place of fixture. The two principal rectangular radii were beams of oak; the arch which lay between their extremities was made of solid wood of a particular kind, and the whole was bound together by twelve beams. It received additional strength from several iron bands, and the arch was covered with plates of brass, for the purpose of receiving the 5400 divisions into which it was to be subdivided. A large and strong pillar of oak, shod with iron, was driven into the ground, and kept in its place by solid



URANIBORG.

mason work. To this pillar the quadrant was fixed in a vertical plane, and steps were prepared to elevate the observer, when stars of a low altitude required his attention. As the instrument could not be conveniently covered with a roof, it was protected from the weather by a covering made of skins; but notwithstanding this and other precautions, it was broken to pieces by a violent storm, after having remained uninjured for the space of five years. — *Brewster.*

A smaller but more serviceable azimuth quadrant of brass gave angles to the nearest minute. He had a copper globe constructed at great expense with the positions of some 1000 stars carefully marked upon it.

The very precision of his observations tended to confirm his scepticism of the Copernican hypothesis, as it seemed incredible that the earth's supposed orbital motion should cause no change which he could detect in the position and brightness of the stars. He was also misled by supposing that the stars had measurable angular magnitude. He was not successful in making any fundamental improvement in the relatively crude methods of time measurement, depending himself on wheel-mechanism without the regulating pendulum, and an apparatus of the sand-glass or clepsydra type.

In 1577 Tycho made observations on a brilliant comet, and drew from them important theoretical inferences; namely, that instead of being an atmospheric phenomenon, the comet was at least three times as remote as the moon, and that it was revolving about the sun at a greater distance than Venus — unimpeded by the familiar crystalline spheres. He was even led, in discussing apparent irregularities of its motion, to suggest that its orbit might be oval — foreshadowing one of Kepler's great discoveries.

According to the current view of his time, comets

were formed by the ascending from the earth of human sins and wickedness, formed into a kind of gas, and ignited by the anger of God. This poisonous stuff falls down again on people's heads, and causes all kinds of mischief, such as pestilence, Frenchmen (!), sudden death, bad weather, etc. — *Dreyer,* Tycho Brahe.

Eleven years later Tycho published a volume on the comet as a part of a comprehensive astronomical treatise which was, however, never completed. About the same time his royal patron died, and the new administration proved less sympathetic with the great astronomer's work and less indulgent with his extravagance and personal eccentricities.

After a series of disagreements, Tycho withdrew from his observatory in 1597, spent the winter in Hamburg, and after negotiations with different sovereigns, accepted the invitation of the Emperor Rudolph to settle in Prague in 1599. Here he again organized a staff of assistants, including, to the great advantage of himself and of his science, the young Kepler, but his further progress was prematurely terminated by death in 1601, at the age of 55.

Tycho's chief services to the progress of astronomy consisted first, in the superior accuracy of his instruments and observations, heightened by repetition and systematic correction of errors; second, in the extension of these observations over a long series of years. In both respects he departed from current practice, and anticipated the modern. In point of accuracy his errors of star-places seem rarely to have exceeded 1′ to 2′, and he even determined the length of the year within one second. While he recomputed almost every important astronomical constant, he accepted the traditional distance of the sun.

Kepler gave striking evidence later of his confidence in Tycho's accuracy by writing: —

> 'Since the divine goodness has given to us in Tycho Brahe a most careful observer, from whose observations the error of 8′ is shewn in this calculation, . . . it is right that we should with gratitude recognize and make use of this gift of God. . . . For if I could have treated 8′ of longitude as negligible I should have already corrected sufficiently the hypothesis . . . discovered in chapter xvi. But as they could not be neglected, these 8′ alone have led the way towards the complete reformation of astronomy, and have made the subject-matter of a great part of this work.' — *Berry.*

On the other hand, Tycho was not strong on the theoretical side. He was never willing to accept the Copernican hypothesis of rotation and orbital motion of the earth — maintaining, for example, that if the earth moved, a stone dropped from the top of a tower must fall at a distance from the foot. Again with reference to the apparent displacement of the stars which would be expected to result from orbital motion of the earth, he says: —

> A yearly motion would relegate the sphere of the fixed stars to such a distance that the path described by the earth must be insignificant in comparison. Dost thou hold it possible that the space between the sun, the alleged centre of the universe, and Saturn amounts to not even $\frac{1}{700}$ of that distance? At the same time this space must be void of stars.

Sensible, however, of the weakness of the Ptolemaic theory, he devised an ingenious compromise in which the planets revolved about the Sun in their respective periods, and the entire heavens about the earth daily — all of which is not mathematically different from the Copernican theory.

We see in him at the same time a perfect son of the sixteenth century, believing the universe to be woven together by mysterious connecting threads which the contemplation of the stars or of the elements of nature might unravel, and thereby lift the veil of the future; we see that he is still, like most of his contemporaries, a believer in the solid spheres and the atmospherical origin of comets, to which errors of the Aristotelean physics he was destined a few years later to give the death-blow by his researches on comets; we see him also thoroughly discontented with his surroundings, and looking abroad in the hope of finding somewhere else the place and the means for carrying out his plans.

As a practical astronomer Tycho has not been surpassed by any observer of ancient or modern times. The splendor and number of his instruments, the ingenuity which he exhibited in inventing new ones and in improving and adding to those which were formerly known, and his skill and assiduity as an observer, have given a character to his labors and a value to his observations which will be appreciated to the latest posterity. — *Brewster.*

Kepler. — Pierre de la Ramée, or Petrus Ramus, a French mathematician and philosopher, impatient with the cumbrous astronomical hypotheses of the ancients, and unsatisfied with Copernicus' proposed simplification, published a work in 1569 expressing the hope

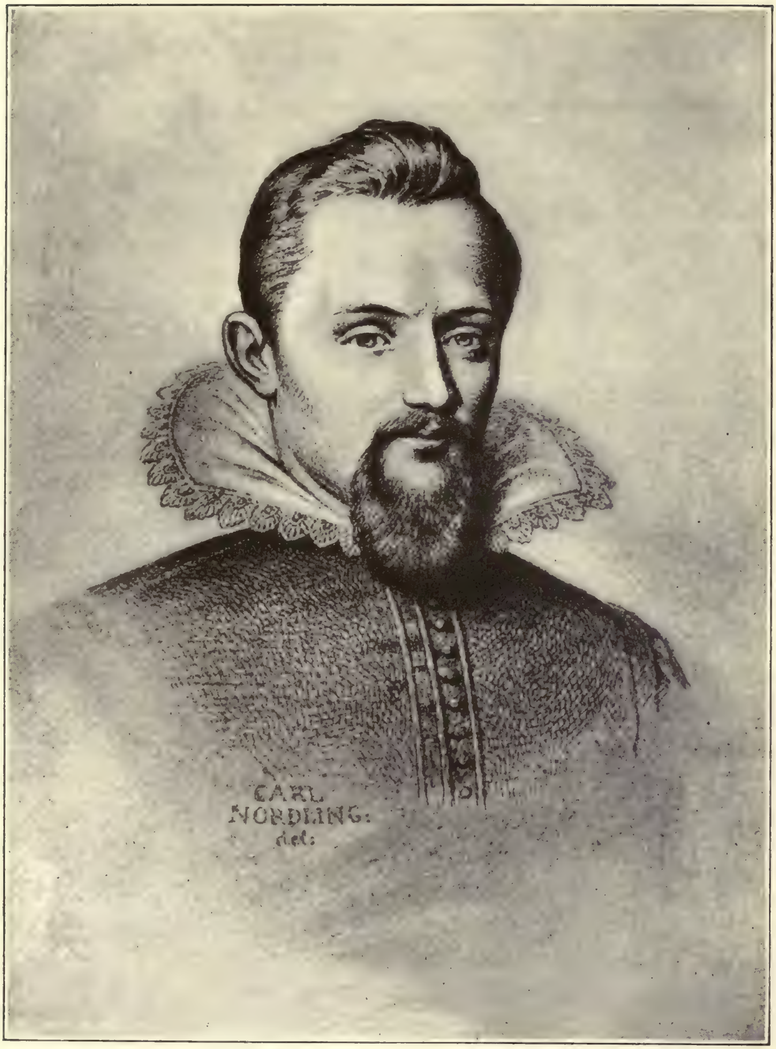
> 'that some distinguished German philosopher would arise and found a new astronomy on careful observations by means of logic and mathematics, discarding all the notions of the ancients.'

Within a few months he discussed the matter at length with Tycho Brahe at Augsburg. Without accepting Ramus' views, the young astronomer did make it his life work to lay the necessary foundation for such a new astronomy. Thirty years later, Mästlin, professor at Tübingen, wrote his former student Kepler — then aged 28 —

> that Tycho 'had hardly left a shadow of what had hitherto been taken for astronomical science, and that only one thing was certain, which was that mankind knew nothing of astronomical matters.'

Born late in 1571 in Würtemberg, of Protestant parents in very straitened circumstances, Johann Kepler's whole life was a struggle against poverty, ill-health, and adverse conditions. In 1594, abandoning with some hesitation theological studies, for which his acceptance of the new Copernican hypothesis disqualified him, he was appointed lecturer on mathematics at Gratz. Students were few, and his duties included the preparation of a yearly almanac, containing, besides what its name implies, a variety of weather predictions and astrological information. "Mother Astronomy," he says, "would surely have to suffer hunger if the daughter Astrology did not earn their bread."

Becoming thus more interested in astronomy, "there were," he says, "three things in particular: viz., the number, the size, and the motion of the heavenly bodies, as to which I searched zealously for reasons why they were as they were and not otherwise." The first result which seemed to him important, though somewhat fantastic from our standpoint, was a crude correspondence be-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Kepler (*Opera omnia*).

tween the planetary orbits and the five regular solids, published in 1596 under a title which may be abridged to Cosmographic Mystery.

> The Earth is the circle, the measure of all. Round it describe a dodecahedron, the circle including this will be Mars. Round Mars describe a tetrahedron, the circle including this will be Jupiter. Describe a cube round Jupiter, the circle including this will be Saturn. Then inscribe in the Earth an icosahedron, the circle inscribed in it will be Venus. Inscribe an octahedron in Venus, the circle inscribed in it will be Mercury.

Kepler declared that he would not renounce the glory of this discovery "for the whole Electorate of Saxony." The correspondence of the dimensions of this fantastic geometrical construction with the distances of members of our solar system is in reality far from close, but both Tycho Brahe and Galileo seem to have been favorably impressed by the book.

The difficulties of Kepler's position as a Protestant in Gratz led him, after a preliminary visit, to accept an engagement as Tycho's assistant at Prague.

> The powers of original genius were then for the first time associated with inventive skill and patient observation, and though the astronomical data provided by Tycho were sure of finding their application in some future age, yet without them, Kepler's speculations would have been vain and the laws which they enabled him to determine would have adorned the history of another century. — *Brewster.*

In 1602 Kepler succeeded Tycho as imperial mathematician. Most fortunately, also, he secured possession of his chief's great collection of observations, though not of the instruments, — a matter of less consequence, since Kepler like Copernicus was a mathematician rather than an observer. To the study of these records he devoted the next 25 years. Among all the planetary observations of Tycho Brahe those of Mars presented the irregularities most difficult of explanation, and it was these which, having been originally assigned to Kepler, engrossed his attention for many years, and in the end led to some of his finest discoveries.

The Copernican theory like the Ptolemaic involved the resolution of the motion of each planet into a main circular motion, modified by superimposing other circular motions — epicycles — successively upon it, each circle being the path of the centre of the next. Even after disentangling the essential irregularities of Mars' orbit from those merely due to irregular motion of the earth, he could still obtain no satisfactory agreement with Tycho's records, of which, as has been said, he refused to doubt the accuracy. Taking advantage of his own failure — as happens to men of true genius — he abandoned the restriction of circular motions, and experimented with other closed curves, of which the ellipse is simplest. Taking the sun at a focus, the problem was at last solved, theory and observation reconciled within due limits of error. At the same time uniform motion was naturally abandoned, for with a non-circular orbit, it was evident that the planet could not describe both equal distances and equal areas in equal times. Here, again, Kepler's scientific imagination led him to the great discovery that the planet traverses its orbit in such a manner that a line joining it to the sun would describe sectors of equal area in equal times, the planet thus moving fastest when nearest the sun.

Of Kepler's celebrated three laws, the first two are:

The planet describes an ellipse, the sun being in one focus.

The straight line joining the planet to the sun sweeps out equal areas in equal intervals of time.

These results were published in 1609 as part of extended Commentaries on the Motions of Mars.

The great problem was solved at last, the problem which had baffled the genius of Eudoxus and had been a stumbling-block to the Alexandrian astronomers, to such an extent that Pliny had called Mars the *inobservabile sidus*. The numerous observations made by Tycho Brahe, with a degree of accuracy never before attained, had in the skilful hand of Kepler revealed the unexpected fact that Mars describes an ellipse, in one of the foci of which the sun is situated, and that the radius vector of the planet sweeps over equal areas in equal times. And the genius and astounding patience of Kepler had proved that not only did this new theory satisfy the observations, but that no other hypothesis could be made to agree with the observations, as every proposed alternative left outstanding errors, such as it was impossible to ascribe to errors of observation. Kepler had therefore, unlike all his predecessors, not merely put forward a new hypothesis which might do as well as another to enable a computer to construct tables of the planet's motion; he had found the actual orbit in which the planet travels through space.

In the history of astronomy there are only two other works of equal importance, the book *De Revolutionibus* of Copernicus and the *Principia* of Newton. The 'astronomy without hypothesis' demanded by Ramus had at last been produced, and well might Kepler proclaim:

'It is well, Ramus, that you have run from this pledge, by quitting life and your professorship; if you held it still, I should, with justice, claim it.'

Resuming later the tendency of his Cosmographic Mystery, he published in 1619 his Harmony of the World, containing his third law: —

The squares of the times of revolution of any two planets (including the earth) about the sun are proportional to the cubes of their mean distances from the sun.

In his delight he exclaims 'Nothing holds me, I will indulge in my sacred fury; I will triumph over mankind by the honest confession that I have stolen the golden vases of the Egyptians to build up a tabernacle for my God, far away from the confines of Egypt.' —

'What sixteen years ago, I urged as a thing to be sought, that for which I joined Tycho Brahe, for which I settled in Prague, for which I have devoted the best part of my life to astronomical contemplations — at length I have brought to light, and recognized its truth beyond my most sanguine expectations. It is not eighteen months since I got the first glimpse of light, three months since the dawn, very few days since the unveiled sun, most admirable to gaze on, burst out upon me.' . . .

Archimedes of old had said "Give me a place to stand on, and I shall move the world." Tycho Brahe had given Kepler the place to stand on, and Kepler did move the world.

It should be borne in mind that Kepler's results depend not on *a priori* theory for their confirmation, but upon actual observations supporting them and interpreted by them. The great further step of showing that the three laws are not independent and empirical, but mathematical consequences of a single mechanical law still awaited the genius of Newton.

Kepler's notions in regard to force and motion are still crude. Thus, for example, having in mind an analogy with magnetism, Kepler says in his Epitome of the Copernican Astronomy, (1618–1621): —

> 'There is therefore a conflict between the carrying power of the sun and the impotence or material sluggishness (inertia) of the planet; each enjoys some measure of victory, for the former moves the planet from its position and the latter frees the planet's body to some extent from the bonds in which it is thus held . . . but only to be captured again by another portion of this rotatory virtue.'

Elsewhere he says: —

> 'We must suppose one of two things: either that the moving spirits, in proportion as they are more removed from the sun, are more feeble; or that there is one moving spirit in the centre of all the orbits, namely, in the sun, which urges each body the more vehemently in proportion as it is nearer; but in more distant spaces languishes in consequence of the remoteness and attenuation of its virtue.'
>
> — *Whewell.*

He recognized the necessity of a force exercised by the sun, but believed it inversely proportional to the distance instead of to the square of the distance. His notions of gravity are expressed in his book on Mars: —

> 'Every bodily substance will rest in any place in which it is placed isolated, outside the reach of the power of a body of the same kind. Gravity is the mutual tendency of cognate bodies to join each other (of which kind the magnetic force is), so that the earth draws a stone much more than the stone draws the earth. Supposing that the earth were in the centre of the world, heavy bodies would not seek the centre of the world as such, but the centre of a round, cognate

body, the earth; and wherever the earth is transported heavy bodies will always seek it; but if the earth were not round they would not from all sides seek the middle of it, but would from different sides be carried to different points. If two stones were situated anywhere in space near each other, but outside the reach of a third cognate body, they would after the manner of two magnetic bodies come together at an intermediate point, each approaching the other in proportion to the attracting mass. And if the earth and the moon were not kept in their orbits by their animal force, the earth would ascend towards the moon one fifty-fourth part of the distance, while the moon would descend the rest of the way and join the earth, provided that the two bodies are of the same density. If the earth ceased to attract the water all the seas would rise and flow over the moon. — *Dreyer.*

Kepler's last important published work was his Rudolphine Tables (1627), embodying the accumulated results of Tycho's work and his own, and remaining a standard for a century. It is noteworthy that during Kepler's work on these tables, mathematical computation was peacefully revolutionized by the introduction of logarithms, newly discovered by Napier and Bürgi.

In 1628, after vain attempts to collect arrears of his salary as imperial mathematician, he even joined Wallenstein as astrologer, but died soon after at Regensburg in 1630.

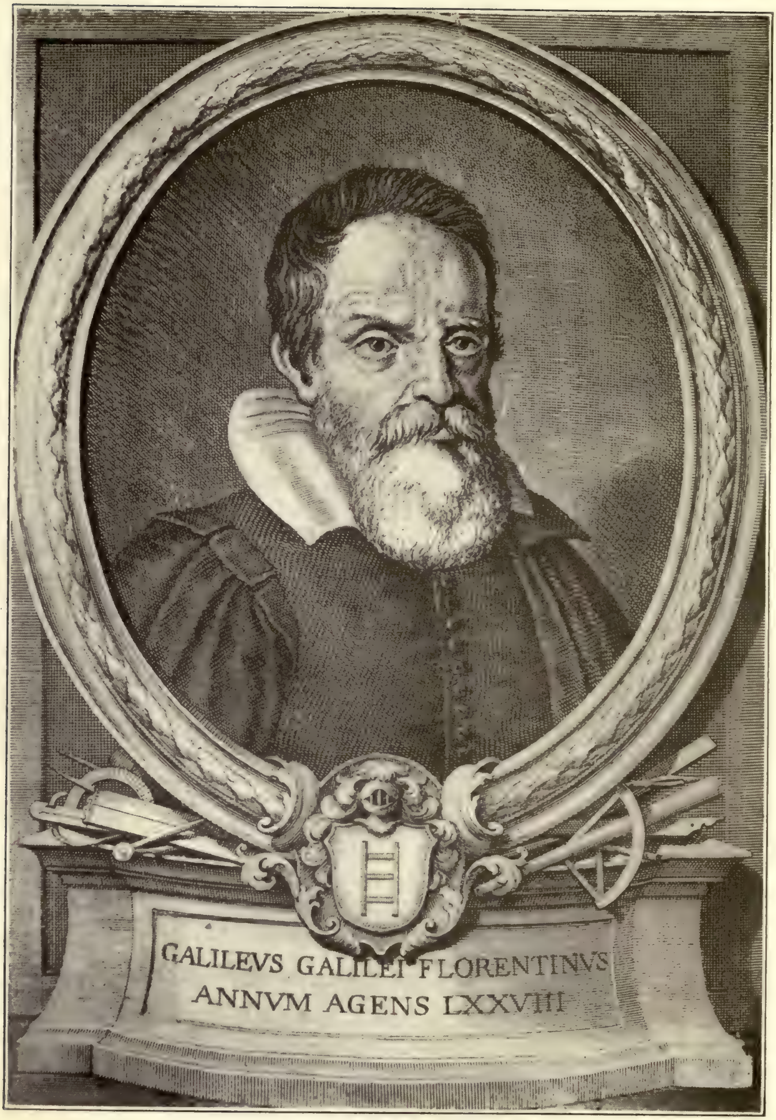
Kepler also wrote an important work on Dioptrics with a mathematical discussion of refraction and the different forms of the newly invented telescope, the whole constituting the foundation of modern optics. In it he develops the first correct theory of vision, "Seeing amounts to feeling the stimulation of the retina, which is painted with the colored rays of the visible world. The picture must then be transmitted to the brain by a mental current, and delivered at the seat of the visual faculty." He supposes that color depends on density and transparency, and that refraction is due to greater resistance of a dense medium. He enunciates the law that intensity of light varies inversely as the square of the distance. "In proportion as the spherical surface from whose centre the light proceeds is greater or smaller, so is the strength or density of the light-rays which fall on the smaller sphere to the strength of those rays which fall on the larger sphere." He explains the estimation of distance by binocular vision. He supposes the velocity of light to be infinite. His more purely mathematical work will be mentioned in a later chapter.

Kepler added Plato's boldness of fancy to his own patient and candid habit of testing his fancies by a rigorous and laborious comparison with the phenomena; and thus his discoveries led to those of Newton. — *Whewell.*

If Kepler had burnt three-quarters of what he printed, we should in all probability have formed a higher opinion of his intellectual grasp and sobriety of judgment, but we should have lost to a great extent the impression of extraordinary enthusiasm and industry, and of almost unequalled intellectual honesty, which we now get from a study of his works.— *Berry.*

Kepler says: 'If Christopher Columbus, if Magellan, if the Portuguese, when they narrate their wanderings, are not only excused, but if we do not wish these passages omitted, and should lose much pleasure if they were, let no one blame me for doing the same.' Kepler's talents were a kindly and fertile soil, which he cultivated with abundant toil and vigor, but with great scantiness of agricultural skill and implements. Weeds and the grain throve and flourished side by side almost undistinguished; and he gave a peculiar appearance to his harvest, by gathering and preserving the one class of plants with as much care and diligence as the other. — *Whewell.*

Endowed with two qualities, which seemed incompatible with each other, a volcanic imagination and a pertinacity of intellect which the most tedious numerical calculations could not daunt, Kepler conjectured that the movements of the celestial bodies must be connected together by simple laws, or, to use his own expression, by harmonic laws. These laws he undertook to discover. A thousand fruitless attempts, errors of calculation inseparable from a colossal undertaking, did not prevent him a single instant from advancing resolutely toward the goal of which he imagined he had obtained a glimpse. Twenty-two years were employed by him in this investigation, and still he was not weary of it! What, in reality, are twenty-two years of labor to him who is about to become the legislator of worlds; who shall inscribe his name in ineffaceable characters upon


GALILEVS GALILEI FLORENTINVS
ANNVM AGENS LXXVIII

Galileo (*Opere*, 1744).

the frontispiece of an immortal code; who shall be able to exclaim in dithyrambic language, and without incurring the reproach of anyone, 'The die is cast; I have written my book; it will be read either in the present age or by posterity, it matters not which; it may well await a reader, since God has waited six thousand years for an interpreter of his words.' — *Arago.*

The philosophical significance of Kepler's discoveries was not recognized by the ecclesiastical party at first. It is chiefly this, that they constitute a most important step to the establishment of the doctrine of the government of the world by law. But it was impossible to receive these laws without seeking for their cause. The result to which that search eventually conducted not only explained their origin, but also showed that, as laws, they must, in the necessity of nature, exist. It may be truly said that the mathematical exposition of their origin constitutes the most splendid monument of the intellectual power of man. — *Draper.*

GALILEO. — Columbus discovered America when Copernicus was but 19, and before the birth of Tycho Brahe, Magellan had completed the proof of the earth's rotundity by actually sailing around it, while Luther had stirred up the great religious revolt of Protestantism. The later years of Kepler and Galileo fell within the period of the Thirty Years' War, of which neither was to witness the close. Permanent English settlements in America had just begun. Galileo (1564–1642), born on the day of Michael Angelo's death, "nature seeming to signify thereby the passing of the sceptre from art to science," and in the same year with Shakespeare, exerted a mighty influence on the development of science in many fields, and in particular laid the foundations of modern dynamics.

It is a remarkable circumstance in the history of science that astronomy should have been cultivated at the same time by three such distinguished men as Tycho, Kepler and Galileo. While Tycho in the 54th year of his age was observing the heavens at Prague, Kepler, only 30 years old, was applying his wild genius to the determination of the orbit of Mars, and Galileo, at the age of 36, was about to direct the telescope to the unexplored regions of space. The diversity of gifts which Providence assigned to these three philosophers was no less remarkable. Tycho was destined to lay the foundation of modern astronomy by a vast series of accurate observations made with the largest and the finest instruments; it was the proud lot of Kepler to deduce the laws of the planetary orbits from the observations of his predecessors; while Galileo enjoyed the more dazzling honor of discovering by the telescope new celestial bodies and new systems of worlds. — *Brewster.*

Coming into a world still dominated by the Aristotelian tradition, Galileo is puzzled by the conflict between his own observations and the accepted theories, but firm and fearless in his convictions, he eagerly and powerfully controverts the older notions, incidentally gaining enemies as well as disciples. What those accepted theories were may be exemplified by the following passages from a work of Daniel Schwenter (1585–1636), professor of mathematics at Altdorf:—

'When a body falls it moves faster the nearer it approaches the earth. The farther it falls the more power it possesses. For everything which is heavy, hastens according to the opinion of philosophers towards its natural place, that is the centre of the earth, just as man returning to his fatherland becomes the more eager the nearer he comes, and therefore hastens so much the more. Still another natural cause contributes to this. The air which is parted by the falling ball, hastens together again behind the ball and drives it always harder.'

If the Copernican theory were true, the bullet remaining two minutes in the air would be left many miles behind by the revolving earth, — a distance which the moving atmosphere could not possibly carry it. The rainbow is "a mirror in which the human understanding can behold its ignorance in broad day." The powder drives the bullet in an oblique line to the highest point of its path, then follows motion in an arc, finally, the natural motion vertically downward.

In his whole point of view and habit of mind Galileo embodied the attitude and spirit of modern science. He was keenly alert in observing, analyzing, and reflecting on natural phenomena, eager and convincing in his expositions, sceptical and intolerant of mere authority, whether in science, philosophy, or theology. It was a true instinct of the conservatives to recognize in him the champion of a principle fatally hostile to their own. Between these antagonistic principles no permanent peace was possible.

While still a mere youth, he discovered the regularity of pendulum vibrations by observing the slow swinging of the cathedral lamp of Pisa (1582). Before he was 25 he published work on the hydrostatic balance (1586), and on the centre of gravity of solids. Only a little later he conducted at the leaning tower simple experiments in falling bodies, which upset world-old notions on this everyday matter, showing that the velocity of descent is not, as was commonly supposed, proportional to weight. And "yet the Aristotelians, who with their own eyes saw the unequal weights strike the ground at the same instant, ascribed the effect to some unknown cause, and preferred the decision of their master to that of nature herself."

He further showed that the hypothesis of uniform acceleration accounted correctly for the observed relations between space, time and velocity, and that the path of a projectile is a parabola. In the words of a recent authority, when Galileo

> deduced by experiment, and described with mathematical precision, the acceleration of a falling body, he probably contributed more to the physical sciences than all the philosophers who had preceded him.

Hearing of the telescope newly invented in Holland, he constructed one for himself, by means of which he discovered sun spots, the mountains of the moon, the satellites of Jupiter, the rings of Saturn, and the phases of Venus. The sensation created by these discoveries is described in the following passages from Fahie's Life of Galileo and Brewster's Martyrs of Science.

> 'As the news had reached Venice that I had made such an instrument, six days ago I was summoned before their Highnesses, the Signoria, and exhibited it to them, to the astonishment of the whole senate. Many of the nobles and senators, although of a great age, mounted more than once to the top of the highest church tower in

Venice, in order to see sails and shipping that were so far off that it was two hours before they were seen, without my spy-glass, steering full sail into the harbour; for the effect of my instrument is such that it makes an object 50 miles off appear as large as if it were only five.'

'But the greatest marvel of all is the discovery of four new planets. I have observed their motions proper to themselves and in relation to each other, and wherein they differ from the motions of the other planets. These new bodies move round another very great star, in the same way as Mercury and Venus, and, peradventure, the other known planets, move round the sun. As soon as my tract is printed, which I intend sending as an advertisement to all philosophers and mathematicians, I shall send a copy to his Highness, the Grand Duke, together with an excellent spy-glass, which will enable him to judge for himself of the truth of these novelties.' — *Fahie.*

Galileo's discoveries on the surface of the moon were ill received by the followers of Aristotle. According to their preconceived opinions, the moon was perfectly spherical and absolutely smooth; and to cover it with mountains and scoop it out into valleys was an act of impiety which defaced the regular forms which Nature herself had imprinted. It was in vain that Galileo appealed to the evidence of observation and to the actual surface of our own globe. The very irregularities on the moon were, in his opinion, a proof of divine wisdom; and had its surface been absolutely smooth, it would have been 'but a vast unblessed desert, void of animals, of plants, of cities, and men — the abode of silence and inaction — senseless, lifeless, soulless, and stripped of all those ornaments which now render it so varied and so beautiful.'

In examining the fixed stars and comparing them with the planets, Galileo observed a remarkable difference in the appearance of their discs. All the planets appeared with round globular discs like the moon; whereas the fixed stars never exhibited any disc at all but resembled lucid points sending forth twinkling rays. Stars of all magnitudes he found to have the same appearance; those of the fifth and sixth magnitude having the same character, when seen through a telescope, as Sirius, the largest of the stars, when seen by the naked eye.

Important and interesting as these discoveries were, they were thrown into the shade by those to which he was led during a careful examination of the planets with a more powerful telescope. On the 7th of January, 1610, at one o'clock in the morning, when he directed his telescope to Jupiter, he observed three stars near the body of the planet, two being to the east and one to the west of him. They were all in a straight line, and parallel to the ecliptic and appeared brighter than other stars of the same magnitude. Believing them to be fixed stars, he paid no great attention to their distances from Jupiter and from one another. On the 8th of January, however, when, from some cause or other, he had been led to observe the stars again, he found a very different arrangement of them; all the three were on the west side of Jupiter, nearer one another than before and almost at equal distances. Though he had not turned his attention to the extraordinary fact of the mutual approach of the stars, yet he began to consider how Jupiter could be found to the east of the three stars, when but the day before he had been to the west of two of them. The only explanation which he could give of this fact was that the motion of Jupiter was direct, contrary to the astronomical calculations and that he had got before these two stars by his own motion.

In this dilemma between the testimony of his senses and the results of calculation, he waited for the following night with the utmost anxiety, but his hopes were disappointed, for the heavens were wholly veiled in clouds. On the 10th, two only of the stars appeared, and both on the east side of the planet. As it was obviously impossible that Jupiter could have advanced from west to east on the 8th of January, and from east to west on the 10th, Galileo was forced to conclude that the phenomenon which he had observed arose from the motion of the stars, and he set himself to observe diligently their change of place. On the 11th there were still only two stars, and both to the east of Jupiter, but the more eastern star was now twice as large as the other one, though on the preceding night they had been perfectly equal. This fact threw a new light upon Galileo's difficulties, and he immediately drew the conclusion, which he considered to be indubitable, 'that there were in the heavens three stars which revolve around Jupiter, in the same manner as Venus and Mercury revolve around the sun.' On the 12th of January he again observed them in new positions, and of different magnitudes; and on the 13th he discovered a fourth star, which completed the four secondary planets with which Jupiter is surrounded. — *Brewster.*

His results were published in 'The Sidereal Messenger,' announcing 'great and very wonderful spectacles, and offering them to the consideration of every one, but especially of philosophers and astronomers; which have been observed by Galileo Galilei . . . by the assistance of a perspective glass lately invented by him; namely in the face of the moon, in innumerable fixed stars in the Milky Way, in nebulous stars, but especially in four planets which revolve around Jupiter at different intervals and periods with a wonderful celerity; which, hitherto not known to any one, the author has recently been the first to detect, and has decreed to call the Medicean stars.'

— *Whewell.*

The reception which these discoveries met with from Kepler is highly interesting, and characteristic of the genius of that great man. He was one day sitting idle and thinking of Galileo, when his friend Wachenfels stopped his carriage at his door to communicate to him some intelligence. 'Such a fit of wonder,' says he, 'seized me at a report which seemed to be so very absurd, and I was thrown into such agitation at seeing an old dispute between us decided in this way, that between his joy, my coloring, and the laughter of both, confounded as we were by such a novelty, we were hardly capable, he of speaking, or I of listening. On our parting, I immediately began to think how there could be any addition to the number of the planets without overturning my Cosmographic Mystery, according to which Euclid's five regular solids do not allow more than six planets round the sun. . . I am so far from disbelieving the existence of the four circumjovial planets, that I long for a telescope, to anticipate you, if possible, in discovering two round Mars, as the proportion seems to require, six or eight round Saturn, and perhaps one each round Mercury and Venus.'

In a very different spirit did the Aristotelians receive the Sidereal Messenger of Galileo. The principal professor of philosophy at Padua resisted Galileo's repeated and urgent entreaties to look at the moon and planets through his telescope; and he even labored to convince the Grand Duke that the satellites of Jupiter could not possibly exist.[1]

'There are seven windows given to animals in the domicile of the head, through which the air is admitted to the tabernacle of the body, to enlighten, to warm, and to nourish it. What are these parts of the microcosmos? Two nostrils, two eyes, two ears, and a mouth. So in the heavens, as in a macrocosmos, there are two favorable stars, two unpropitious, two luminaries, and Mercury undecided and indifferent. From this and many other similarities in nature, such as the seven metals, etc. which it were tedious to enumerate, we gather that the number of planets is necessarily seven. Moreover, these satellites of Jupiter are invisible to the naked eye, and therefore can exercise no influence on the earth, and therefore would be useless, and therefore do not exist. Besides, the Jews and other ancient nations, as well as modern Europeans, have adopted the division of the week into seven days, and have named them after the seven planets. Now, if we increase the number of the planets, this whole and beautiful system falls to the ground.' — *Fahie.*

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[1] 'As I wished to show the satellites of Jupiter to the professors in Florence, they would neither see them nor the telescope. These people believe there is no truth to seek in nature, but only in the comparison of texts.'

It was inevitable that such a man as Galileo should accept the Copernican hypothesis. He writes to Kepler in 1597: —

'I esteem myself fortunate to have found so great an ally in the search for truth. It is truly lamentable, that there are so few who strive for the true and are ready to turn away from wrong ways of philosophizing. But here is no place for bewailing the pitifulness of our times, instead of wishing you success in your splendid investigations. I do this the more gladly, since I have been for many years an adherent of the Copernican theory. It explains to me the cause of many phenomena which under the generally accepted theory are quite unintelligible. I have collected many arguments for refuting the latter, but I do not venture to bring them to publication.

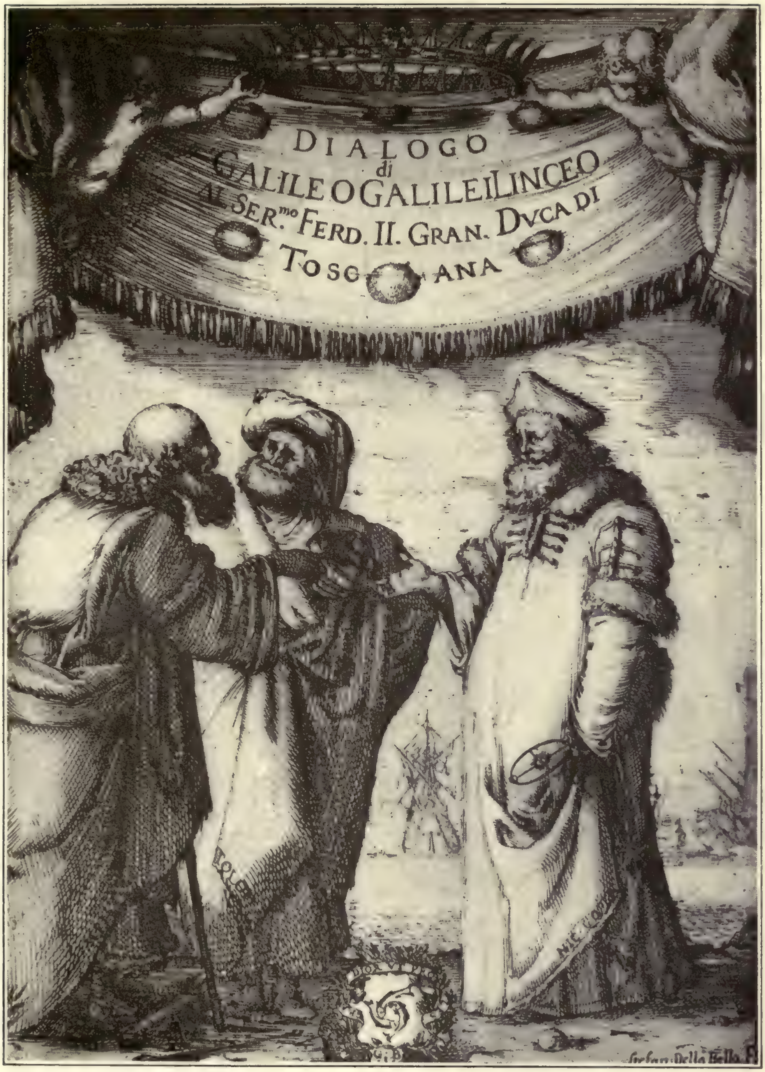
'That the moon is a body like the earth I have long been assured. I have also discovered a multitude of previously invisible fixed stars, outnumbering more than ten times those which can be seen by the naked eye, — forming the Milky Way. Further I have discovered that Saturn consists of three spheres which almost touch each other.'

While none of Galileo's astronomical discoveries were either necessary or sufficient to confirm the Copernican theory, their support was exceedingly important. Thus the slow motion of the sun spots across the disc and their subsequent reappearance showed rotation of that body, the satellites of Jupiter and particularly the phases of Venus, analogous to those shown by the moon, obviously harmonized with the Copernican theory. This implied at least that the planets shone by reflected sunlight, and it had indeed been insisted against that theory that Venus and Mercury under it must show phases till then undiscovered.

In 1632 Galileo published his celebrated Dialogue on the Two Chief Systems of the World, the Ptolemaic and the Copernican, a work comparable in magnitude and importance with Copernicus' Revolutions. In the curious preface he says: —

> 'Judicious reader, there was published some years since in Rome a salutiferous Edict, that, for the obviating of the dangerous Scandals of the present Age, imposed a reasonable Silence upon the Pythagorean Opinion of the Mobility of the Earth. There want not such as unadvisedly affirm, that the Decree was not the production of a sober Scrutiny, but of an illformed passion; and one may hear some mutter that Consultors altogether ignorant of Astronomical observations ought not to clipp the wings of speculative wits with rash prohibitions. My zeale cannot keep silence when I hear these inconsiderate complaints. I thought fit, as being thoroughly acquainted with that prudent Determination, to appear openly upon the Theatre of the World as a Witness of the naked Truth. . . . I hope that by these considerations the world will know that if other Nations have Navigated more than we, we have not studied less than they; and that our returning to assert the Earth's stability, and to take the contrary only for a Mathematical *Capriccio*, proceeds not from inadvertency of what others have thought thereof, but (had one no other inducements), from these reasons that Piety, Religion, the Knowledge of the Divine Omnipotency, and a consciousness of the incapacity of man's understanding dictate unto us.'

In the first of the four conversations into which the work is divided, the Aristotelian theory of the peculiar character of the heavenly bodies is subjected to destructive criticism, with emphasis on such phenomena as the appearance of new stars, of comets and of sun spots, the irregularities of the moon's surface, the phases of Venus, the satellites of Jupiter, etc.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Galileo's Dialogue.

> 'When we consider merely the vast dimensions of the celestial sphere in comparison with the littleness of our earth . . . and then think of the speed of the motion by which a whole revolution of the heavens must be accomplished in one day, I cannot persuade myself that the heavens turn while the earth stands fast.'

Adducing not merely the sun spots themselves, but their rapid variation, he insists that the universe is not rigid and permanent, but constantly changing or, as science has more and more emphasized since his day, passing through consecutive, related phases or *evolving*.

> 'I can listen only with the greatest repugnance when the quality of unchangeability is held up as something preëminent and complete in contrast to variability. I hold the earth for most distinguished exactly on account of the transformations which take place upon it.'

He begins to see the fallacy of the objections that if the earth rotated, a body dropped from a masthead would be left behind by the ship and that movable objects could be thrown off centrifugally at the equator. As positive arguments in support of the Copernican system, he urges particularly the retrogressions and other irregularities of the planets, and also the tides.

Of the famous controversy of Galileo with the Inquisition, it may here suffice to quote the judgment of the court (see Appendix): —

> 'The proposition that the sun is in the centre of the world and immovable from its place is absurd, philosophically false and formally heretical; because it is expressly contrary to the Holy Scriptures,' etc.

and a passage from the biographer already cited at so much length: —

> For over fifty years he was the knight militant of science, and almost alone did successful battle with the hosts of Churchmen and Aristotelians who attacked him on all sides — one man against a world of bigotry and ignorance. If then, . . . once, and only once, when face to face with the terrors of the Inquisition, he, like Peter, denied his Master, no honest man, knowing all the circumstances, will be in a hurry to blame him.

Of Galileo's still more remarkable services to physics and dynamics, something will be added in a later chapter.

MEDICAL AND CHEMICAL SCIENCES. — These were still at the low medieval level. There was as yet no scientific medicine, and no chemistry but alchemy, which was now in its final stage, iatro (medical) chemistry. Here one great name is that of Paracelsus (1493–1541), erratic and radical Swiss physician and alchemist, whose chief merit is his courage in opposing mere authority in science, and whose influence long after caused "salt, sulphur, and mercury" to be highly regarded and carefully studied. He also introduced and insisted upon the importance of antimony as a remedy, and is said to have been the first to use that tincture of opium which is still known by his name for it; viz. laudanum. Paracelsus, on the other hand, in spite of the fact that he was a popular surgeon, rejected the study of anatomy, taught medical knowledge through scanning of the heavens, and considered diseases as spiritual in origin. "The true use of chemistry," he said, "is not to make gold but to prepare medicines."

Another name worthy of remembrance in the chemistry of the sixteenth century is that of Landmann (Latin, *Agricola*) whose great work on Metallurgy (*De Re Metallica, 1546*) is the most important of this period, and who must also be regarded as the first mineralogist of modern times.

ANATOMY. VESALIUS. — Hardly less important, meantime, than the studies of Copernicus, Tycho Brahe, Galileo and Kepler upon the heavenly bodies were those of the Belgian anatomist, Andreas Vesalius, upon the human body. For more than 1000 years there had been almost no progress in anatomy or medicine, Hippocrates and Galen being still regarded as the final authorities in these matters up to the middle of the sixteenth century. Vesalius (1514–1564), born in Brussels and educated in Paris, was the first in modern times to dissect the human body, and to publish excellent drawings of his dissections. It was said that he opened the body of a nobleman before the heart had entirely ceased beating, and thereby incurring the displeasure of the Inquisition, was sentenced to perform a penitential journey to Jerusalem. At all events, he went to Jerusalem and was shipwrecked and lost while returning.

After Vesalius the study of human anatomy was vigorously and successfully prosecuted in Italy as was natural, since it was in Italy that Humanism and the revival of learning first took firm hold of Christian Europe. One of Vesalius' Italian contemporaries, Eustachius, whose name is still familiarly associated with the passage or "tube" connecting the throat and the middle ear, is hardly less famous in the history of anatomy than is Vesalius himself. The name of Fallopius, professor at Pisa in 1548 and at Padua in 1551, is also similarly associated with the human oviducts, — the so-called Fallopian tubes. His disciple Fabricius of Acquapendente discovered the valves in the veins, and was the teacher of William Harvey. A Spanish anatomist of note, Michael Servetus, — born 1509, — perished as a martyr at the stake in 1553 because of heretical writings abhorrent alike to the Inquisition and to Calvin.

Of physiology we have as yet little or no account. Doubtless all the anatomists just mentioned and many other "philosophers" had pondered, as did Aristotle and his predecessors, on the workings of the animal, and especially the human, mechanism. But from Aristotle (B.C. 322) to William Harvey (1578–1657) no real progress was made. It is a melancholy commentary on superstition and human prejudice that long after the brilliant work of Vesalius and the Italian anatomists, no proper "anatomy acts" existed to make lawful dissection either possible or easy, so that for several centuries afterward anatomists, surgeons, and medical students felt themselves at times obliged to resort to "body-snatching."

NATURAL HISTORY AND NATURAL PHILOSOPHY. — No great progress was made in this field after the observations of Aristotle, Theophrastus, Xenophanes, and Pythagoras until the sixteenth century. Fossils mostly remained unexplained or were regarded as "freaks" of nature. Animals and plants were comparatively neglected and, if studied, considered either as the raw material for supposed remedies or medicines, or else as treated by Aristotle. The twenty-six books *De Animalibus*, of Albertus Magnus (*d.* 1282) were not printed until 1478, but were apparently well known in manuscript copies. No great worker appears in this almost neglected field until we come to Conrad Gesner (or Gessner) (1516–1565), the first famous naturalist of modern times, on account of his vast erudition surnamed "the German Pliny." Professor of Greek at Lausanne and later of Natural History at Basel, he was almost as prolific an author as was della Porta fifty years later, for he wrote extensively upon plants, animals, milk, medicine, and theology, as well as various classical subjects. Yet he ranks high in the history of biology, both for the extent and the quality of his work in zoölogy and botany. It is significant that Gesner was a Swiss, and as such probably safe from persecution at a time when William Turner, an English ornithologist, worked and published in Cologne.

At the end of the fifteenth and beginning of the sixteenth centuries Leonardo da Vinci (1452–1519) turned his attention in part from art to science, engineering, and inventions, making interesting studies in architecture, hydraulics, geology, etc. He is regarded as the first engineer of modern times, and has been called " the world's most universal genius." Palissy, "the Potter," later examined minutely various fossils and took the then advanced ground (as Xenophanes and Pythagoras had done, however, some two thousand years earlier) that these are in reality what they appear to be, *i.e.* petrified remains of plant and animal life, and not "freaks of nature." Palissy's bold stand on this subject marks one of the first steps in modern times toward rational geology.

It was not until the end of the sixteenth century, when William Gilbert, an eminent practising physician of Colchester, England (1540–1603), published his now famous work on the magnet (*De Magnete*) that further progress was made through the first rational treatment of electrical and magnetic phenomena. To him is due the name electricity (*vis electrica*). He regarded the earth as a great magnet and, accepting the Copernican theory, attributed the earth's rotation to its magnetic character.

He even extended this idea to the heavenly bodies, with an animistic tendency. Gilbert is also reputed to have done important work in chemistry, but none of this has survived.

> His work is one of the finest examples of inductive philosophy that has ever been presented to the world. It is the more remarkable because it preceded the *Novum Organum* of Bacon, in which the inductive method of philosophizing was first explained. — *Thomas Thomson.*

The most prolific writer on natural philosophy and physical science of the sixteenth century was G. della Porta (1543–1615), a native of Naples and a resident of Rome, founder of an early scientific academy there, and afterwards of the famous *Accademia dei Lincei* of Rome. His writings are voluminous and in many books, of which we need mention here only his *Magia Naturalis,* (1569), *De Refractione* (1593), *Pneumatica* (1691), *De Distillatione* (1604), *De Munitione* (1608) and *De Aeris Transmutationibus* (1609).

In his Natural Magic, della Porta is the first to describe a camera obscura, besides touching on many interesting properties of lenses, and referring to spectacles, some forms of which had long been known. His work On Refraction deals largely with binocular vision, and is a criticism of the work of Euclid and Galen on that subject. The author hints also at a crude telescope, and may have known some form of stereoscope. Della Porta's compositions range all the way from natural magic to Italian comedies, and entitle him to high rank as a tireless and original, if not especially fruitful, thinker and worker.

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DREYER. *Tycho Brahe; Planetary Systems.* Chapters XII–XVI.
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GILBERT. *On the Magnet.*
LOCY. *Biology and its Makers.*
LODGE. *Pioneers of Science.*

CHAPTER XI

PROGRESS OF MATHEMATICS AND MECHANICS IN THE SIXTEENTH CENTURY

IT was not alone the striving for universal culture which attracted the great masters of the Renaissance, such as Brunelleschi, Leonardo da Vinci, Raphael, Michael Angelo and especially Albrecht Dürer, with irresistible power to the mathematical sciences. They were conscious that, with all the freedom of the individual phantasy, art is subject to necessary laws and, conversely, with all its rigor of logical structure, mathematics follows esthetic laws. — *Rudio.*

The miraculous powers of modern calculation are due to three inventions: the Arabic Notation, Decimal Fractions and Logarithms. — *Cajori.*

The invention of logarithms and the calculation of the earlier tables form a very striking episode in the history of exact science, and, with the exception of the *Principia* of Newton, there is no mathematical work published in the country which has produced such important consequences, or to which so much interest attaches as to Napier's *Descriptio.* — *Glaisher.*

It is Italy, which is the fatherland of Archimedes, whose creative power embraces all domains of the mechanical science, the land of the Renaissance, from out of which those mighty waves of new ideas and new impulses in science and art have come forth into the world — the fatherland of Galileo the creator of experimental physics, of Leonardo da Vinci the engineer, of Lagrange who has given its form to modern analytical mechanics. — *W. v. Dyck.*

Dynamics is really a product of modern times, and affords the rare example of a development fulfilled in a single great personage — Galileo. Nothing is finer than how he, beginning in the Aristotelian spirit, gradually frees himself from its bondage and, instead of empty metaphysics, introduces well-directed methodical investigations of nature. — *Timerding.*

The period from the invention of printing about 1450 to that of analytic geometry in 1637 was one of very great importance for mathematics and mechanics as well as for astronomy. At the beginning, Arabic numerals were known, but the mathematics even of the universities hardly extended beyond the early books of Euclid and the solution of simple cases of quadratic equations in rhetorical form. At the end of the period the foundations of modern mathematics and mechanics were securely laid.

AIMS AND TENDENCIES OF MATHEMATICAL PROGRESS. — In the centuries just preceding, the chief applications of mathematics had connected themselves with the relatively simple needs of trade, accounts and the calendar, with the graphical constructions of the architect and the military engineer, and with the sines and tangents of the astronomer and the navigator. During the period in question some of these applications became increasingly important, and at the same time mathematics was more and more cultivated for its own sake. Mathematicians became gradually a more and more distinctly differentiated class of scholars; mathematical text-books took shape. The beginnings of this evolution have been dealt with already; its further progress is now to be traced.

The larger achievements and tendencies of the period in mathematical science were the following: —

In Arithmetic, decimal fractions and logarithms were introduced, regulating and immensely simplifying computation; a general theory of numbers was developed; in Algebra, a compact and adequate symbolism was worked out, including the use of the signs $+$, $\div$, $\times$, $-$, $=$, $()$, $\sqrt{\ }$, and of exponents; equations of the third and fourth degree were solved, negative and imaginary roots accepted, and many theorems of our modern theory of equations discovered.

In Geometry, the computation of π was carried to many decimals, the beginnings of projective geometry were made, and a so-called method of indivisibles developed, foreshadowing the integral calculus; in plane and spherical Trigonometry, the theorems and processes now in use were worked out, and extensive tables computed.

In Mechanics, ideas about force and motion, equilibrium and centre of gravity, were gradually clarified.

Underlying some of these new developments are the dawning fundamental concepts: function, continuity, limit, derivative, infinitesimal, on which our modern mathematics has been built up. Descartes, Newton and Leibnitz are soon to make their revolutionary discoveries in analytic geometry and the calculus.

We have seen that up to about 1500 the chief stages in the development of mathematics have been the introduction and improvement of Arabic arithmetic for commercial purposes (though accounts were kept in Roman numerals until 1550 to 1650), the rediscovery of Greek geometry, and the improvement of trigonometry in connection with its increasing use in astronomy, navigation and military engineering. The development of science has been powerfully promoted by the general intellectual emancipation of the Renaissance, while mathematical progress, beginning earlier, has been both a cause and a consequence of the general advance. The diffusion and the preservation of scientific knowledge have derived immense advantage from the new art of printing and from expanding commercial intercourse. Algebra, almost helpless in Greek times because, for lack of proper symbolism, expressed only in geometrical or rhetorical form, has been converted by a process of abbreviation, at first into a syncopated form, intermediate between the rhetorical and our modern purely symbolic notation.

Pacioli. — The earliest printed book on arithmetic and algebra was published at Venice in 1494 by Lucas Pacioli, a Franciscan monk born in Tuscany about 1450. Rules are here given for the fundamental operations of arithmetic, and for extracting square roots. Commercial arithmetic is treated at considerable length by the newer algoristic or Arabic methods. The method of arbitrary assumption corrected by proportion is used effectively, for example: —

> To find the original capital of a merchant who spent a quarter of it in Pisa and a fifth of it in Venice, who received on these transactions 180 ducats, and who has in hand 224 ducats.
>
> Assume that his original capital was 100 ducats; then the surplus would be $100 - 25 - 20 = 55$, but this is $\frac{5}{4}$ of his actual surplus $224 - 180$, therefore his original capital was $\frac{4}{5}$ of $100 = 80$ ducats.

Some of Pacioli's commercial problems are exceedingly complicated. He solves numerical equations of the first and second degree, but admits only positive roots and considers the solution of cubic equations, as well as the squaring of the circle, impossible. Addition is denoted by p or $\bar{p}$, equality sometimes by *ae*, a beginning of syncopated algebra. The introduction of the radical sign with indices $\sqrt{2}$, $\sqrt{3}$ and of the signs + and − date from about this time.

In geometry Pacioli, like Regiomontanus, employs algebraic methods. Among other problems he determines a triangle from the radius of the inscribed circle and the segments into which it divides one of the sides. His solution, though highly esteemed at the time, is much less simple than he might have obtained by the formulas at his command.

In the spirit of the Renaissance he brings the feeble mathematics of the universities into fruitful relations with the practical mathematics of the artist and the architect. The inscribed hexagon and the equilateral triangle play their part as gild secrets in the development of Gothic architecture. The question is not "How to prove," but "How to do."

On the other hand, the current tendency to drift into mystical interpretation is exemplified by the following extract from Pacioli: —

> There are three principal sins, avarice, luxury, and pride; three sorts of satisfaction for sin, fasting, almsgiving, and prayer; three persons offended by sin, God, the sinner himself, and his neighbour; three witnesses in heaven, *Pater, verbum,* and *spiritus sanctus;* three degrees of penitence, contrition, confession, and satisfaction, which Dante has represented as the three steps of the ladder that leads to purgatory, the first marble, the second black and rugged stone, and the third red porphyry. There are three sacred orders in the church militant, *subdiaconati, diaconati,* and *presbyterati;* there are three parts not without mystery, of the most sacred body made by the priest in the mass; and three times he says *Agnus Dei,* and three times, *Sanctus;* and if we well consider all the devout acts of Christian worship, they are found in a ternary combination; if we wish rightly to

partake of the holy communion, we must three times express our contrition, *Domine non sum dignus;* but who can say more of the ternary number in a shorter compass, than what the prophet says, *tu signaculum sanctae trinitatis.* There are three Furies in the infernal regions; three Fates, Atropos, Lachesis, and Clotho. There are three theological virtues; *Fides, spes,* and *charitas. Tria sunt pericula mundi: Equum currere; navigare, et sub tyranno vivere.* There are three enemies of the soul: the Devil, the world, and the flesh. There are three things which are of no esteem: the strength of a porter, the advice of a poor man, and the beauty of a beautiful woman. There are three vows of the Minorite Friars; poverty, obedience, and chastity. There are three terms in a continued proportion. There are three ways in which we may commit sin: *corde, ore, ope.* Three principal things in Paradise: glory, riches, and justice. There are three things which are especially displeasing to God: an avaricious man, a proud poor man, and a luxurious old man. And all things in short, are founded in three; that is, in number, in weight, and in measure.

GEOMETRY IN ART. — Brunelleschi (1377–1446), the famous architect of the early Renaissance, made a perspective view of the Signoria in Florence in a sort of box with clouds. The famous doors of the Baptistery by his contemporary Ghiberti show the development of perspective in the marked contrast between the earlier and the later panels. Raphael in his School of Athens includes himself and Bramante in a group of mathematicians. Painters were even called for a time perspectivists — *prospettivi.*

Leonardo da Vinci (1452–1519), one of the intellectual giants of the Renaissance, eminent alike in art, science and engineering, gave the first correct explanation of the partial illumination of the darker part of the moon's disc by reflection from the earth. He calls mechanics the paradise of the mathematical sciences, because through it one first gains the fruit of these sciences. He denies the possibility of perpetual motion, saying "Force is the cause of motion and motion the cause of force." He discusses the lever, the wheel and axle, bodies falling freely or on inclined planes, foreshadowing Galileo. Contrary to the Aristotelian tradition he asserts that everything tends to continue in its given state, and he even enunciates the fundamental principle that for simple machines forces in equilibrium are inversely as the virtual velocities.

> 'Whoever,' he says, 'appeals to authority applies not his intellect but his memory.' 'While Nature begins with the cause and ends with the experiment, we must nevertheless pursue the opposite plan, beginning with the experiment and by means of it investigating the cause.' 'No human investigation can call itself true science, unless it comes through mathematical demonstration.' 'He who scorns the certainty of mathematics will not be able to silence sophistical theories which end only in a war of words.'

Unfortunately his work in this field remained unpublished, and therefore relatively unfruitful.

Leonardo and other great artists of his time — notably Albrecht Dürer of Nuremberg (1471–1528) — developed the geometrical theory of perspective. For the purpose of accurately representing the human head Dürer made both plans and elevation. "Intelligent painters and accurate artists," he says, "at the sight of works painted without regard to true perspective must laugh at the blindness of these people, because to a right understanding nothing impresses more disagreeably than falsehood in a painting, regardless of the diligence with which it has been made. That such painters, however, are pleased with their own mistakes is due to the fact that they have not learned the art of measurement, without which no one can become a true workman." All this had importance both for modern art and modern geometry.

Characteristic of this period is the so-called *Margarita Philosophica* published in many editions from 1503 to 1600. It was the first modern encyclopædia printed, and gives in its twelve books "a compendium of the trivium, the quadrivium, and the natural and moral sciences."

A younger contemporary of Pacioli, Michael Stifel (1487–1567), a German monk converted to Lutheranism, developed a fantastic arithmetical interpretation of the Bible, identifying Pope Leo X with the beast in Revelation and predicting the immediate end of the world, — with results disastrous to his person as well as his reputation.

> He relates . . . that whilst a monk at Esslingen in 1520, and when infected by the writings of Luther, he was reading in the library of his convent the 13th Chapter of Revelations, it struck his mind that the Beast must signify the Pope, Leo X; He then proceeded in pious hope to make the calculation of the sum of the numeral letters in *Leo decimus,* which he found to be M, D, C, L, V, I; the sum which these formed was too great by M, and too little by X; but he bethought him again, that he has seen the name written Leo X; and that there were ten letters in *Leo decimus,* from either of which he could obtain the deficient number, and by interpreting the M to mean *mysterium,* he found the number required, a discovery which gave him such unspeakable comfort, that he believed that his interpretation must have been an immediate inspiration of God. — *Peacock.*

Stifel's writings on arithmetic and algebra embody some improvements of current notation. He introduced for example the symbols $1A$, $1AA$, $1AAA$ for what we should denote by x, x^2, x^3.

The low state of computation at this time is illustrated with startling clearness by a bulletin on the blackboard at Wittenberg, in which Melanchthon urgently invited the academic youth to attend a course on arithmetic, adding that the beginnings of the science are very easy, and even division can with some diligence be comprehended.

ROBERT RECORDE (1510–1558) studied at Oxford and graduated in medicine at Cambridge in 1545, later becoming "royal physician." His "Grounde of Artes" or arithmetic, one of the earliest mathematical books printed in English (1540), ran through more than 27 editions and exerted a great influence on English education. In the "Preface to the Loving Reader" he says: —

> Sore ofttimes have I lamented with myself the unfortunate condition of England, seeing so many great Clerks to arise in sundry other parts of the World, and so few to appear in this our Nation; whereas for pregnancy of natural wit (I think) few Nations do excell English-men. But I cannot impute the cause to any other thing, then to the contempt or misregard of Learning. For as English-men are inferiour to no men in mother Wit, so they pass all men in vain Pleasures, to which they may attain with great pain and labour; and

> are slack to any never so great commodity, if there hang of it any painfull study or travelsome labour.

The book itself is in the form of a dialogue or catechism beginning: —

> The Scholar speaketh.
>
> 'Sir, such is your authority in mine estimation, that I am content to consent to your saying, and to receive it as truth, though I see none other reason that doth lead me thereunto; whereas else in mine own conceit it appeareth but vain, to bestow any time privately in learning of that thing that every Child may and doth learn at all times and hours, when he doth any thing himself alone, and much more when he talketh or reasoneth with others.'

He employs the symbol + "whyche betokeneth too muche, as this line − plaine without a crosse line betokeneth too little."

In 1557 he published an algebra under the alluring title "Whetstone of Witte," using the sign = for equality, which he says he selected because "noe 2 thynges can be moare equalle" than two parallel straight lines.

ALGEBRAIC EQUATIONS OF HIGHER DEGREE.—Two great Italian mathematicians vied with each other in giving a powerful impetus to the development of algebra in the sixteenth century.

Niccolo Fontana or TARTAGLIA (1500–1557) a man of the humblest origin, lectured at Verona and Venice, and first won fame by successfully meeting a challenge to solve mathematical problems, all of which proved, as he had anticipated, to involve cubic equations.

His *Nova Scienza* (1537) discusses falling bodies, and many problems of military engineering and fortification, the range of projectiles, the raising of sunken galleys, etc.

> The title-page is chiefly occupied by a large plate, which represents the courts of Philosophy, to which Euclid is doorkeeper, Aristotle and Plato being masters of an inmost court, in which Philosophy sits throned, Plato declaring by a label that he will let nobody in who does not understand Geometry. In the great court there is a cannon being fired, all the sciences looking on in a crowd — such as Arithmetic,

Geometry, Music, Astronomy, Cheiromancy, Cosmography, Necromancy, Astrology, Perspective, and Prestidigitation! A wonderfully modest-looking gentleman, with his hand upon his heart, stands among the number, with a you-do-me-too-much-honour look upon his countenance; Arithmetic and Geometry are pointing to him, and under his feet his name is written — Nicolo Tartalea. — *Morley,* Jerome Cardan.

The *Inventioni* (1546) gives his solution of the cubic equation. A treatise on Numbers and Measures (1556, 1560) gives a method for finding the coefficients in the expansion of $(1 + x)^n$ for $n = 2$, ... 6. It contains also a wide range of problems from commercial arithmetic and a collection of mathematical puzzles. The following examples may illustrate these: —

'Three beautiful ladies have for husbands three men, who are young, handsome, and gallant, but also jealous. The party are travelling, and find on the bank of a river, over which they have to pass, a small boat which can hold no more than two persons. How can they pass, it being agreed that, in order to avoid scandal, no woman shall be left in the society of a man unless her husband is present?'

'A ship carrying as passengers 15 Turks and 15 Christians encounters a storm, and the pilot declares that in order to save the ship and crew one half of the passengers must be thrown into the sea. To choose the victims, the passengers are placed in a circle, and it is agreed that every 9th man shall be cast overboard, reckoning from a certain point. In what manner must they be arranged so that the lot may fall exclusively upon the Turks?'

'Three men robbed a gentleman of a vase containing 24 ounces of balsam. Whilst running away they met in a wood with a glass-seller of whom in a great hurry they purchased three vessels. On reaching a place of safety they wish to divide the booty, but they find that their vessels contain 5, 11, and 13 ounces respectively. How can they divide the balsam into equal portions?' — *Ball.*

There is no other treatise that gives as much information concerning the arithmetic of the sixteenth century, either as to theory or application. The life of the people, the customs of the merchants, the struggles to improve arithmetic, are all set forth here by Tartaglia in an extended but interesting fashion.

Tartaglia, anticipating Galileo, taught that falling bodies of different weight traverse equal distances in equal times, and that a body swung in a circle if released flies off tangentially.

GIROLAMO CARDAN (1501–1576) led a life of wild and more or less disgraceful adventure, strangely combined with various forms of scientific or semi-scientific activity, — particularly the practice of medicine. He studied at Pavia and Padua, travelled in France and England, and became professor at Milan and Pavia.

His *Ars Magna* (1545) contains the solution of the cubic equation fraudulently obtained from his rival Tartaglia. After its publication the aggrieved Tartaglia challenged Cardan to meet him in a mathematical duel. This took place in Milan, August 10, 1548, but Cardan sent his pupil Ferrari in his place. Tartaglia relates that he was accompanied only by his brother, Ferrari by many friends. Cardan had left for parts unknown. As Tartaglia began to explain to the crowd the origin of the strife and to criticise Ferrari's 31 solutions, he was interrupted by a demand that judges be chosen. Knowing no one present he declines to choose; all shall be judges. Being finally allowed to proceed he convicts his opponent of an erroneous solution, but is then overwhelmed by tumultuous clamor with demands that Ferrari must have the floor to criticise his solution. In vain he insists that he be allowed to finish, after which Ferrari may talk to his heart's content. Ferrari's friends are vehement; he gains the floor and chatters about a problem which he claims Tartaglia has not been able to solve till the dinner hour arrives and Tartaglia, apprehending still worse treatment, withdraws in disgust.

Ferrari (1522–1565), this disciple of Cardan, even succeeded in giving a general solution of the equation of the fourth degree, beyond which, as has been shown only in quite recent times, the solution can in general no longer be similarly expressed.

Some idea of the difficulty of these sixteenth century achievements may be conveyed by the corresponding modernized solutions. If the given equation is $ax^3 + bx^2 + cx + d = 0$ the coefficient of the first term is made 1 by division and that of the second is made 0 by the substitution $x = y - \frac{b}{3a}$. The new equation having the form $y^3 + ey + f = 0$ we now put $y = z - \frac{e}{3z}$, whence $z^3 - \frac{e^3}{27z^3} + f = 0$, a quadratic equation in z^3. The solution of the original equation of degree three is thus made to depend on that of an equation of degree one less.

Similarly if the given equation of the fourth degree is in our notation $ax^4 + bx^3 + cx^2 + dx + e = 0$ the coefficient of the first term is made 1 by division and that of the second is made 0 by the substitution $x = y - \frac{b}{4a}$

The new equation having the form

$$y^4 + fy^2 + gy + h = 0.$$

We put $y^4 + fy^2 + gy + h = (y^2 - ay + \beta)(y^2 + ay + \gamma)$

whence

$$f = \beta + \gamma - a^2$$

$$g = (\beta - \gamma)\,a$$

$$h = \beta\gamma.$$

We obtain a, β, γ from these three equations by eliminating two and solving the cubic equation obtained for the other; that is, the solution of the original equation of degree four is made to depend on that of a new equation of degree one less.

One of Cardan's scientific inventions was an improved suspension of the compass needle. He was also eminent as an astrologer.

SYMBOLIC ALGEBRA: VIETA. — Of still greater importance in the history of algebra is F. Vieta (1540–1603) a lawyer of the French court. He won the interest of Henry IV by solving a complicated problem proposed by an eminent mathematician, as was the custom of the time, as a challenge to the learned world. This involved an equation of the 45th degree which he succeeded in solving by a trigonometric device. Later he was employed to interpret the cipher despatches of the hostile Spaniards. His *In Artem Analyticam Isagoge* is the earliest work on *symbolic* algebra. In it known quantities are denoted by consonants, unknown by vowels, the use of homogeneous equations is recommended, the first six powers of a binomial given, and a special exponential notation introduced. He shows that the celebrated classical problems of trisecting a given angle and duplicating a cube involve the solution of the cubic equation, and makes important discoveries in the general theory of equations — for example resolving polynomials into linear factors and deriving from a given equation other equations having roots which differ from those of the first by a constant or by a given factor. He solves Apollonius' famous problem of determining the circle tangent to three given circles, and expresses π by an infinite series. He devises systematic methods for the solution of spherical triangles.

DEVELOPMENT OF TRIGONOMETRY. — Many circumstances combined to promote the development of trigonometry at this period. It was needed by the military engineer, the builder of roads, the astronomer, the navigator, and the mapmaker whose work was tributary to all of these.

Rheticus (George Joachim, 1514–1576), — "the great computer whose work has never been superseded," — worked out a table of natural sines for every 10 seconds to fifteen places of decimals. We owe to him our familiar formulas for sin $2x$ and sin $3x$. The notation sin, tan, etc. and the determination of the area of a spherical triangle date from about this time. To this period belong also the very important work of Mercator on map-making and the reform of the calendar by Pope Gregory XIII.

MAP-MAKING. — Mercator (Gerhard Krämer, 1512–1594) devoted himself in his home city, Louvain, to mathematical geography, and gained his livelihood by making maps, globes and astronomical instruments, combined in later life with teaching. His great world map, completed in 1569, marks an epoch in cartography. The first "Atlas" was published by his son in 1595. He gives a mathematical analysis of the principles underlying the projection of a spherical surface on a plane.

> 'If,' he says, 'of the four relations subsisting between any two places in respect to their mutual position, namely difference of latitude, difference of longitude, direction and distance, only two are regarded, the others also correspond exactly, and no error can be committed as

must so often be the case with the ordinary marine charts and so much the more the higher the latitude.'

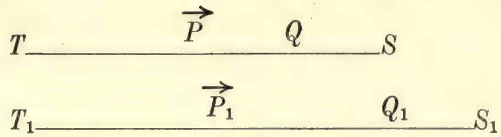
Mercator's geometrical method amounts to projecting the spherical surface of the earth on a cylinder tangent to the earth along the equator and having the same axis with the earth. Under this method of projection, angles are preserved in magnitude, but areas remote from the equator are disproportionately expanded. A straight line on the chart corresponds with the course of a ship steering a constant course.

THE GREGORIAN CALENDAR.—Until 1582 the Julian calendar (p. 143) remained in force with 365¼ days each year and a gradually increasing error amounting at this time to ten days. Under the auspices of Pope Gregory the days from October 5 to 15, 1572, were dropped and the number of leap-years in 400 reduced from 100 to 97. Religious jealousies prevented the adoption of this reform in Protestant Germany for a century, while England postponed it until 1752.

A NEW INVENTION FOR COMPUTATION.—The invention of logarithms would appear to have been a natural sequel of any adequate theory and notation for exponents. Thus Stifel in his arithmetic (1544) had tabulated small integral powers of 2—from ¼ to 64—and shown the correspondence between multiplication of these powers and addition of the indices or exponents, but his use of exponents was too limited, he lacked the apparatus of decimal fractions necessary for the practical application of the method and probably had no conception of the vast labor-saving possibilities so near at hand.

In 1614 John Napier published at Edinburgh his *Mirifici Logarithmorum Canonis Descriptio,* for which the time was so fully ripe that an enthusiastic reception was at once assured. Napier as a devout Protestant, stimulated by fear of an impending Spanish invasion, busied himself with inventions "proffitabill & necessary in theis dayes for the defence of this Iland & withstanding of strangers enemies of God's truth & relegion." Among these were a mirror for burning distant ships, and a sort of armored chariot. Impressed by the tremendous calculations then in progress by

Rheticus, Kepler, and others in connection with the development of the new astronomy, Napier made a vastly more important invention. His definition of a logarithm rests on the following kinetic basis: —

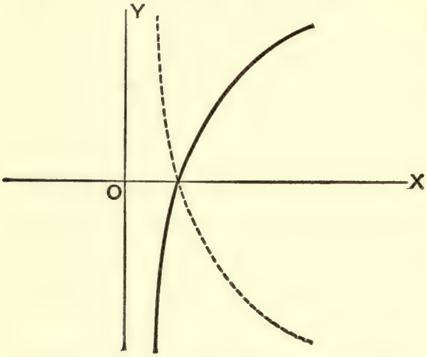


TS is a straight line of definite length; T_1S_1 extends to the right indefinitely. Moving points P and P_1 start from T and T_1 with equal initial speeds; the latter continues at the same rate, the former is retarded so that its speed is always proportional to its distance from S. If equal intervals are taken on T_1S_1 the corresponding intervals in TS will grow smaller to the right. When P is at any position Q the logarithm of QS is represented by the corresponding length T_1Q_1 on the other line. It may be shown in fact that if in our notation $PS = x$, $T_1P_1 = y$, $TS = l$, $\frac{dx}{dy} = -\frac{x}{l}$. This conception involving a functional relation between two variables went much deeper than the comparison of discrete numbers by Stifel.

Napier's conception of a logarithm involved a perfectly clear apprehension of the nature and consequences of a certain functional relationship, at a time when no general conception of such a relationship had been formulated, or existed in the minds of mathematicians, and before the intuitional aspect of that relationship had been clarified by means of the great invention of coördinate geometry made later in the century by René Descartes. A modern mathematician regards the logarithmic function as the inverse of an exponential function; and it may seem to us, familiar as we all are with the use of operations involving indices, that the conception of a logarithm would present itself in that connection as a fairly obvious one. We must however remember that, at the time of Napier, the notion of an index, in its generality, was no part of the stock of ideas of a mathematician, and that the exponential notation was not yet in use.

— *Hobson.*

Independent tables were computed by the astronomer Bürgi and published at Prague in 1620. Both Napier and Bürgi, basing their work on the relation which we should express by the equivalent equations $x = a^y$ and $y = \log_a x$, avoid fractional values of y by taking values of a near 1, their actual values being $a = .9999999$ and $a = 1.0001$ respectively. In choosing a base less than 1, Napier is also influenced by his desire that sines and cosines as proper fractions shall have positive logarithms. If we introduce our modern graphical interpretation of $y = \log_a x$, Bürgi is concerned with the determination of abscissas of points where the exponential curve is met by the horizontal straight lines $y = c$ where c takes successive integral values. Choosing a base a near 1 naturally gives values of x near each other. Napier's choice of a base less than 1 would correspond with the same curve inverted.



In 1615 Henry Briggs, afterwards Savilian Professor of Geometry at Oxford, wrote of Napier "I hope to see him this summer, if it please God, for I never saw book which pleased me better, or made me more wonder." In connection with this and later visits it was soon discovered that great simplification in the practical use of logarithms would result from taking $\log 1 = 0$ and $\log 10 = 1$ and giving up the restriction of logarithms to integral values, thus making the decimal parts of all logarithms depend wholly on the sequence of digits. Napier had been so predominantly interested in trigonometric applications that his table consisted not of logarithms of abstract numbers, but of 7-place logarithms of the trigonometric functions for each minute. In connection with his change of the base, Briggs developed interesting methods of interpolating and testing the accuracy of logarithms. He gives the logarithms from 1 to 20,000 and from 90,000 to 100,000 to 14 places, computing also 10-place trigonometric tables with an angular interval of 10 seconds.

> Kepler recognized immediately the enormous significance of the new logarithmic method and addressed an enthusiastic panegyric to Napier in 1620, not knowing that he had died in 1617. What if logarithms had been invented in time to save Kepler his vast computations?
>
> A few years ago we have been shown in a rectorial address what the telescope has meant for observational astronomy. An equally great significance attached to logarithms for the computing astronomer.
>
> — *Gutzmer.*

Vlacq of Leyden soon after filled the gap in Briggs' table, and this is the basis for the tables since published. The first tables to base *e*, commonly called Napierian, were published in 1619. In more recent times methods of interpolation have been employed which are more powerful and less laborious, while ordinary computation has been simplified by avoiding the use of too many decimal places, and by the mechanical device of the slide-rule. The modern computing machine naturally tends to supersede the logarithmic method. Among the remarkable computations characteristic of the sixteenth century may be mentioned Ludolph von Ceulen's achievement in computing π to 35 decimal places, using regular polygons of 96 and 192 sides. German writers in consequence have sometimes attached his name to this important constant.

In England Thomas Harriott (1560–1621) and William Oughtred (1575–1660) rendered important services in introducing the most recent advances in arithmetic, algebra and trigonometry. The former rejected negative and imaginary roots indeed, but used the signs $>$ and $<$, denotes a^2 by $a\ a$, etc. Oughtred uses the symbols $\times$ and $::$, also the contractions for sine, cosine, etc.

"TWO NEW BRANCHES OF SCIENCE." — Even after Galileo's condemnation by the Inquisition, though old, infirm, and nearly blind, his scientific ardor was unquenched, and in 1638 he published (at Leyden) a work on mechanics under the title, Conversations and Mathematical Demonstrations on two New Branches of Science, which constituted the most notable progress in mechanics since Archimedes. He says:

My purpose is to set forth a very new science dealing with a very ancient subject. There is, in nature, perhaps nothing older than motion, concerning which the books written by philosophers are neither few nor small; nevertheless I have discovered by experiment some properties of it which are worth knowing and which have not hitherto been either observed or demonstrated. Some superficial observations have been made, as, for instance, that the free motion (*naturalem motum*) of a heavy falling body is continuously accelerated; but to just what extent this acceleration occurs has not yet been announced; for so far as I know, no one has yet pointed out that the distances traversed, during equal intervals of time, by a body falling from rest, stand to one another in the same ratio as the odd numbers beginning with unity.

It has been observed that missiles and projectiles describe a curved path of some sort; however no one has pointed out the fact that this path is a parabola. But this and other facts, not few in number or less worth knowing, I have succeeded in proving; and what I consider more important, there have been opened up to this vast and most excellent science, of which my work is merely the beginning, ways and means by which other minds more acute than mine will explore its remote corners.

This discussion is divided into three parts; the first part deals with motion which is steady or uniform; the second treats of motion as we find it accelerated in nature; the third deals with the so-called violent motions and with projectiles. . . .

Throughout this work Galileo depends on results of experiment rather than on mere speculation. He recognizes that air has weight and that water can be raised but a certain height by the ordinary pump,[1] but he still accepts the ancient notion that

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[1] 'This pump worked perfectly so long as the water in the cistern stood above a certain level; but below this level the pump failed to work. When I first noticed this phenomenon I thought the machine was out of order; but the workman whom I called in to repair it told me the defect was not in the pump but in the water which had fallen too low to be raised through such a height; and he added that it was not possible, either by a pump or by any other machine working on the principle of attraction, to lift water a hair's breadth above eighteen cubits; whether the pump be large or small this is the extreme limit of the lift. Up to this time I had been so thoughtless that, although I knew a rope, or rod of wood, or of iron, if sufficiently long, would break by its own weight when held by the upper end, it never occurred to me that the same thing would happen, only much more easily, to a

"nature abhors a vacuum" as an explanation. He shows experimentally that a body descends an inclined plane with uniformly accelerated motion.

> In a board 12 ells in length a groove half an inch wide was made. It was drawn straight and lined with very smooth parchment. The board was then raised at one end, first one ell, then two. Then Galileo let a polished brass ball roll through the groove and determined the time of descent for the whole length of the groove. If on the other hand he let the ball roll through only one quarter of the length, this required just half the time. . . . The distances were to each other as the squares of the times,

a law verified by hundredfold repetitions for all sorts of distances and slopes. The time was still determined by weighing water escaping through a small orifice. He shows by ingenious experiments the dependence of velocity on height alone, and that a freely falling body has the necessary energy to reach its original level. The whole theory of the falling body is now easily deduced.

> When, therefore, I observe a stone initially at rest falling from an elevated position and continually acquiring new increments of speed, why should I not believe that such increases take place in a manner which is exceedingly simple and rather obvious to everybody? If now we examine the matter carefully we find no addition or increment more simple than that which repeats itself always in the same manner. This we readily understand when we consider the intimate relationship between time and motion; for just as uniformity of motion is defined by and conceived through equal times and equal spaces (thus we call a motion uniform when equal distances are traversed during equal time-intervals), so also we may, in a similar manner, through equal time-intervals, conceive additions of speed as taking place without complication; thus we may picture to our mind a motion as uniformly and continuously accelerated when, during any equal intervals of time whatever, equal increments of speed are given to it. . . .
>
> Hence the definition of motion which we are about to discuss may

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column of water. And really is not that thing which is attracted in the pump a column of water attached at the upper end and stretched more and more until finally a point is reached where it breaks, like a rope, on account of its excessive weight? . . .'

be stated as follows: A motion is said to be uniformly accelerated, when starting from rest, it acquires, during equal time-intervals, equal increments of speed. . . .

The time in which any space is traversed by a body starting from rest and uniformly accelerated is equal to the time in which that same space would be traversed by the same body moving at a uniform speed whose value is the mean of the highest speed and the speed just before acceleration began. . . .

The spaces described by a body falling from rest with a uniformly accelerated motion are to each other as the squares of the time-intervals employed in traversing these distances. . . .

Galileo passes from falling bodies to pendulums, in which the friction of the inclined plane is absent and air resistance negligible. He appreciates the possibility of utilizing the pendulum for time measurement, and devises a simple apparatus for the purpose, foreshadowing the invention of the clock. He discovers that the time of vibration of the pendulum varies as the square root of the length.

He analyzes correctly the component motions of a projectile, recognizing the law of the parallelogram of motion, as distinguished from the parallelogram of forces discovered by Newton. He shows that whether the initial direction of aim is horizontal or not, the path described is a parabola with axis vertical, explicitly neglecting air resistance and change of direction of vertical force.

I now propose to set forth those properties which belong to a body whose motion is compounded of two other motions, namely, one uniform and one naturally accelerated; these properties, well worth knowing, I propose to demonstrate in a rigid manner. This is the kind of motion seen in a moving projectile; its origin I conceive to be as follows: . . .

A projectile which is carried by a uniform horizontal motion compounded with a naturally accelerated vertical motion describes a path which is a semi-parabola. — *Galileo,* Two New Sciences.

All this Dynamics was practically pioneer work of enormous importance for the future of mechanics.

In Statics Galileo had somewhat more from the ancients to build upon. To him we owe the formulation of the law of virtual velocities — applying dynamical ideas to problems of statics. If two forces are in equilibrium they are proportional to the corresponding paths, or: What one by any machine gains in power is lost in distance. The parallelogram or triangle of forces in equilibrium however escapes him, and his ideas about impulse though remarkably in advance of his time were not fully worked out.

He investigates strength of materials under tension and fracture, with reference to practical applications in construction. He draws just inferences in regard to the relation between strength and size of plants and animals as well as machines, comparing for example hollow bones and straws with solid bodies of similar mass. He derives an important formula for the stiffness of a horizontal beam supported at one end and regarded as a lever. He discusses the curve formed by a cord suspended between two points, recognizing that it is not a parabola.

In Hydrostatics he reviews the known work of Archimedes and corrects the error of the Aristotelians in regard to the dependence of floating on specific gravity. He develops the modern theory that the fundamental factor in the mechanics of fluids is that they consist of freely moving particles yielding to the slightest force. He makes effective application of the principle of virtual velocities to fluids. At the close of his third conversation he expresses his modest confidence in the great future of his new ideas.

> The theorems set forth in this brief discussion will, when they come into the hands of other investigators, continually lead to wonderful new knowledge. It is conceivable that in such a manner a worthy treatment may be gradually extended to all the realms of nature

— a prediction magnificently fulfilled in succeeding generations.

Among other branches of physics in which Galileo accomplished work of value may be mentioned the expansion by heat — the beginnings of thermometry, experiments on the acoustics of vibrating cords and plates, discovering the dependence of harmony on the ratio of the rates of vibration, and the relations of length, thickness, and tension of cords. He explains resonance and dissonance. He assumes light to have a finite velocity, but does not succeed in measuring it.

> Let each of two persons take a light contained in a lantern, or other receptacle, such that by the interposition of the hand, the one can shut off or admit the light to the vision of the other. Next let them stand opposite each other at a distance of a few cubits and practice until they acquire such skill in uncovering and occulting their lights that the instant one sees the light of his companion he will uncover his own. After a few trials the response will be so prompt that without sensible error the uncovering of one light is immediately followed by the uncovering of the other, so that as soon as one exposes his light he will instantly see that of the other. Having acquired skill, at this short distance, let the two experimenters, equipped as before, take up positions separated by a distance of two or three miles and let them perform the same experiment at night, noting carefully whether the exposures and occultations occur in the same manner as at short distances; if they do, we may safely conclude that the propagation of light is instantaneous; but if time is required at a distance of three miles which, considering the going of one light and the coming of the other, really amounts to six, then the delay ought to be easily observable. If the experiment is to be made at still greater distances, say eight or ten miles, telescopes may be employed, each observer adjusting one for himself at the place where he is to make the experiment at night; then although the lights are not large and are therefore invisible to the naked eye at so great a distance, they can readily be covered and uncovered since by aid of the telescopes, once adjusted and fixed, they will become easily visible. . . .

He seeks to apply to astronomical phenomena the new discoveries in magnetism.

Everywhere the mathematical and inductive method became manifest in this man. Almost all domains of science received therefrom the most powerful impulse. And above all the whole field of science was freed from the outgrowths of metaphysical modes of thought with which it had been previously so overrun. Galileo's

> individual method consisted namely in always conforming to the limits of scientific investigation, and confining his attention to seizing the phenomena sharply in their progress and in their relation with allied processes, without wandering into a fruitless search after the ultimate bases of the phenomena. — *Dannemann.*

Such a limitation has been of the highest value for the renewal of natural science as it followed at the beginning of the seventeenth century.

Galileo was not chiefly interested in mathematics, but he emphasizes the dependence of other sciences upon it.

> True philosophy expounds nature to us; but she can be understood only by him who has learned the speech and symbols in which she speaks to us. This speech is mathematics, and its symbols are mathematical figures. Philosophy is written in this greatest book, which continually stands open here to the eyes of all, but cannot be understood unless one first learns the language and characters in which it is written. This language is mathematics and the characters are triangles, circles and other mathematical figures.

He gives an acute discussion of infinite, infinitesimal and continuous quantities leading up to the conclusion "that the attributes 'larger,' 'smaller,' and 'equal' have no place either in comparing infinite quantities with each other or in comparing infinite with finite quantities." Again "the finite parts of a *continuum* are neither finite nor infinite but correspond to every assigned number."

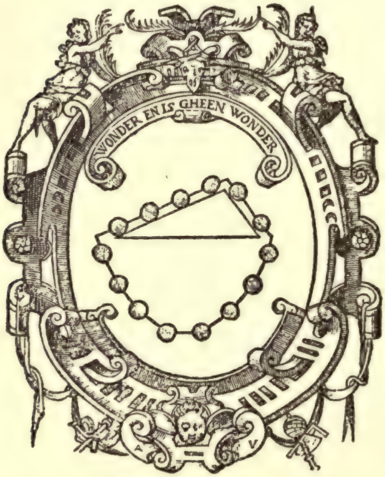
In commenting on Galileo's achievements, Lagrange the great mathematician of the eighteenth century says: —

> These discoveries did not bring to him while living as much celebrity as those which he had made in the heavens; but to-day his work in mechanics forms the most solid and the most real part of the glory of this great man. The discovery of Jupiter's satellites, of the phases of Venus, and the Sun-spots, etc., required only a telescope and assiduity; but it required an extraordinary genius to unravel the laws of nature in phenomena which one has always under the eye, but the explanation of which, nevertheless, had always baffled the researches of philosophers.

Leonardo da Vinci likens a scientific conquest to a military victory in which theory is the field marshal, experimental facts the soldiers. The philosophers who preceded Galileo had, in the main, been trying to fight battles without soldiers. — *Crew.*

A Pioneer in Mechanics. Stevinus. — Even before Galileo, Stevinus of Bruges (1548–1620), a man who thought independently on mechanical problems, made the first really important advances since Archimedes, eighteen centuries earlier. Besides engaging in mercantile pursuits he was quartermaster-general of the Dutch army, and an authority on military engineering. He was influential in improving methods of public statistics and accounting, and advocated decimal weights and measures. Appreciating the possibilities of the decimal fraction he asserted (1585) that fractions are quite superfluous, and every computation can be made with whole numbers, but he did not realize the simplest notation. The honor of this great invention he shares with Bürgi of Cassel. Another of his inventions was a sailing carriage carrying 28 people and outstripping horses.

In a treatise on Statics and Hydrostatics (1586) he introduced comparatively new and powerful geometrical methods for dealing with mechanical problems. Among the most interesting is his discussion of the inclined plane by means of an endless chain hanging freely over a triangle with unequal sides. Excluding the inadmissible hypothesis of perpetual motion, the uniform chain must be in equilibrium in any position. The hanging portion is by itself in equilibrium, therefore the two inclined sections must balance each other, and either would be balanced by a vertical force corresponding to the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Stevinus' Triangle.

altitude of the triangle. Arriving thus at the parallelogram of forces in equilibrium, he expresses his astonishment by exclaiming "Here is a wonder and yet no wonder."

In studying pulleys and their combinations he arrives at the far-reaching result that in a system of pulleys in equilibrium "the products of the weights into the displacements they sustain are respectively equal" — a remark containing the principle of virtual displacement. He reaches correct results in regard to basal and lateral pressure by reasoning analogous to that about the chain, and by assuming on occasion that a definite portion of the liquid is temporarily solidified. By ingenious experiments he proves the dependence of fluid pressure on area and depth, and takes proper account of upward and lateral pressure. He studies the conditions of equilibrium for floating bodies, showing that the centre of gravity of the body in question must lie in a perpendicular with that of the water displaced by it, and that the deeper the centre of gravity of the floating body the more stable is the equilibrium.

In analyzing the lateral pressure of a fluid Stevinus anticipates the calculus point of view by dividing the surface into elements on each of which the pressure lies between ascertainable values. Increasing the number of divisions, he says it is manifest that one could carry this process so far that the difference between the containing values should be made less than any given quantity however small — all quite in harmony with our present definitions of a *limit*.

Stevinus' work and that of Galileo seem to have been quite independent of each other, the former confining his theory to statics, the latter laying a solid foundation for the new science of dynamics. Torricelli, a disciple of Galileo best known for his invention of the mercurial barometer, extended dynamics to liquids, studying the character of a jet issuing from the side of a vessel.

Throughout this period the universities lagged. In Italy Galileo lectured to medical students who were supposed to need astronomy for medical purposes — *i.e.* astrology. At Wittenberg there was a professor for arithmetic and the sphere, and one for Euclid, Peurbach's planetary theory and the Almagest, but their students were few. . . . So, says a German writer, we face the extraordinary fact that the most educated of the nation were as helpless in the problems of daily life as a marketwoman of to-day. The university lectures in mathematics were mainly confined to the most elementary computation, — matters taught more thoroughly in the commercial schools, particularly after the invention of printing.

GIORDANO BRUNO (1548–1600). — In the Appendix will be found the judgment and sentence of the Inquisition upon Galileo, together with his recantation, — one of the darkest pages in the history of Science. Another victim of the Inquisition was Bruno, an Italian philosopher, who, having joined the Dominican order at the age of fifteen, was later accused of impiety and subjected to persecution. Bruno fled from Rome to France, and later to England, where at Oxford he disputed on the rival merits of the Copernican and the so-called Aristotelian systems of the universe. In 1584 he published an exposition of the Copernican theory. Bruno, moreover, attacked the established religion, jeered at the monks, scoffed at the Jewish records, miracles, etc., and after revisiting Paris, and residing for a time in Wittenberg, rashly returned to Italy, where he was apprehended by the Inquisition and thrown into prison. After seven years of confinement he was excommunicated and, on Feb. 17, 1600, burnt at the stake. In 1889 a statue in his honor was unveiled in Rome at the place of his execution, the Square of the Flower Market. Thus was the end of the sixteenth century illuminated by the flames of martyrdom.

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CHAPTER XII

NATURAL AND PHYSICAL SCIENCE IN THE SEVENTEENTH CENTURY

THE CIRCULATION OF THE BLOOD: HARVEY (1578–1657).—The blood has always been regarded as one of the principal parts of the body. Hippocrates considered it one of his four great "humors," and in the Hebrew Scriptures it is stated that "the blood . . . is the life." Yet up to the seventeenth century nothing definite was known of its movements throughout the body. That it was under pressure must have been known, for it flowed or "escaped" freely from wounds, and flow results only from pressure of some sort, while escape is relief from detention. The arteries had been misinterpreted for centuries and were early considered to be air tubes, because they were studied only after death when as we now know they are empty. Even the dissections of the anatomists of the sixteenth century had failed to reveal the complete and true office of the arteries, and it remained for Harvey, an English pupil of the Italian anatomist Fabricius, to make — largely through the vivisection of animals and observation of the heart and arteries in actual operation — discoveries of basic importance in anatomy, physiology, embryology, and medicine (see Appendix).

While working in Italy, Harvey learned of and doubtless saw the *valves in the veins* which were discovered by Fabricius. These valves are thin flaps of tissue so placed as to check the flow of blood in one direction while offering no resistance to that flowing the other way. On his return to England, Harvey apparently pondered on the function of these valves and saw that they could be of use only by permitting the flow of the blood in one direction while preventing its movement in the opposite direction. At this time it was supposed that the blood simply oscillated, or moved back and forth like a pendulum, a view which, if the valves had any meaning, was now plainly untenable. Harvey therefore set to work to study the beating of the heart and the flowing of the blood, and soon came to the conclusion that there must be a steady flow or streaming *in one direction*, and not an oscillation back and forth as was generally supposed. But to prove was here, as always, harder than to believe, and much time and labor were required to settle the question. At length, however, by dissections and vivisections of the lower animals, and after publishing (in 1628) a brochure presenting his facts and meeting objections, Harvey succeeded, with the result that his name justly stands to-day beside those of the Greek and Alexandrian Fathers of Medicine, Hippocrates and Galen. It is one of the ironies of fate that while Harvey rightly reasoned from circumstantial evidence that the blood must steadily flow from the arteries to the veins, he himself never actually saw that flowing, — a sight which any schoolboy may now see, but impossible before the introduction of the microscope, and first enjoyed by Malpighi in 1661, only four years after Harvey's death.

In embryology, also, Harvey proved himself an original and penetrating observer. In his day and earlier it was supposed that the embryo, in the hen's egg, for example, exists even at the very outset as a perfect though extremely minute chick, with all its parts complete. This "*preformation*" theory was opposed by Harvey, whose doctrine of "*epigenesis*" was substantially that of modern embryology: viz. that the embryo chick is gradually formed by processes of growth and differentiation from comparatively simple and undifferentiated matter, somehow set apart and prepared in the body of the parents.

ATMOSPHERIC PRESSURE: TORRICELLI'S BAROMETER. — The problem of the existence and nature of voids and vacua had always been an interesting puzzle for philosophers. The Greeks assumed the existence of empty spaces or "voids," and as late as the age of Elizabeth it was the orthodox belief that "nature abhors a vacuum." Galileo, even, held to it in 1638. (Cf. p. 246.)

Evangelista Torricelli (1608–1647), inspired by the Dialogues of Galileo (1638), published on Motion and other subjects in 1644.

He resided with Galileo and acted as his amanuensis from 1641 until Galileo's death. In experimenting with mercury he found that this did not rise to 33 feet, but instead to hardly as many inches. He next proved, by comparing the specific gravity of water and mercury, that the same "pressure" was at work in both cases, and boldly affirmed that this pressure was that of the atmosphere. The tube of mercury used in his experiments was what we now call a barometer (*baros*, weight), but it was for a long time called "the Torricellian Tube," as the empty space above the mercury is still called the "Torricellian vacuum." This invention or discovery of Torricelli's was one of the most fertile ever made, for at one blow it demolished the ancient superstition that "nature abhors a vacuum," explained very simply two ancient puzzles (why water rises in a pump, and why it rises only 33 feet), determined accurately the weight of the atmosphere, proved it possible to make a vacuum, and gave to mankind an entirely new and invaluable instrument, the barometer. Torricelli's results and explanations were received at first with incredulity, but were soon confirmed, notably by Pascal (1623–1662) in a treatise, New Experiments on the Vacuum. In one of these Pascal used wine instead of water or mercury in the Torricellian tube, with satisfactory results, and in another, reasoning that if Torricelli were right, liquids in the tube should stand lower on a mountain than in a valley, persuaded his brother-in-law, Perier, to ascend the Puy de Dôme (near Clermont, France) in September, 1648, on which mountain the column was found to be much shorter. This and other brilliant work by Pascal have given him a high rank among natural philosophers.

Since it was now easy to obtain a vacuum by the Torricellian experiment, fresh attempts were made to produce vacua otherwise. Von Guericke, burgomaster of Magdeburg in Hannover, after many failures, finally succeeded in pumping the air out of a hollow metallic globe. It was in this experiment that the air-pump was introduced. Guericke found that his globe had to be very strong to resist crushing by the atmospheric pressure, and in the popular demonstration now known as that of the Magdeburg hemispheres he showed that eight horses on either side were unable to overcome this pressure on a particular globe which he had constructed and exhausted of air. These various experiments and discoveries relating to atmospheric pressure led to the investigations and laws of Boyle, Mariotte, and others and, less than a century later, to the steam-engine of Watt, in which steam was at first used only to produce a vacuum, — atmospheric pressure being employed as the moving force.

FURTHER STUDIES OF THE ATMOSPHERE: GASES. — Meantime the chemical composition of the atmosphere was being no less eagerly studied. Robert Boyle (1627–1691) published at Oxford in 1660, New Experiments Physico-Mechanical touching the Spring of the Air and its Effects, and in his Sceptical Chymist gives an interesting and instructive picture of the chemical ideas of his time. He was the first to insist on the difference between compounds and mixtures, and probably the first to use the pneumatic trough for the collection and study of gases.

The word "gas" was introduced by Van Helmont (1577–1644), who by virtue of the following remarkable statement deserves to be remembered as the principal chemist of the earlier half of the seventeenth century: —

> Charcoal and in general those bodies which are not immediately resolved into water, disengage by combustion *spiritum sylvestrum.* From 62 lbs. of oak charcoal 1 lb. of ash is obtained, therefore the remaining 61 lbs. are this *spiritum sylvestre.* This spirit, hitherto unknown, I call by the new name of *gas.* It cannot be enclosed in vessels or reduced to a visible condition. There are bodies which contain this spirit and resolve themselves entirely into it: in these it exists in a fixed or solidified form, from which it is expelled by fermentation, as we observe in wine, bread, etc.

It has been well said that

> this passage is remarkable not only for the explicit mention of carbonic acid gas (as we now call it) as a product of fermentation, and for the introduction of the word *gas* for the first time, but also for its appeal to the *balance,*

the formal introduction of which into chemistry was only made a century later by Lavoisier. Van Helmont also points out that his *gas sylvestre* is produced by the action of acids on shells, is engendered in putrefaction and combustion, and is present in caves, mines, and mineral waters. In these ideas and passages we find an agreeable departure from the mysticism of the alchemists and the wild surmises of Paracelsus. At the same time Van Helmont's ideas in other directions were crude enough, since he is credited with a recipe for the artificial production of mice from "corn and sweet basil."

FROM PHILOSOPHY TO EXPERIMENTATION. — The seventeenth century differs from all before it in the increasing attention paid to experimental science. From the philosophizing of Paracelsus and Gilbert it is agreeable to pass to the experimental work of Harvey, Torricelli and in chemical inquiries to Van Helmont, whose logical successor is Robert Boyle (1627–1691), already mentioned for his work on the resistance, or "spring," of the atmosphere, etc. Among many other ingenious experiments Boyle worked on evaporation, in air and *in vacuo;* on boiling and on freezing; and on the effects of exposing animals to the diminished atmospheric pressure produced by the air-pump. In this direction he was the first to prove that fishes require air dissolved in the water in which they live. He also studied the rusting of metals — a problem then widely discussed — and from all his studies concludes that there is in the atmosphere some vital substance which plays a principal part in such phenomena as combustion, respiration, and fermentation. When this substance has once been consumed, flame is instantly extinguished, and yet the air from which it has gone seems nearly intact. He wrote a treatise entitled, Fire and Flame weighed in a Balance, in which he described the increase of weight of metals on calcination. But as he got about the same results whether the crucible was open or shut, he was misled into the belief that the air had little to do with his results, which he attributed rather to the fixation of the "fire" by the porous crucibles. In these and Boyle's other experiments it is plain that we are rapidly moving from alchemical and iatro-chemical stages toward the modern experimental period of chemistry, of which he and Van Helmont are the pioneers. Neither, however, while working on air greatly advanced our ideas of atmospheric chemistry.

The atmosphere in its relation to combustion and respiration was further studied by an English physician, Dr. John Mayow (1645–1679), who made many experiments upon the shrinkage of air-volume during the burning of camphor and other substances and during the confinement of mice under a bell-glass. The dying of the mice and the cessation of the combustion, which after a time ensued, he attributed to the exhaustion of some ingredient in the air indispensable to life and combustion. This ingredient, which we now call oxygen, Mayow named "fire-air."

Very soon, however, a new theory of combustion (and as it turned out a false theory) began to absorb the attention of natural philosophers.

FROM ALCHEMY TO CHEMISTRY. — The saying is attributed to Liebig that "Alchemy was never at any time different from chemistry." In one sense this is undoubtedly true. The search for "the philosopher's stone," "the elixir of life," "potable gold," and the "transmutation of metals," consisted of necessity in the use of processes such as boiling, baking, wetting, drying, evaporating, condensing, burning, calcifying, decalcifying, acidifying, freezing, melting, and the like, mostly tending towards chemical changes and the formation of new mixtures and compounds. But even if Liebig's saying were true, chemistry has passed through three principal stages; viz. first, purely empirical experimenting, mostly for practical purposes, whether metallurgical or other; second, an iatro-chemical or medico-chemical phase; and finally the really scientific period of to-day, the way for which may be said to have been cleared by the Sceptical Chymist of Robert Boyle,[1] first published in English in Oxford in 1661. In this remarkable little book Boyle by means of a dialogue discusses and sharply criticises the chemistry of the "hermetick" (*i.e.* Aristotelian) natural philosophers, and also "the vulgar Spagyrists" (*i.e.* the medico-chemists of the Paracelsus type) and questions the value of terms then hazy in their meaning, such as "element" and "principle," as used in alchemy. He does not himself propound any new theories of consequence, but he does insist on more knowledge, more experimentation, and less groundless speculation. We quote from Professor Pattison Muir's valuable introductory essay to the "Everyman" edition: —

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[1] The Hon. Robert Boyle was one of the most active, perhaps the most so, of that remarkable group of scientific investigators who, in the reign of Charles II., raised England to the foremost place among European nations in the pursuit of science, and gave their period a renown which has caused it to be often spoken of, and very justly, as the classical age of English science. . . . Boyle had been since 1646 engaged in chemical researches in London, being then connected with the earlier

> *The Sceptical Chymist* embodies the reasoned conceptions which Boyle had gained from the experimental investigations of many physical phenomena. . . . The book is more than an elegant and suggestive discourse on chemico-physical matters; it is an elucidation of the true method of scientific inquiry. . . . At that time the alchemical scheme of things dominated most of those who were inquiring into the transmutations of material substances. That scheme was based on a magical conception of the world. . . . When a magical theory of nature prevails, the impressions which external events produce on the senses of observers are corrected, not by careful reasoning and accurate experimentation, but by inquiring whether they fit into the scheme of things which has already been elaborated and accepted as the truth. Natural events become as clay in the hands of the intellectual potter for whom 'there is nothing good or bad but thinking makes it so.' . . . An alchemical writer of the seventh century said: 'Copper is like a man; it has a soul and a body.' . . . It is not possible to attach any definite, clear, meanings to alchemical writings about the four elements. Their indefiniteness was their strength. . . . As the plain man to-day is soothed and made comfortable by the assurance that certain phrases to which he attaches no definite meanings are really scientific, so, when Boyle lived, the plain man rested happily in the belief that the four elements were the last word of science regarding the structure of the materials of the world. . . .

group of scientific inquirers in London known as the 'Invisible College' . . . Boyle, too, we must observe, was above all things unprejudiced. He had leanings towards alchemy and never quite repudiated a belief in the possibility of transmuting metals. In medical matters, which greatly interested him, he showed perfect tolerance towards those whom the profession called quacks. — *J. F. Payne.*

Boyle found the same fault with the 'Principles' of the 'Vulgar Spagyrists' as he found with the 'Elements' of the 'hermetick philosophers.' 'Tell me what you mean by your Principles and your Elements,' he cried; 'then I can discuss them with you as working instruments for advancing knowledge.'

'Methinks the Chymists in their search after truth are not unlike the navigators of Solomon's Tarshish Fleet, who brought home from their long and perilous voyages not only gold and silver and ivory but apes and peacocks too: for so the writings of several (I say not all) of your hermetick philosophers present us, together with diverse substantial and noble experiments, theories which, either like peacocks' feathers, make a great show, but are neither solid nor useful, or else like apes, if they have some appearance of being rational, are blemished with some absurdity or other that, when they are attentively considered, makes them appear ridiculous.'

The fact that at the middle of the seventeenth century criticism of this sort seemed to Boyle to be needed shows how little real progress toward modern scientific chemistry had even then been made; and, as often happens, truth had to be reached through further error.

A FALSE THEORY OF COMBUSTION: PHLOGISTON. — Two German contemporaries of Boyle, Becher (1625–1682), and Stahl (1660–1734), as a result of studies on combustion and the calcining of metals, departed from the four elements of antiquity and assumed the participation in these processes of a something dispelled by heating. To this something Stahl gave the name *phlogiston*, "the combustible substance, a principle of fire, but not fire itself." And because from a metallic calx (oxide) the metal could be recovered by burning with charcoal, the metal was held to have absorbed "phlogiston" in the process from the charcoal, which, having mostly disappeared, was regarded as almost pure phlogiston. Conversely, when the metal was calcined (or oxidized) by burning without charcoal, it was held to have lost its phlogiston. This theory, which to-day seems bizarre, satisfied the chief requirement imposed on any new theory: viz. that of accounting for the facts (as then known), and was therefore naturally accepted and advocated by natural philosophers for the next hundred years. It was not until new facts had been accumulated which were not explained by the theory of Becher and Stahl, and especially the fact revealed by the use of the balance, that substances calcined often *gained* weight (making it necessary to assume that phlogiston possessed negative gravity, or "levity" since its loss increased weight), that the theory became plainly untenable and was abandoned. This, however, only happened late in the eighteenth century, and before this time much progress had been made in chemistry in other directions. Meanwhile, in spite of its falsity, the theory of phlogiston had done good service. It had, for example, effectually turned the attention of chemists away from magic, from potable gold, and from the making of medicines, to speculations on composition, decomposition, and chemical change, — topics not only more worthy but more fruitful.

BEGINNINGS OF ORGANIC CHEMISTRY. — Meantime, a kind of organic chemistry was initiated by Hermann Boerhaave (1668–1738), a physician of Leyden. In the seventeenth and eighteenth centuries the term "organic" stood more than it does to-day for the living world and its products which were then regarded as things altogether apart from the lifeless or inorganic world. To-day organic chemistry hardly means more than the chemistry of the carbon compounds, but at that time it meant the chemistry of bodies found in or produced by living things. Medical men had long been interested in alchemy, and in more modern times in iatrochemistry, so that it was natural enough that Boerhaave, a physician, should undertake to subject organic substances to chemical processes. And this he did, though more in the fashion of the pharmaceutical, than the analytical, chemist of to-day. Boerhaave was a famous teacher of medicine and of botany, and crowds of students attended his lectures, thereby testifying to the now rapidly growing popularity of scientific learning. His Elements of Chemistry, published in 1732, was widely used and marks an epoch in the history of chemistry.

At about the same time, Dr. Stephen Hales (1677–1761), an English clergyman of a strongly scientific bent, did similar work in England. In addition, Hales made important studies on the atmosphere, and invented the *manometer,* which he applied to the measurement of the arterial blood pressure in the horse, and the upward root pressure in plants, besides accomplishing much other good work. Hales will perhaps be longest remembered in chemistry for his skilful use of the *pneumatic trough,* a simple but indispensable laboratory appliance for the easy collection of gases in a closed vessel over water, and especially for his studies on air.

MEDICAL SCIENCE AND MEDICAL THEORY IN THE SEVENTEENTH CENTURY. THOMAS SYDENHAM. — In the middle of the seventeenth century medical theory took a long step forward under the influence of Thomas Sydenham (1624–1689), often called "the English Hippocrates" because of the naturalism and rationalism which he urged in medicine and because of the sanity of his opinions and theories. Setting aside magic, mysticism, and the medical chemistry of Paracelsus, and insisting on a material basis (*materies morbi*) for the causes of disease, Sydenham laid the foundations of modern scientific medical philosophy and practice. He was a close friend of Locke, the philosopher, — by whose materialistic and rationalistic ideas he was doubtless influenced, — and was also a correspondent of Boyle. His famous definition of disease as, "An effort of nature, striving with all her might to restore the patient by the elimination of morbific matter," is still interesting for its implication of the modern idea of disease as a struggle for existence between pathogenic matters (such as microbes) and the inner forces of the body.

It is, however, to Vesalius and Harvey, to Leeuwenhoek and Kircher and Malpighi and the other microscopists of the seventeenth century, and their successors, *i.e.* to the experimenters and laboratory workers, rather than to Sydenham or his successors, that medical science is chiefly indebted, since no great progress could be made in sound medical theory or rational medical practice until anatomy, physiology and microscopy had paved the way for a more scientific pathology.

THE BEGINNING OF MODERN IDEAS OF LIGHT AND OPTICS. — The nature of light, darkness and vision are very old problems.

It is easy, even for savages, to account for daylight as sunlight, and the corresponding nightlight as moonlight and starlight, but for more highly developed man to explain just what the light is which comes from sun, moon and stars, is not so easy. Obviously, since "luminaries" — sun, moon, stars, firebrands and torches — produce light which is weaker as distance from the source increases, a kind of "emission" theory of light is natural and reasonable. It was even held by the ancients that we see by means of light emitted from our own eyes, and that light is a more or less palpable substance. A similar error was held concerning heat, which until the end of the eighteenth century was generally regarded as a peculiar material body or substance, "caloric," which when absorbed from other bodies produced a state of heat, and when emitted caused, by its absence, cold.

How men could have believed for ages that objects are rendered visible by something projected from the eye itself — so that the organ of sight was supposed to be analogous to the tentacula of insects, and sight itself a mere species of touch — is most puzzling. They seem not till about 350 B.C. to have even raised the question: If this is how we see, Why cannot we see in the dark? or, more simply; What is darkness? The former of these questions seems to have been first put by Aristotle.

The ancients probably understood that light travels in straight lines, and they must have known something about reflection and refraction of light, for they knew about images in still water, and had mirrors of polished metal, and burning glasses of spherical glass shells, or balls of rock crystal. To Hero of Alexandria we owe the important deduction from the Greek geometers that the course of a reflected ray is the shortest possible (p. 123).

The perfection of gem cutting among the ancients has also been held to prove their acquaintance with lenses. But it was not until the seventeenth century that modern ideas of light and optics began to be formulated, with the work of Snellius, Descartes and Newton on reflection and refraction, and of Römer on the velocity, of light.

Every student should read the earlier parts of Newton's Optics in which are described the fundamental experiments on the decomposition of white light. — LORD RAYLEIGH.

The work of Christian Huygens, towards the end of the seventeenth century, second only to that of Newton, both in extent and importance, touched upon a great variety of subjects, including some in the natural sciences. As a young man he wrote upon geometry; in early middle life he invented the cycloidal pendulum. He was the first to apply pendulums to clocks and spiral springs to watches, and to devise the achromatic eye-piece which still bears his name. He also made a telescope and, finally, at the age of fifty, observed the phenomena of polarization and, most important of all, proposed the modern wave theory of light.

THE FIRST SCIENTIFIC INSTRUMENTS: TELESCOPE, BAROMETER, THERMOMETER, AIR-PUMP, MICROSCOPE, MANOMETER. — The complete history of the origin of the telescope, the thermometer and the microscope is not known. The account usually given of the invention of the telescope makes it accidental and due to the children of a Dutch spectacle maker, named Jansen, who while at play happened to bring together two lenses in such a way that a distant church spire seen through them looked magnified and near. The father, whose attention was drawn to the phenomenon, seeing in the arrangement a source of profit, thereupon made and sold the combination as a toy or "wonder," under which form it was on sale in 1609, becoming known to Galileo, who instantly realized its importance and made improvements in it. It appears that soon after 1609 Galileo had a fairly good instrument, magnifying 8 diameters, with which he was quickly and easily able to make some of his most splendid astronomical discoveries.

The early history of the telescope shows that the effect of combining two lenses was understood by scientists long before any particular use was made of this knowledge; and that those who are accredited with introducing perspective glasses to the public hit by accident upon the invention. Priority was claimed by two firms of spectacle-makers in Middelburg, Holland, namely Zacharias, miscalled

Jansen, and Lippershey. Galileo heard of the contrivance in July 1609 and soon furnished so powerful an instrument of discovery that . . . he was able to make out the mountains in the moon, the satellites of Jupiter in rotation, the spots on the revolving sun . . .

About 1639, Gascoigne, a young Englishman, invented the micrometer which enables an observer to adjust a telescope with very great precision.

The history of the microscope is closely connected with that of the telescope. In the first half of the seventeenth century the simple microscope came into use. It was developed from the convex lens . . . Leeuwenhoek before 1673 had studied the structure of minute animal organisms and ten years later had even obtained sight of bacteria. Very early in the same century Zacharias had presented Prince Maurice, the commander of the Dutch forces, and the Archduke Albert, Governor of Holland, with compound microscopes. Kircher (1601–1680) made use of an instrument that represented microscopic forms at one thousand times larger than their actual size. — Libby. *Introduction to the History of Science.*

The name of Galileo goes also with the invention of the thermometer, an air, or more strictly a water, thermometer having been introduced by him about 1597. Mercury was not substituted for water until 1670, but alcohol thermometers, also introduced by Galileo, were used much earlier. The freezing and boiling of water were supposed to take place at variable temperatures and it was not until the end of the seventeenth century that it was realized that the freezing and the boiling points are invariable. (For Galileo's other work in physics, see pp. 246–250.) Pendulum clocks, "aërial" telescopes and the achromatic eye-pieces which bear his name were introduced by Huygens, the first in 1657 and the others about 1680.

The invention of the barometer by Torricelli has already been described above (p. 257). The air-pump, though merely the application of an ordinary pump to air instead of water, was so rich in its results that it deserves a high place among the other and more important inventions of this remarkable scientific era.

About the origin of the (compound) microscope there is much the same obscurity as about that of the telescope. Simple microscopes such as "magnifiers," burning glasses, spectacles, and other lenses, had long been known, — some of them from antiquity, — but the compound microscope, which consists of two lenses or combinations of lenses so placed as to coöperate in the production of one highly magnified image of a near and minute object (the telescope doing the same for large and distant objects), first appears about 1650. Some of the earliest microscopists are Kircher, Leeuwenhoek, Malpighi, and Grew. The two former apparently saw with the microscope and made drawings of bacteria, besides many other micro-organisms and cellular structures. The two latter are the founders of microscopic anatomy, Malpighi of that of animals, Grew of that of plants. Malpighi's work is especially notable, since he for the first time actually observed the passage of blood cells from arteries to veins, and that in 1661 only four years after Harvey's death. Malpighi's name is also familiar to students of human anatomy and physiology in connection with those parts of the kidneys and the spleen which bear his name. The versatile and accomplished Englishman Dr. Robert Hooke (1635–1703), who flourished in this century and did ingenious, extensive, and often remarkable work at the basis of almost every branch of modern science, was the first to discover by the microscope the cellular structure of living things. Hooke was one of the original members of the Royal Society, with which Leeuwenhoek also corresponded.

The most remarkable fact connected with the invention of the compound microscope is that, because of its physical imperfections, and in spite of some use as just described, it was virtually abandoned for almost a century and a half, and only re-introduced after the invention and perfection of the *achromatic objective* in the first quarter of the nineteenth century. The truth appears to be that owing to excessive spherical and chromatic aberration the compound microscope of the seventeenth and eighteenth centuries was of limited value, and that microscopists often preferred the less powerful, but more perfect, simple microscope.

The manometer was apparently first used by Stephen Hales, who measured with it the blood pressure of a horse, the root pressure of plants, etc. It is described in his Statical Essays (1727) and Haemostaticks (1733).

ORGANIZATION OF THE FIRST SCIENTIFIC ACADEMIES AND SOCIETIES. — The Academy of Plato (fifth century B.C.), and the Lyceum of Aristotle, the Museum at Alexandria (third century B.C.), and the so-called Academy of Alcuin (in the eighth century A.D.) may be regarded as precursors of the academies and societies of the Renaissance, but — with the possible exception of an academy formed by Leonardo da Vinci in the fifteenth century — the first devoted chiefly to science was probably that founded by della Porta at Naples in 1560 and named *Academia Secretorum Naturae.* The requirement for membership was to have made some discovery in natural science. Della Porta fell under ecclesiastical suspicion as a practitioner of the black arts, and though acquitted was ordered to close his "Academy." The *Accademia dei Lincei* (of the Lynx), founded at Rome in 1603, included both della Porta and Galileo among its early members, and still flourishes. Its device is a lynx with upturned eyes.

The Royal Society of London, like many other societies, was the outgrowth of meetings of friends for discussion and was chartered in 1662. (For Boyle's *Invisible College* see above, p. 261.)

Among the earlier members of the Royal Society were Boyle and Hooke, Mayow, Huygens, Ray, Grew, Malpighi, Leeuwenhoek, and Isaac Newton. A well-known passage quoted by Huxley from Dr. Wallis, one of the first members, is of special interest since it shows what subjects were most dwelt upon by men of science at the time of Cromwell and the Restoration: —

> Some twenty years before the outbreak of the plague (1665), says Huxley, a few calm and thoughtful students banded themselves together for the purpose, as they phrased it, of 'improving natural knowledge.' The ends they proposed to attain cannot be stated more clearly than in the words of one of the founders of the organisation: —
>
> 'Our business was (precluding matters of theology and state affairs) to discourse and consider of philosophical enquiries, and such as related thereunto: — as Physick, Anatomy, Geometry, Astronomy, Navigation, Staticks, Magneticks, Chymicks, Mechanicks, and

Natural Experiments; with the state of these studies and their cultivation at home and abroad. We then discoursed of the circulation of the blood, the valves in the veins, the *venæ lacteæ,* the lymphatic vessels, the Copernican hypothesis, the nature of comets and new stars, the satellites of Jupiter, the oval shape (as it then appeared) of Saturn, the spots on the sun and its turning on its own axis, the inequalities and selenography of the moon, the several phases of Venus and Mercury, the improvement of telescopes and grinding of glasses for that purpose, the weight of air, the possibility or impossibility of vacuities and nature's abhorrence thereof, the Torricellian experiment in quicksilver, the descent of heavy bodies and the degree of acceleration therein, with divers other things of like nature, some of which were then but new discoveries, and others not so generally known and embraced as now they are; with other things appertaining to what hath been called the New Philosophy, which from the times of Galileo at Florence, and Sir Francis Bacon (Lord Verulam) in England, hath been much cultivated in Italy, France, Germany, and other parts abroad, as well as with us in England.'

The learned Dr. Wallis, writing in 1696, narrates in these words what happened half a century before, or about 1645.

Among the first publications of the Royal Society of London were the works of Malpighi, the Italian microscopical anatomist, in 1669, and others by Leeuwenhoek, the Dutch microscopist.

The French Academy (*Acadêmie des sciences*) began its meetings in 1666, and the corresponding Berlin Academy in 1700.

The oldest American association for the promotion of science is the American Philosophical Society Held at Philadelphia for Promoting Useful Knowledge, proposed by Benjamin Franklin in 1743 and finally organized in 1769. Franklin himself presided over it from 1769 until his death in 1790.

THE NEW PHILOSOPHY: BACON AND DESCARTES.—It has been shown above how the all-inclusive philosophy of their predecessors began with Plato and Aristotle to be divisible into general and "natural" philosophy,—a differentiation which continued to exist and to increase slowly through the Middle Ages and the Renaissance.

We have also shown how the mariner's compass, the invention of printing, the discovery of the New World, the heliocentric hypothesis, the idea of the earth as a magnet, the exploration of the human body, the Reformation, and the progress of mathematical science, were already widely opening men's minds, so that by the end of the sixteenth century it is not surprising to find the new knowledge reacting upon the old philosophy. With this movement two great names will always be associated: viz. those of Francis Bacon and René Descartes.

Bacon, because of his official position and immense philosophical and literary ability, was able to draw universal attention to the methods of science and especially to the method of investigation by induction, so that his indirect service to science was great. Bacon's true place in science was, however, well understood by his contemporaries, for one of the greatest, Harvey, discoverer of the circulation of the blood, remarks that, "the Lord Chancellor writes of science like — a Lord Chancellor."

Descartes, far more important than Bacon in respect to his contributions to various branches of science, likewise stirred the intellect of Europe and helped to bring about those changes in the old philosophy which in the minds of many made it new. Descartes was not only a mathematician of the first rank but an ingenious and original worker in many branches of scientific inquiry such as music, anatomy, physiology, optics, etc. It is to him that we owe the first ideas of mechanism in living bodies, his notion of a "man machine" being highly original and suggestive.

Science, says Descartes, may be compared to a tree; metaphysics is the root, physics the trunk, and the three chief branches are mechanics, medicine, and morals.

Here are my books, he is reported to have told a visitor, as he pointed to the animals which he had dissected.

The conservation of health, he writes in 1646, has always been the principal end of my studies.

Bacon and Descartes were methodologists, both urging the fundamental importance to progress, of method and its right use in investigation and inquiry, and Descartes, younger by almost a generation, admired and to some extent imitated his predecessor in this direction.

PROGRESS OF NATURAL AND PHYSICAL SCIENCE IN THE SEVENTEENTH CENTURY. — A mere glance at the Tabular View of Chronology in the Appendix will suffice to show the immense superiority of the seventeenth to any preceding century in the number as well as the productivity of the workers devoted to the mathematical, and likewise to the natural and physical, sciences. The achievements of this century in natural philosophy are especially notable both for their fundamental character and their wide range. A century which began with a Galileo and ended with a Huygens and a Newton; which witnessed the introduction of the telescope, the barometer, the thermometer, the air-pump, the manometer, and the microscope, as well as the organization of the greatest and most useful scientific societies the world has hitherto known, must be forever famous. And when to the names and works of Galileo and Huygens and Newton we add those of Kepler, Harvey, Torricelli, Halley, Descartes, Boyle, Hales, Boerhaave, Leeuwenhoek, and Malpighi, we have a brilliant company indeed.

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CHAPTER XIII

BEGINNINGS OF MODERN MATHEMATICAL SCIENCE

. . . . All the sciences which have for their end investigations concerning order and measure, are related to mathematics, it being of small importance whether this measure be sought in numbers, forms, stars, sounds, or any other object; that, accordingly, there ought to exist a general science which should explain all that can be known about order and measure, considered independently of any application to a particular subject, and that, indeed, this science has its own proper name, consecrated by long usage, to wit, *mathematics*. And a proof that it far surpasses in facility and importance the sciences which depend upon it is that it embraces at once all the objects to which these are devoted and a great many others besides. . . . — *Descartes.*

As long as algebra and geometry proceeded along separate paths, their advance was slow and their applications limited. But when these sciences joined company, they drew from each other fresh vitality and thenceforward marched on at a rapid pace toward perfection.
— *Lagrange.*

The application of algebra has far more than any of his metaphysical speculations, immortalized the name of Descartes, and constitutes the greatest single step ever made in the progress of the exact sciences. — *Mill.*

The idea of *coördinates* which forms the indispensable scheme for making all processes visible, with its many-sided and stimulating applications in all branches of daily life, — whether medicine, physical geography, political economy, statistics, insurance, the technical sciences — the first beginnings of the calculus in their historical evolution, the development of the ideas of function and limit in connection with the elementary theory of curves, these are things without which in the present day not the slightest comprehension of the phenomena of nature can be attained, of which, however, the knowledge enables us as by magic to gain an insight with which in depth and range, but above all in certainty, scarcely any other can be compared. — *Voss.*

How many celebrate the names of Newton and Leibnitz! How few have a real appreciation of that which these men have created of permanent value! Here lie the roots of our present-day knowledge, here the true continuation of the strivings of antique wisdom.
— *Lindemann.*

The invention of the differential calculus marks a crisis in the history of mathematics. The progress of science is divided between periods characterized by a slow accumulation of ideas and periods, when, owing to the new material for thought thus patiently collected, some genius by the invention of a new method or a new point of view, suddenly transforms the whole subject on to a high level.
— *Whitehead.*

MATHEMATICAL PHILOSOPHY. ANALYTIC GEOMETRY. DESCARTES. — The invention of analytic geometry by Descartes in 1637 and the almost contemporary introduction of integral calculus as the method of "indivisibles" may be regarded as the real beginning of modern mathematical science. Thanks to these fruitful ideas the science has during the three centuries that have since elapsed made extraordinary progress both in its own internal development and in its application throughout the range of the physical sciences.

Descartes was born in Touraine in 1596, and after the education appropriate for a youth of family and some years of fashionable life in Paris, entered the army, then in Holland. His military career continued till 1621 with incidental opportunity for his favorite speculations in mathematics and philosophy. Some of his most fruitful ideas dated from dreams and his best thinking was habitually done before rising.

It is impossible not to feel stirred at the thought of the emotions of men at certain historic moments of adventure and discovery — Columbus when he first saw the Western shore, Franklin when the electric spark came from the string of his kite, Galileo when he first turned his telescope to the heavens. Such moments are also granted to students in the abstract regions of thought, and high among them must be placed the morning when Descartes lay in bed and invented the method of coördinate geometry. — *Whitehead.*

In order to devote himself more completely to his favorite studies he settled in Holland in 1629, devoting the next four years to writing a treatise, entitled *Le Monde,* upon the universe. In 1637 he published his great *Discourse on the Method of Good Reasoning and of Seeking Truth in Science.*[1] This begins: —

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[1] Discours de la Méthode pour bien conduire sa raison et chercher la vérité dans les sciences.

If this discourse seems too long to be read all at once, it can be divided into six parts. In the first will be found various considerations concerning the sciences; in the second, the chief rules of the method which the author has sought; in the third, some of those of ethics which he has deduced by this method; in the fourth, the reasons by which he proves the existence of God and of the human soul, which are the foundations of his metaphysics; in the fifth, the order of questions of physics which he has sought, and particularly the explanation of the movement of the heart and of some other difficulties which belong to medicine; also the difference which exists between our soul and that of the beasts; and in the last, what things he believes necessary in order to go farther in the investigation of nature than has been done, and what reasons have made him write.

Good sense is the most widely distributed commodity in the world, for every one thinks himself so well supplied with it that even those who are hardest to satisfy in every other respect are not accustomed to desire more of it than they have. In this it is not probable that all men are mistaken, but rather this testifies that the power of good judgment and of discriminating between the true and the false, which is properly what one calls good-sense or reason, is naturally equal in all men; and thus that the diversity of our opinions is not due to the fact that some are more reasonable than others, but only that we conduct our thought along different channels, and do not consider the same things. For it is not enough to have a good mind, but the principal thing is to apply it well. The greatest souls are capable of the greatest vices as well as of the greatest virtues: and those who only progress very slowly can advance much more, if they follow always the straight road than do those who run, departing from it.

His four cardinal precepts were: —

Never to receive anything for true which he did not recognize to be evidently so; that is, to avoid carefully precipitancy and prejudgment. Second, to divide each of the difficulties which he should examine into as many pieces as possible. Third, to conduct his thoughts in order, beginning with the simplest objects. The last, to make everywhere enumerations so complete and reviews so general that he should be assured of omitting nothing.

Three appendices dealt with optics, meteors, and geometry, the last containing the beginnings of analytic geometry. The relation of his philosophy to mathematics may be indicated in the following passages.

> Considering that, among all those who up to this time made discoveries in the sciences, it was the mathematicians alone who had been able to arrive at demonstrations — that is to say, at proofs certain and evident — I did not doubt that I should begin with the same truths that they have investigated, although I had looked for no other advantage from them than to accustom my mind to nourish itself upon truths and not to be satisfied with false reasons.
>
> When . . . I asked myself why was it then that the earliest philosophers would admit to the study of wisdom only those who had studied mathematics, as if this science was the easiest of all and the one most necessary for preparing and disciplining the mind to comprehend the more advanced, I suspected that they had knowledge of a mathematical science different from that of our time. . . .
>
> I believe I find some traces of these true mathematics in Pappus and Diophantus, who, although they were not of extreme antiquity, lived nevertheless in times long preceding ours. But I willingly believe that these writers themselves, by a culpable ruse, suppressed the knowledge of them; like some artisans who conceal their secret, they feared, perhaps, that the ease and simplicity of their method, if become popular, would diminish its importance, and they preferred to make themselves admired by leaving to us, as the product of their art, certain barren truths deduced with subtlety, rather than to teach us that art itself, the knowledge of which would end our admiration.
>
> Those long chains of reasoning, quite simple and easy, which geometers are wont to employ in the accomplishment of their most difficult demonstrations, led me to think that everything which might fall under the cognizance of the human mind might be connected together in a similar manner, and that, provided only that one should take care not to receive anything as true which was not so, and if one were always careful to preserve the order necessary for deducing one truth from another, there would be none so remote at which he might not at last arrive, nor so concealed which he might not discover.

Descartes had attempted the solution of a historic geometrical problem propounded by Pappus. From a point P perpendiculars are dropped on m given straight lines and also on n other given lines. The product of the m perpendiculars is in a constant ratio to the product of the n; it is required to determine the locus of P. Pappus had stated without proof that for $m = n = 2$ the locus is a conic section, Descartes showed this algebraically, — Newton afterwards conquering the difficulty by unaided geometry.

Descartes distinguished geometrical curves for which x and y may be regarded as changing at commensurable rates, or as we should say, curves for which the slope is an algebraic function of the coördinates, from curves which do not satisfy this condition. These he called "mechanical," and did not discuss further. For the accepted definition of a tangent as a line between which and the curve no other line can be drawn, he introduced the modern notion of limiting position of a secant. In connection with this he considered a circle meeting the given curve in two consecutive points, a perpendicular to the radius of the circle being a common tangent to the circle and the given curve. The circle was not however that of curvature, but had its centre on an axis of symmetry of the given curve. He recognized the possibility of extending his methods to space of three dimensions, but did not work out the details. His geometry contained also a discussion of the algebra then known, and gave currency to certain important innovations, in particular the systematic use of a, b, and c, for known, x, y, and z, for unknown quantities; the introduction of exponents; the collection of all terms of an equation in one member; the free use of negative quantities; the use of undetermined coefficients in solving equations; and his rule of signs for studying the number of positive or negative roots of equations. He even fancied that he had found a method for solving an equation of any degree.

It is important to distinguish just what Descartes contributed to mathematics in his analytic geometry. Neither the combination of algebra with geometry nor the use of coördinates was new. From the time of Euclid quadratic equations had been solved geometrically, while latitude and longitude involving a system of coördinates are of similar antiquity. The great step made by Descartes was his recognition of the equivalence of an equation and the geometrical locus of a point whose coördinates satisfy that equation. On this foundation facts known or ascertainable about geometry may be translated into algebra and conversely. The advantage is comparable with that conferred by the possession of two arms or eyes, or even two senses, under a common will. The intricate but powerful machinery of algebra becomes available for solving geometrical problems, while, on the other hand, the geometrical illustration makes the algebra visible and concrete.

Later works dealt with philosophy and physical science, in particular with a theory of vortices. Descartes enunciates ten natural laws, the first two corresponding with the first two of Newton's. He argues that all matter is in motion and that this must result in the formation of vortices. The sun is the centre of one great vortex, each planet of its own, thus approximating vaguely the future nebular hypothesis. Newton thought it worth while to refute this theory, which was chiefly notable as a bold attempt to interpret the phenomena of the universe by means of a single mechanical principle.

> Lord Kelvin has expressed, with all his force, that the sole satisfactory explanation of the phenomena of nature is that which leads them back in the last analysis to motion in a continuous incompressible fluid. This however was the guiding thought with Descartes.
>
> —*Timerding.*

Descartes's achievements in mathematics leave no doubt of his exceptional intellectual power. He had neither the data nor the scientific method for accomplishing similar results in other branches of science, and in mathematics he would doubtless have accomplished much more had he not expended his energies so widely in over-confident reliance on his logical method. He died at Stockholm in 1650.

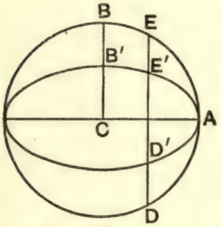
Indivisibles. Cavalieri. — While Descartes was thus as it were incidentally laying the foundations of modern geometrical analysis, his Italian contemporary, Cavalieri (1598–1647) was rendering a similar service to the integral calculus in developing his theory of indivisibles.

The problem of measuring the length of a curve or the area of a figure having a curved boundary, or the volume of a solid bounded by a curved surface goes back indeed to comparatively ancient Greek times. Most notable in this direction was the work of Archimedes. Kepler, attempting to resolve astronomical difficulties by the hypothesis of elliptical orbits, is confronted at once with the problem of determining the circumference of an ellipse. He gives the approximation $\pi\ (a + b)$ where a and b are the semi-axes. This is close if a and b are nearly equal, as in most of the planetary orbits. Interesting himself in current methods of measuring the capacity of casks, he published in 1615 his *Nova Stereometria Doliorum Vinariorum,* in which he determines the volumes of many solids bounded by surfaces of revolution. The Greek method had in case of the circle, etc., depended on an "exhaustion" process of inscribing and circumscribing polygons differing less and less from the curve both in boundary and in area. Kepler however divided his solid into sections, determined the area of a section and then sought the sum. He lacked an adequate system of coördinates, a clearly defined conception of a limit, and an effective method of summation. In view of the intrinsic difficulty of this important problem, however, the extent of his success is remarkable.

He also sought to determine the most economical proportions for casks, etc., expressing his view of the underlying mathematical theory by the theorem "In points where the transition from a less to the greatest and again to a less takes place, the difference is always to a certain degree imperceptible."

Cavalieri, in 1635, adopted the form of statement that a line consists of an infinite number of points, a surface of an infinity of lines, a solid of an infinity of surfaces, but later revised this on the basis of the assumption "that any magnitude may be divided into an infinite number of small quantities which can be made to bear any required ratios one to the other." On this basis, open as it was to criticism, were solved simple area problems involving the parabola and the hyperbola.

The principle of comparing areas by comparing lengths of a system of parallel lines crossing them is easily illustrated in the case of the ellipse by comparing it with the circle having as its diameter the (horizontal) major axis of the ellipse. If a and b are the semi-axes of the ellipse the two curves are known to be so related that every vertical chord of the circle is in a fixed ratio $a:b$ to the part of it lying within the ellipse. The area of the circle must bear the same relation to the area of the ellipse. The transition from length to area while not rigorously worked out by Cavalieri does not necessarily involve the false assumption that area consists of the sum of parallel lines. A similar method is evidently applicable to volumes. Thus was anticipated one of the most interesting and important processes of modern mathematics, — integration as a summation.



$$\frac{DE}{D'E'} = \frac{CB}{CB'} = \frac{a}{b}$$

Similarly Cavalieri determined volumes by a consideration of the thin sections or elements into which they may be resolved by parallel planes. The principle that "two bodies have the same volume if sections at the same level have the same area" is still known by his name.

Descartes's work with tangents seems not to have led him to develop the fundamental ideas of the differential calculus, and it appeared that the integral calculus would be evolved first from the work of Cavalieri.

PROJECTIVE GEOMETRY: DESARGUES. — Hardly less interesting than the new ideas of Descartes and Cavalieri are those of their contemporary Desargues (1593–1662), an engineer and architect of Lyons, who made important researches in geometry. But for the still more brilliant geometrical achievements of Descartes, these might have led to the immediate development of projective geometry, the elements of which are contained in Desargues's work. In general this geometry instead of dealing with definite triangles, polygons, circles, etc., in the Euclidean manner, is based on a consideration of all points of a straight line, of all lines through a common point and of the possible effects of setting up an orderly one-to-one correspondence between them. In particular, Desargues makes a comparative study of the different plane sections of a given cone, deducing from known properties of the circle analogous results for the other conic sections.

In his chief work Desargues enunciates the propositions: —

1. A straight line can be considered as produced to infinity and then the two opposite extremities are united.
2. Parallel lines are lines meeting at infinity and conversely.
3. A straight line and a circle are two varieties of the same species.

On these he bases a general theory of the plane sections of a cone.

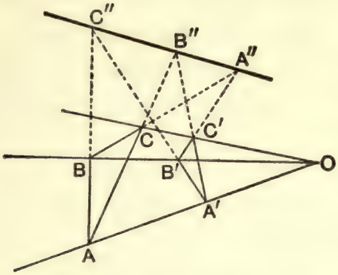
Desargues contented himself with enunciating general principles, remarking: — "He who shall wish to disentangle this proposition will easily be able to compose a volume." He met Descartes while employed by Cardinal Richelieu at the siege of Rochelle, and they with others met regularly in Paris for the discussion of the new Copernican theory and other scientific problems.

> He says 'I freely confess that I never had taste for study or research either in physics or geometry except in so far as they could serve as a means of arriving at some sort of knowledge of the proximate causes . . . . for the good and convenience of life, in maintaining health, in the practice of some art, . . . having observed that a good part of the arts is based on geometry, among others the cutting of stones in architecture, that of sun-dials, that of perspective in particular.'

Perceiving that the practitioners of these arts had to burden themselves with the laborious acquisition of many special facts in geometry, he sought to relieve them by developing more general methods and printing notes for distribution among his friends.

An interesting theorem bearing his name and typical of projective geometry is as follows: — If two triangles ABC and A′B′C′

are so related that lines joining corresponding vertices meet in a point O, then the intersections of corresponding sides will lie in a straight line A″B″C″. It remained for Monge, the inventor of descriptive geometry (p. 335) and others more than a century later to carry this development forward. Desargues's work was indeed practically lost until Poncelet in 1822 proclaimed him the Monge of his century.



Theory of Numbers and Probability: Fermat, Pascal. — But little younger than Descartes and Cavalieri was Pierre de Fermat (1601–1665) a man of quite exceptional position in mathematical history. Devoting to mathematics such leisure as his public duties afforded, he nevertheless published almost nothing, many of his results being known to us only in the form of brief marginal notes without proof. In editing Diophantus he enunciated numerous theorems on integers, for example,

> An odd prime can be expressed as the difference of two square integers in one and only one way.
>
> No integral values of x, y, z can be found to satisfy the equation $x^n + y^n = z^n$, if n be an integer greater than 2.

This seemingly simple theorem has been verified for so wide a range of values of n, that its truth can hardly be doubted, but no general proof has yet been given in spite of a prize of 100,000 marks awaiting him who either proves or disproves it. Some writers even credit Fermat with a substantial share in the invention of the new analytic geometry, in which he had certainly done independent work for some years before Descartes's publication. Laplace indeed calls Fermat "the true inventor of the differential calculus."

He discusses problems of maxima and minima, and passing to concrete phenomena, enunciates the interesting theorem: that Nature, the great workman which has no need of our instruments and machines, lets everything happen with a minimum of outlay, — an idea not indeed strange to some of the Greeks. The law of refraction of a ray of light he deals with correctly as a particular case of the principle of economy, a principle which exerted a potent influence in the scientific philosophy of the following century. Thus for example Euler says in 1744: —

> Since the organization of the world is the most excellent, nothing is found in it, out of which some sort of a maximum or minimum property does not shine forth. Therefore no doubt can exist, that all action in the world can be derived by the method of maxima and minima as well as from the actual operating causes.

Fermat's work in the theory of probability is fundamental. He discusses the case of two players, A and B, where A wants two points to win and B three points. Then the game will certainly be decided in the course of four trials. Take the letters *a* and *b*, and write down all the combinations that can be formed of four letters. These combinations are 16 in number, namely *aaaa, aaab, aaba, aabb, abaa, abab, abba, abbb, baaa, baab, baba, babb, bbaa, bbab, bbba, bbbb.* Now every combination in which *a* occurs twice or oftener represents a case favorable to A, and every combination in which *b* occurs three times or oftener represents a case favorable to B. Thus, on counting them, it will be found that there are 11 cases favorable to A, and 5 cases favorable to B; and, since these cases are all equally likely, A's chance of winning the game is to B's chance as 11 is to 5.

Like Descartes, Pascal (1623–1662) devoted but a fraction of his great talent to mathematical science.

> I have spent much time in the study of the abstract sciences, — but the paucity of persons with whom you can communicate on such subjects gave me a distaste for them. When I began to study man, I saw that these abstract studies were not suited to him, and that in diving into them, I wandered farther from my real track than those who were ignorant of them, and I forgave men for not having attended to these things. But I thought at least I should find many companions in the *study of mankind,* which is the true and proper study of man. Again I was mistaken. There are yet *fewer students of Man than of Geometry.*

Learning geometry surreptitiously at 12 years, he had at 18 written an essay on conic sections and constructed the first computing machine. While most of his later life was devoted to religion, theology, and literature, he undertook a wide range of physical experimentation, and made important contributions to the then new theories of numbers and probability, besides a discussion of the cycloid. The juvenile essay on conic sections contains the beautiful theorem since named for him that the opposite sides of a hexagon inscribed in a conic section meet in a straight line. Of geometry and logic Pascal says: —

> Logic has borrowed the rules of geometry without understanding its power. . . . I am far from placing logicians by the side of geometers who teach the true way to guide the reason. . . . The method of avoiding error is sought by every one. The logicians profess to lead the way, the geometers alone reach it, and aside from their science there is no true demonstration.

His work on probability connected itself with the problem of two players of equal skill wishing to close their play, of which Fermat's solution has been given above.

> The following is my method for determining the share of each player when, for example, two players play a game of three points and each player has staked 32 pistoles.
>
> Suppose that the first player has gained two points and the second player one point; they have now to play for a point on this condition, that if the first player gain, he takes all the money which is at stake, namely 64 pistoles; while if the second player gain, each player has two points, so that they are on terms of equality, and if they leave off playing, each ought to take 32 pistoles. Thus if the first player gain, then 64 pistoles belong to him, and if he lose, then 32 pistoles belong to him. If therefore the players do not wish to play this game, but separate without playing it, the first player would say to the second, 'I am certain of 32 pistoles, even if I lose this point, and as for the other 32 pistoles, perhaps I shall have them and perhaps you will have them; the chances are equal. Let us then divide these 32 pistoles equally, and give me also the 32 pistoles of which I am certain.' Thus the first player will have 48 pistoles and the second 16 pistoles.

By similar reasoning he shows that if the first player has gained two points and the second none, the division should be 56 to 8; while if the first has gained one point, the second none, it should be 44 and 20.

> The calculus of probabilities, when confined within just limits, ought to interest, in an equal degree, the mathematician, the experimentalist, and the statesman. From the time when Pascal and Fermat established its first principles, it has rendered, and continues daily to render, services of the most eminent kind. It is the calculus of probabilities, which, after having suggested the best arrangements of the tables of population and mortality, teaches us to deduce from those numbers, in general so erroneously interpreted, conclusions of a precise and useful character; it is the calculus of probabilities which alone can regulate justly the premiums to be paid for assurances; the reserve funds for the disbursements of pensions, annuities, discounts, etc. It is under its influence that lotteries and other shameful snares cunningly laid for avarice and ignorance have definitely disappeared. — *Arago.*

With this work connected itself his arithmetical triangle in which successive diagonals contain the coefficients which occur in expansions by the binomial theorem, which Newton was soon to generalize.

1	1	1	1	1	1
1	2	3	4	5	
1	3	6	10		
1	4	10			
1	5				
1					

He applies the method of indivisibles successfully to the cycloid (the curve generated by a point on the rim of a rolling wheel).

Pascal invented in 1645 an arithmetical machine, writing the Chancellor in regard to it:

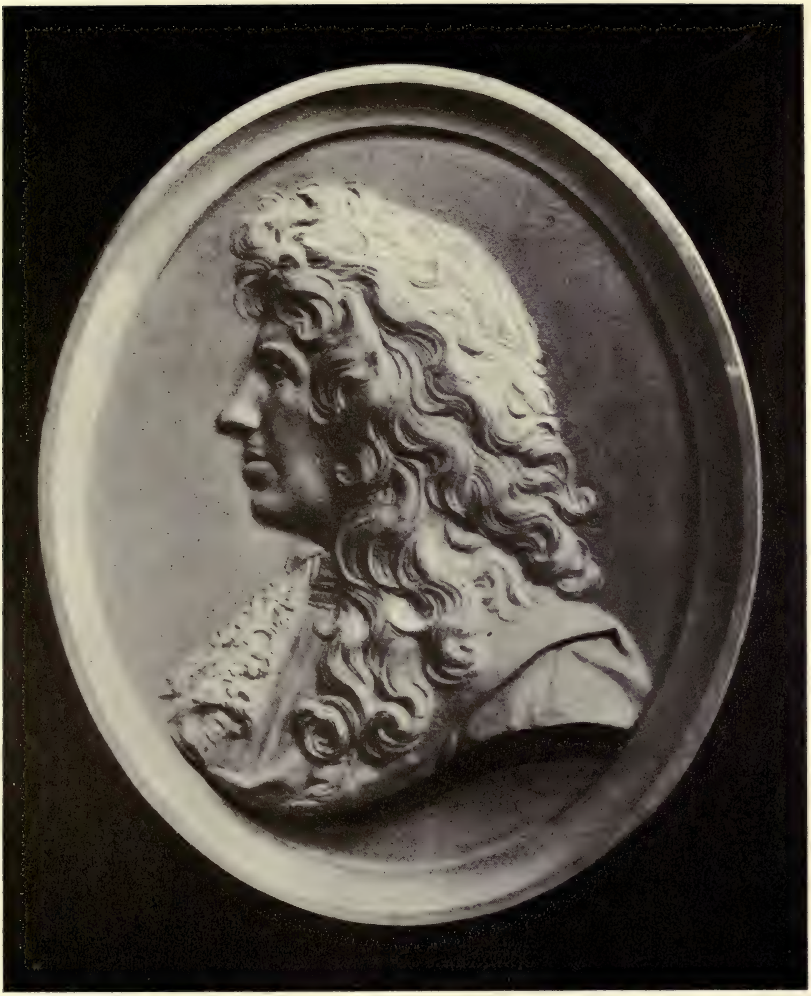
> Sir: If the public receives any advantage from the invention which I have made to perform all sorts of rules of arithmetic in a manner as novel as it is convenient, it will be under greater obligation to your

Highness than to my small efforts, since I should only have been able to boast of having conceived it, while it owes its birth absolutely to the honor of your commands. The length and difficulty of the ordinary means in use have made me think on some help more prompt and easy to relieve me in the great calculations with which I have been occupied for several years in certain affairs which depend on the occupations with which it has pleased you to honor my father for the service of his Majesty in Normandy. I employed for this investigation all the knowledge which my inclination and the labor of my first studies in mathematics have gained for me, and after profound reflection, I recognized that this aid was not impossible to find.

MECHANICS AND OPTICS: HUYGENS. — Most notable among the successors of Galileo in mechanics before we reach Newton was Huygens of Holland (1629–1695) who combined mathematical power with exceptional practical ingenuity. He first (in 1655) explained as a ring the excrescences of Saturn which had been misunderstood by Galileo and others, publishing his discovery in the occult form $a^7c^5d^1e^5g^1h^1i^7l^4m^2n^9o^4p^2q^1r^2s^1t^5u^5$. (*Annulo cingitur tenui, plano, nusquam cohærente ad eclipticam inclinato.*) He also discovered Saturn's largest moon. About the same time he made his great invention of the pendulum clock. Accepting a call to Paris by Colbert at the founding of the French Academy, he remained there from 1666 to 1681.

In optics he developed and maintained even in opposition to the authority of Newton the undulatory or wave theory which only found general acceptance a century later. The velocity of light Galileo had failed to measure by means of signal lanterns, and Descartes had likewise been unable to ascertain it by comparing the observed and computed instants of a lunar eclipse. Huygens points out that even this latter test does not prove instantaneous transmission. Römer's conclusive report on observations of a satellite of Jupiter dates from 1675. On this basis Huygens estimated the velocity of light at 600,000 times that of sound, — a result about one-third too small.

The medium in which light waves travel Huygens named the ether, attributing to its particles three properties in comparison



HUYGENS FROM *ŒUVRES COMPLÈTES*, 1899

with air: extreme minuteness, extreme hardness, extreme elasticity. On this basis he worked out a consistent theory for reflection and refraction. His discussion of the newly discovered phenomenon of double refraction in Iceland spar has been characterized as an "unsurpassed example of the combination of experimental investigation and acute analysis." The attendant phenomenon of polarisation did not escape him, but his theory of wave motion was not sufficiently developed to enable him to explain the matter adequately.

In 1673 Huygens published his great work on the pendulum (*Horologium oscillatorium sive de motu pendulorum*), displaying wonderful skill in his geometrical treatment of the mechanical problems involved. The use of wheel mechanisms with weights for measuring time had been more or less familiar for several centuries, but no effective means for regulating this motion had been devised. Galileo, for example, observing the regularity of pendulum vibrations, had depended on repeated impulses by hand to maintain the motion. Huygens first made the fortunate combination of the two elements, without however inventing the modern escapement. He studied the cycloidal pendulum for which the time of an oscillation would be independent of the amplitude and made precise determinations of the length of the seconds-pendulum at Paris and the corresponding value of the important constant g. The most remarkable achievement in his treatise on the pendulum is the correct analysis of the compound pendulum based on the definition:—

> The centre of oscillation of any figure whatever is that point in the line of gravity, whose distance from the point of suspension is the same as the length of the simple pendulum having the same time of vibration as the figure.

In the course of the discussion he formulates the important law, afterwards somewhat generalized by others:

> Whenever any heavy bodies are set in motion under the action of their own weight, their common centre of gravity cannot rise higher than it was at the beginning of the motion.

In computing the position of the centre of oscillation he arrives at a fraction of the form $\frac{\Sigma mr^2}{\Sigma mr}$, where m denotes the mass of a particle, r its distance from the point of suspension. The numerator is the so-called "moment of inertia," the denominator the "statical moment" of later mechanics. He shows that the point of suspension and the centre of oscillation are interchangeable.

Finally he discusses the theory of centrifugal force, proving that it varies as the square of the velocity and inversely as the radius. This subject he also treated more fully in a special monograph, published after his death when Newton had already given a more general theory. His theorems are: —

> 1. When equal bodies move with the same velocity in unequal circles, the centrifugal forces are to each other inversely as the diameters, so that in the smaller circle the said force is greater.
> 2. When equal movable bodies travel in the same or equal circles with unequal velocities, the centrifugal forces are to each other as the squares of the velocities.

By experiments on a revolving sphere of clay which as he anticipated assumed a spheroidal form, he explains the observed polar flattening of Jupiter. He infers that the earth must also be flattened, and makes a numerical estimate in anticipation of future verification. He explains the effect on a clock pendulum of transporting it from Paris to an equatorial locality, where its weight is opposed by an increased centrifugal force.

Like Wallis (p. 290) and Sir Christopher Wren he accepted the invitation of the Royal Society to attack the general problem of impact. This led ultimately to the publication eight years after his death of his On the Motion of Bodies under Percussion. The theorems enunciated deal with various cases of central impact, one of the most notable being: —

> By mutual impact of two bodies the sum of the products of the masses into the squares of their velocities is the same before and after impact.

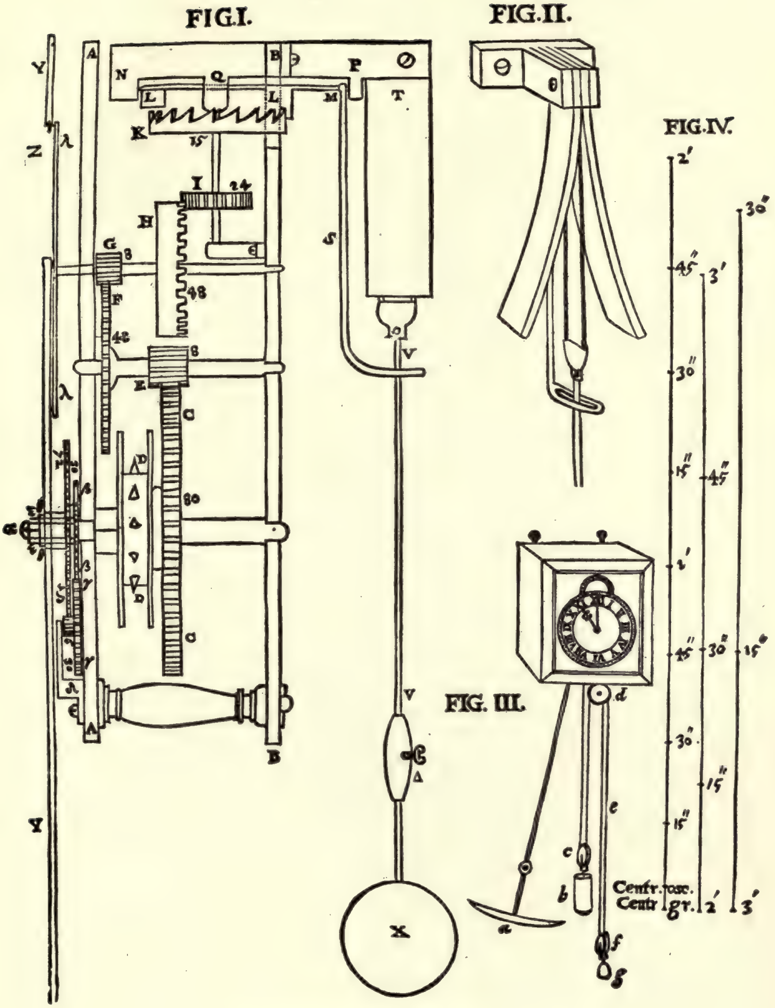


FIG.I.
FIG.II.
FIG.IV.
FIG. III.
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g
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30″
15″
1′
45″
30″
15″
3′
45″
30″
15″
30″
15″
Centr. osc.
Centr. gr.
2′
3′

Huygens' Clock
(*Horologium Oscillatorium*, 1673)

— the first formulation (1669) of the most comprehensive law of mechanics, the conservation of *vis viva.*

Huygens visited England in 1689, but made no use of Newton's new calculus in his published work. In his History of the Mathematical Theories of Attraction and the Figure of the Earth, Todhunter says of Huygens: —

> To him we owe the important condition of fluid equilibrium, that the resultant force at any point of the free surface must be normal to the surface at that point; and this has indirectly promoted the knowledge of our subject. But Huygens never accepted the great principle of the mutual attraction of particles of matter; and thus he contributed explicitly only the solution of a theoretical problem, namely the investigation of the form of the surface of rotating fluid under the action of a force always directed to a fixed point.

WALLIS AND BARROW. — Before attempting to discuss the extraordinary work of Sir Isaac Newton in the whole field of mathematical science a few words should be added concerning two slightly older English mathematicians, John Wallis (1616–1703), Savilian professor at Oxford, and Isaac Barrow (1630–1677), Lucasian professor at Cambridge.

Wallis in his Arithmetic of The Infinites (1656) developed Cavalieri's summation ideas effectively, employing the new Cartesian geometry and a process equivalent to integration for simple algebraic cases. In particular, he explains negative and fractional exponents in the modern sense, and then proceeds to find the area bounded by OX, the curve $y = ax^m$, and any ordinate $x = h$, — or as we should say, he integrates the function ax^m. He develops ingenious methods of interpolation.

In his Treatise on Algebra he says: —

> It is to me a theory unquestionable, That the Ancients had somewhat of like nature with our Algebra; from whence many of their prolix and intricate Demonstrations were derived. . . . But this their Art of Invention, they seem very studiously to have concealed: contenting themselves to demonstrate by Apagogical Demonstrations, (or reducing to Absurdity, if denied,) without showing us the method, by

which they first found out those Propositions, which they thus demonstrate by other ways. . . . Nonius 'O how well it had been if those Authors, who have written in Mathematics, had delivered to us their Inventions, in the same way, and with the same Discourse, as they were found out! And not as Aristotle says of Artificers in Mechanics who show us the Engines they have made, but conceal the Artifice, to make them the more admired!'

His Analytical Conic Sections (1665) made Descartes's geometrical ideas much more intelligible, and his Algebra (1686) marks an important step forward in its systematic use of formulas. He also wrote A Summary Account . . . of the General Laws of Motion, enunciating the formulas for velocity after impact of masses m_1 and m_2 with velocities v_1 and v_2: —

$$v = \frac{m_1v_1 + m_2v_2}{m_1 + m_2}$$

Barrow, after varied adventures, became first Lucasian professor at the University of Cambridge, but resigned his chair six years later to his pupil Newton. His work on optics and geometry contains a notable discussion of the tangent problem and of what he calls the differential triangle, so important in modern elementary differential calculus. His general point of view is illustrated by the following passage: —

> Now as to what pertains to these Surd numbers (which, as it were by way of reproach and calumny, having no merit of their own are also styled Irrational, Irregular, and Inexplicable) they are by many denied to be numbers properly speaking, and are wont to be banished from arithmetic to another Science (which yet is no science), viz. algebra.

Isaac Newton, — was born within a year after Galileo's death, a century after that of Copernicus — December 25, 1642 (O.S.). Destined at first to become a farmer, he was fortunately sent at 17 to the university, where he quickly and eagerly mastered the mathematical work of Euclid, Descartes and Wallis, and Kepler's Dioptrics. His discovery of the general binomial theorem dates from this time, and he even ventured to attack the great problem of gravitation by carefully comparing the motion of the moon with that of a falling body near the earth — for which however his data were not yet sufficiently accurate.

Newton took his B. A. degree in the Lent Term, 1665. In that spring the plague appeared, and for a couple of years he lived mostly at home, though with occasional residence at Cambridge. Probably at this time his creative powers were at their highest. His use of fluxions may be traced back to May, 1665; his theory of gravitation originated in 1666; and the foundation of his optical discoveries would seem to be only a little later. In an unpublished memorandum made some years later (cancelled, but believed to be correct in the part here quoted), he thus described his work of this time: 'In the beginning of the year 1665 I found the method of approximating Series and the Rule for reducing any dignity of any Binomial into such a series. The same year, in May, I found the method of tangents of Gregory and Slusius, and in November had the direct method of Fluxions, and the next year in January had the Theory of Colours, and in May following I had entrance into the inverse method of Fluxions. And the same year I began to think of gravity extending to the orb of the Moon, and . . . from Kepler's Rule of the periodical times of the Planets being in a sesquialterate proportion of their distances from the centers of their orbs I deduced that the forces which keep the Planets in their orbs must (be) reciprocally as the squares of their distances from the centers about which they revolve: and thereby compared the force requisite to keep the Moon in her orb with the force of gravity at the surface of the earth, and found them answer pretty nearly. All this was in the two plague years of 1665 and 1666, for in those days I was in the prime of my age for invention, and minded Mathematicks and Philosophy more than at any time since.' — *Ball,* Mathematical Gazette, July, 1914.

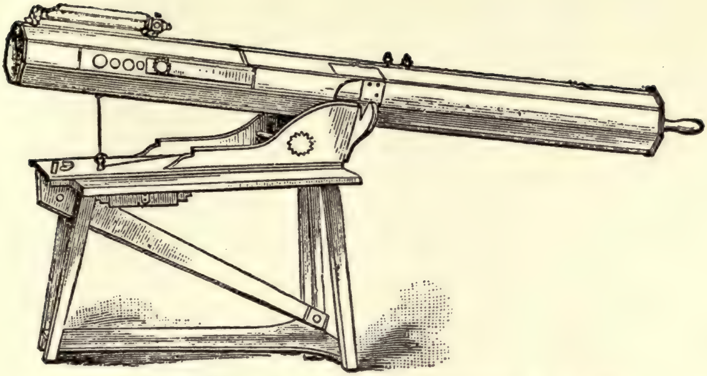
Optics. — Interesting himself in the telescope, Newton succeeded in eliminating the disturbing chromatic aberration due to unequal refraction of the different colors by constructing a reflecting telescope with a concave mirror in place of a convex lens. On the other hand, turning his attention to the colors of the solar spectrum, he wrote his Opticks or a Treatise of the Reflections, Refractions, Inflections and Colours of Light, published in 1704.

Disclaiming any intention of setting up speculative hypotheses, he discusses the observed phenomena of refracted light, speaking of his discovery of the different refrangibility of the rays of light as "in my judgment the oddest if not the most considerable detection which hath hitherto been made in the operations of nature." While he does not insist upon it, he seems always to have the underlying idea that light itself consists of minute particles — the degree of fineness corresponding with the color — a theory which held the field — thanks to his potent authority with his too subservient followers — against the better undulatory theory of Huygens until the nineteenth century. In the experiments on which this work is based Newton not only decomposes light by a refracting prism or series of prisms, but also succeeds in recombining the component colors to reproduce the original white.

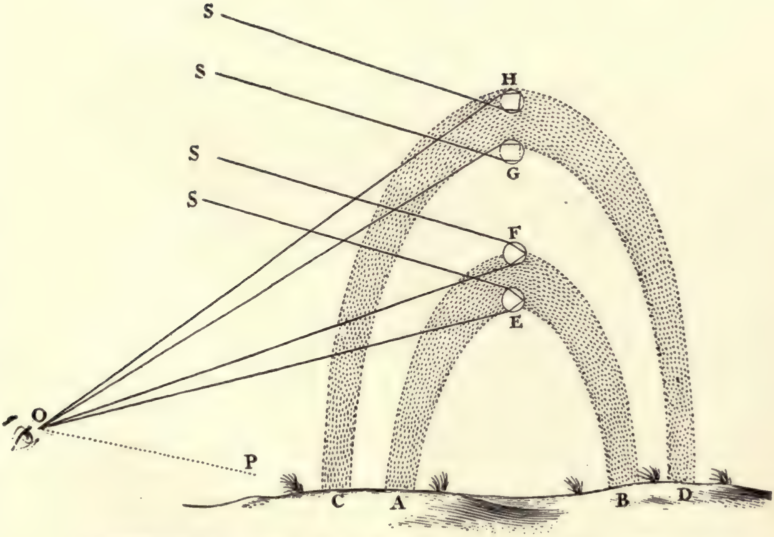
> The colors of objects, he says, are nothing more than their power to reflect one or another kind of ray. And in the rays again is nothing other than the power to transmit this motion into our organ of sense, in which last finally arises the sensation of these motions in the form of colors.

He solves at last the problem of the rainbow. All this constitutes an immense advance over the current Aristotelian notions.

THE THEORY OF GRAVITATION: *Principia* — In 1682 Newton returned to his attempt of 16 years earlier to explain the moon's motion by means of the assumed influence of gravitation. During this long interval French geographers, testing the supposedly spherical shape of the earth, had made a new and more precise triangulation — with the first use of telescopic instruments. Newton's earlier data had led to a determination of the acceleration due to gravity at the distance of the moon as 13½ feet per minute. The new data changed this result to 15, in agreement with his hypothesis that the force varied inversely as the square of the distance. Stirred to the inmost depths of his usually calm nature by his realization that he was approaching a solution of the great problem, he had to beg a friend to complete his calculations. The new astronomy founded by Copernicus, built up by Tycho Brahe,



Newton's Telescope (Great Astronomers, R. S. Ball).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Newton's Theory of the Rainbow (Opticks, 1704).

Kepler, and Galileo, was now to be completely formulated and mathematically interpreted by Newton's crowning discovery of a single mechanical principle governing the whole.

It was now a question of verifying the correctness of this principle by applying it to all measured or measurable astronomical phenomena. The investigation was gradually extended to the planets, the moons of Jupiter, the tides, and even the comets. Everywhere the law was verified that attraction varies as the product of the masses and inversely as the square of the distance.

The whole theory was elaborated in Newton's monumental *Principia Philosophiæ Naturalis Mathematica* published in 1687. He begins this treatise with a series of definitions and laws:

> 1. The quantity of matter is the measure of the same, arising from its density and bulk conjunctly.
> 2. The quantity of motion is the measure of the same, arising from the velocity and quantity of matter conjunctly.
> 3. The innate force of matter is a power of resisting, by which every body, as much as in it lies, endeavours to persevere in its present state, whether it be of rest, or of moving uniformly forward in a right line.
> 4. An impressed force is an action exerted upon a body, in order to change its state, either of rest, or of moving uniformly forward in a right line.
> 5. A centripetal force is that by which bodies are drawn or impelled, or any way tend, towards a point as to a centre.

These and succeeding definitions are followed by the famous Laws of Motion:

> I. Every body perseveres in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed thereon.
>
> II. The alteration of motion is ever proportional to the motive force impressed, and is made in the direction of the right line in which that force is impressed.
>
> III. To every action there is always opposed an equal reaction: or the mutual actions of two bodies upon each other are always equal, and directed to contrary parts. Corollary I continues: A body by two

forces conjoined will describe the diagonal of a parallelogram, in the same time that it would describe the sides, by those forces apart.

Of these laws, Pearson in his Grammar of Science remarks:

> The Newtonian laws of motion form the starting-point of most modern treatises on dynamics, and it seems to me that physical science, thus started, resembles the mighty genius of an Arabian tale emerging amid metaphysical exhalations from the bottle in which for long centuries it has been corked down.

Passing to variable forces he discusses in particular the motion of a body acted on by a central attractive force — *i.e.* a force attracting it towards a fixed point. He derives the law by which equal areas are described in equal times, and shows that conversely, if equal areas are so described in a plane, the determining force must be a central one. Turning to the consideration of orbits, he deals with the hypothesis of an elliptical orbit with the attracting force at one of the foci, and shows that the attractive force must vary inversely as the square of the distance from that focus. The same result is obtained for the other conic sections. These theorems applying to particles, he next shows that the action of a homogeneous sphere on an external particle is the same as if its mass were concentrated at its centre, so that the action of two such spheres on each other is subject to the laws already derived.

Comparing planetary motions with those of projectiles which had been treated by Galileo, Newton says: —

> That the planets can be held in their paths is evident from the motions of projectiles. A stone thrown is deflected from the straight line by its weight and falls describing a curved line to the earth. If thrown with greater velocity it goes farther, and it could happen that it described a curve of 10,100,1000 miles, and at last went outside the boundaries of the earth, and never fell back. . . .

The force of gravity for small distances being sensibly constant in direction, causes motion in a path approximately parabolic. For greater and greater ranges the change of direction of the force must be taken account of and the path recognized as elliptical or hyperbolic. The laws as stated deal with the relations of only two mutually attracting bodies. Newton of course appreciates that such a case is purely ideal and that, since every body attracts every other, the result of dealing with only two is merely a first approximation to the reality.

All planets he says are mutually heavy, therefore, for example; Jupiter and Saturn will attract each other in the vicinity of their conjunction and perceptibly disturb each other's motion. Similarly the Sun will disturb the motion of the Moon, and Sun and Moon will disturb our ocean.

Newton prefaced these applications of the theory with four rules which should guide scientific men in making hypotheses. These in their final shape, are to the following effect: (1) We should not assume more causes than are sufficient and necessary for the explanation of observed facts. (2) Hence, as far as possible, similar effects must be assigned to the same cause; for instance, the fall of stones in Europe and America. (3) Properties common to all bodies within reach of our experiments are to be assumed as pertaining to all bodies; for instance, extension. (4) Propositions in science obtained by wide induction are to be regarded as exactly or approximately true, until phenomena or experiments show that they may be corrected or are liable to exceptions. The substance of these rules is now accepted as the basis of scientific investigation. Their formal enunciation here serves as a landmark in the history of thought. — *Mathematical Gazette, July,* 1914.

Every new satellite, says Brewster in his Life of Newton, every new asteroid, every new comet, every new planet, every new star circulating round its fellow, proclaims the universality of Newton's philosophy, and adds fresh lustre to his name. It is otherwise however in the general history of science. The reputation achieved by a great invention is often transferred to another which supersedes it, and a discovery which is the glory of one age is eclipsed by the extension of it in another. . . . It is the peculiar glory of Newton, however, that every discovery in the heavens attests the universality of his laws, and adds a greener leaf to the laurel chaplet which he wears.

Shrinking always from publicity[1] and controversy, Newton like Copernicus had gradually perfected his great work, but, like Copernicus, Newton might never have published it but for the fortunate urgency of a faithful disciple, Edmund Halley.

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[1] In one instance he authorized publication of one of his works "so it be without my name to it: for I see not what there is desirable in public esteem, were I able

NEWTON'S MATHEMATICS: FLUXIONS. — Newton's services to mathematics itself were not less original and momentous than to celestial mechanics.

> His extraordinary abilities . . . enabled him within a few years to perfect the more elementary . . . processes, and to distinctly advance every branch of mathematical science then studied, as well as to create several new subjects. There is hardly a branch of modern mathematics which cannot be traced back to him and of which he did not revolutionize the treatment.
>
> In pure geometry Newton did not establish any new methods, but no modern writer has ever shown the same power in using those of classical geometry, and he solved many problems in it which had previously baffled all attempts. In algebra and the theory of equations he introduced the system of literal indices, established the binomial theorem . . ., and created no inconsiderable part of the theory of equations. . . . He always by choice, avoided using trigonometry in his analysis, . . . In analytical geometry he introduced the modern classification of curves into algebraical and transcendental; and established many of the fundamental properties of asymptotes, multiple points and isolated loops. He illustrated these by an exhaustive discussion of cubic curves. — *Ball.*

Newton's greatest mathematical achievement was of course the invention of the fluxional or infinitesimal calculus. In his Treatise of the Method of Fluxions and Infinite Series he says: —

> 1. Having observed that most of our modern Geometricians neglecting the synthetical Method of the Ancients, have applied themselves chiefly to the analytical Art, and by the Help of it have overcome so many and so great Difficulties, that all the Speculations of Geometry seem to be exhausted, except the Quadrature of Curves, and some other things of a like Nature which are not yet brought to Perfection: To this End I thought it not amiss, for the sake of young

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to acquire and maintain it: it would perhaps increase my acquaintance, the thing which I study chiefly to decline." Again in 1675 he writes "I was so persecuted with discussions arising out of my theory of light, that I blamed my own imprudence for parting with so substantial a blessing as my quiet, to run after a shadow."

> Students in this Science, to draw up the following Treatise; wherein I have endeavored to enlarge the Boundaries of Analyticks, and to make some Improvements in the Doctrine of Curved Lines.

— surely a sufficiently modest introduction of perhaps the most important step in the progress of mathematical science.

Something further as to the evolution of his theory of Fluxions may be indicated, without too much technical detail, by the following passages from Brewster: —

> Having met with an example of the method of Fermat, in Schooten's Commentary on the Second Book of Descartes, Newton succeeded in applying it to affected equations, and determining the proportion of the increments of indeterminate quantities. These increments he called *moments*, and to the velocities with which the quantities increase he gave the names of *motions, velocities* of *increase*, and *fluxions*. He considered quantities not as composed of indivisibles, but as generated by motion; and as the ancients considered rectangles as generated by drawing one side into the other, that is, by moving one side upon the other to describe the area of the rectangle, so Newton regarded the areas of curves as generated by drawing the ordinate into the abscissa, and all indeterminate quantities as generated by continual increase. Hence, from the flowing of time and the moments thereof, he gave the name of *flowing quantities* to all quantities which increase in time, that of *fluxions* to the velocities of their increase, and that of *moments* to their parts generated in moments of time.
>
> Newton then proceeds to show the application of the propositions to the solution of the twelve following problems, many of which were at that time entirely new:
>
> 1. To draw tangents to curve lines.
> 2. To find the quantity of the crookedness of lines.
> 3. To find the points distinguishing between the concave and convex portions of curved lines.
> 4. To find the point at which lines are most or least curved.
> 5. To find the nature of the curve line whose area is expressed by any given equation.
> 6. The nature of any curve line being given, to find other lines whose areas may be compared to the area of that given line.
> 7. The nature of any curve line being given, to find its area when

it may be done; or two curved lines being given, to find the relation of their areas when it may be.

8. To find such curved lines whose lengths may be found, and also to find their lengths.

9. Any curve line being given, to find other lines whose lengths may be compared to its length, or to its area, and to compare them.

10. To find curve lines whose areas shall be equal or have any given relations to the length of any given curve line drawn into a given right line.

11. To find the length of any curve line when it may be.

12. To find the nature of a curve line whose length is expressed by any given equation when it may be done.

Such were the improvements in the higher geometry which Newton had made before the end of 1666.

Such is a brief account of the mathematical writings of Sir Isaac Newton, not one of which was voluntarily communicated to the world by himself. The publication of his Universal Arithmetic is said to have been made by Whiston against his will; and, however this may be, it was an unfinished work, never designed for the public. The publication of his Quadrature of Curves, and of his Enumeration of Curve Lines, was in Newton's opinion rendered necessary, in consequence of plagiarisms from the manuscripts of them which he had lent to his friends, and the rest of his analytical writings did not appear till after his death.

An account of Newton's very important work in analytic geometry and the theory of algebraic equations lies outside the range of the present work.

Much of Newton's reluctance to publish his more revolutionary theories may be attributed to his distaste for controversy, and he was unfortunately involved not only in such issues as to priority as his own reticence invited, but also in defending himself against attacks on philosophic grounds. The character of some of these may be illustrated by the following passages from an eminent critic, Bishop Berkeley: —

> He who can digest a second or third fluxion, a second or third difference, need not, methinks, be squeamish about any point in Divinity.

> And what are these fluxions? The velocities of evanescent increments. And what are these same evanescent increments? They are neither finite quantities, nor quantities infinitely small, nor yet nothing. May we not call them ghosts of departed quantities?

In regard to the controversy between the friends of Newton and those of Leibnitz as to priority in the invention of the calculus, Newton himself says in a celebrated scholium: —

> The correspondence which took place about ten years ago, between that very skilful geometer G. G. Leibnitz and myself, when I had announced to him that I possessed a method of determining maxima and minima, of drawing tangents, and of performing similar operations, which was equally applicable to surds and to rational quantities, and concealed the same in transposed letters, involving this sentence, (*Data Aequatione quotcunque Fluentes quantitates involvente, Fluxiones invenire, et vice versa*), this illustrious man replied that he also had fallen on a method of the same kind, and he communicated his method, which scarcely differed from my own, except in the forms of words and notation (and in the idea of the generation of quantities).

Of the controversy as a whole Newton's biographer Brewster remarks:

> The greatest mathematicians of the age took the field, and statesmen and princes contributed an auxiliary force to the settlement of questions upon which, after the lapse of nearly 200 years, a verdict has not yet been pronounced.
>
> Although the honour of having invented the calculus of fluxions, or the differential calculus, has been conferred upon Newton and Leibnitz, yet, as in every other great invention, they were but the individuals who combined the scattered lights of their predecessors, and gave a method, a notation, and a name to the doctrine of quantities infinitely small.
>
> In studying this controversy, after the lapse of nearly a century and a half, when personal feelings have been extinguished, and national jealousies allayed, it is not difficult, we think, to form a correct estimate of the claims of the two rival analysts, and of the spirit and temper with which they were maintained. The following are the results at which we have arrived:

1st — That Newton was the first inventor of the Method of Fluxions; that the method was incomplete in its notation; and that the fundamental principle of it was not published to the world till 1687, twenty years after he had invented it.

2d — That Leibnitz communicated to Newton in 1677 his Differential Calculus, with a complete system of notation, and that he published it in 1684, three years before the publication of Newton's Method.

— *Brewster.*

It is said that when the Queen of Prussia asked Leibnitz his opinion of Sir Isaac Newton, he replied that taking mathematicians from the beginning of the world to the time when Sir Isaac lived, what he had done was much the better half; and added that he had consulted all the learned in Europe upon some difficult points without having any satisfaction and that when he applied to Sir Isaac, he wrote him in answer by the first post, to do so and so, and then he would find it.

The exalted estimation in which Newton's genius has been held in later times may be illustrated by the following passages.

The great Newtonian Induction of Universal Gravitation is indisputably and incomparably the greatest scientific discovery ever made, whether we look at the advance which it involved, the extent of the truth disclosed, or the fundamental and satisfactory nature of this truth. — *Whewell.*

The efforts of the great philosopher . . . were always superhuman; the questions which he did not solve were incapable of solution in his time. — *Arago.*

Newton was the greatest genius that ever existed, and the most fortunate, for we cannot find more than once a system of the world to establish. — *Lagrange.*

His own attitude is sufficiently indicated in his statements: —

I do not know what I may appear to the world, but, to myself, I seem to have been only like a boy playing on the seashore, and diverting myself in now and then finding a smoother pebble or a prettier shell than ordinary, whilst the great ocean of truth lay all undiscovered before me.

If I have seen farther than Descartes, it is by standing on the shoulders of giants. (See also Appendix.)

Leibnitz. — Newton's great contemporary and scientific rival Leibnitz has been called the Aristotle of the seventeenth century. Born in 1646 at Leipsic, he took his doctor's degree at 20 and was immediately offered a university professorship. Alchemy, diplomacy, philosophy, mathematics, all shared his energetic attention. In the last he invented a calculating machine and the differential calculus. In regard to the machine Klein says: —

> merely the formal rules of computation are essential, for only these can be followed by the machine. It cannot possibly have an intuitive conception of the meaning of the numbers. It is thus no accident that a man so great as Leibnitz was both father of purely formal mathematics and inventor of the first calculating machine.

He became librarian at Hannover, founded the Academy of Sciences in Berlin, and was instrumental in the organization of similar bodies in St. Petersburg, Dresden, and Vienna. His advanced ideas on education may be inferred from his remark —

> We force our youths first to undertake the Herculean labor of mastering different languages, whereby the keenness of the intellect is often dulled, and condemn to ignorance all who lack knowledge of Latin.

But for the overshadowing genius of Newton, Leibnitz' service to the progress of science would have been even greater than it actually was. Comparing their work in mathematics where their competition was keenest, it should be appreciated that while Newton's work in mathematical science was incomparably greater in range, it was Leibnitz who gave to the differential calculus the better form and notation out of which our own has grown.

> It appears that Fermat, the true inventor of the differential calculus, considered that calculus as derived from the calculus of finite differences by neglecting infinitesimals of higher orders as compared with those of a lower order . . . Newton, through his method of fluxions, has since rendered the calculus more analytical, he also simplified and generalized the method by the invention of his binomial theorem. Leibnitz has enriched the differential calculus by a very happy notation. — *Laplace.*

Leibnitz' sense of mathematical form was also well exemplified by his important algebraic invention of determinants. In regard to numbers he says: —

> The imaginary numbers are a fine and wonderful refuge over the divine spirit, almost an *amphibium* between being and not being.

> Leibnitz's discoveries lay in the direction in which all modern progress in science lies, in establishing order, symmetry, and harmony, *i.e.* comprehensiveness and perspicuity, — rather than in dealing with single problems, in the solution of which followers soon attained greater dexterity than himself. — *Merz.*

> Leibnitz believed he saw the image of creation in his binary arithmetic, in which he employed only two characters, unity and zero. Since God may be represented by unity, and nothing by zero, he imagined that the Supreme Being might have drawn all things from nothing, just as in the binary arithmetic all numbers are expressed by unity with zero. This idea was so pleasing to Leibnitz, that he communicated it to the Jesuit Grimaldi, President of the Mathematical Board of China, with the hope that this emblem of the creation might convert to Christianity the reigning emperor who was particularly attached to the sciences. — *Laplace.*

HALLEY: PREDICTION OF COMETS. — In applying Newton's theories to known comets his friend and disciple Halley made the astonishing discovery that some of them instead of visiting the solar system once for all, actually described elliptical orbits of vast extent and great eccentricity about the sun. Among these he found one which having appeared in 1531, 1607, and 1682, should, if his identifications were correct, return in 1759. This bold prediction was fulfilled, and Halley's comet has not only reappeared in 1835 and 1910, but has even been traced back almost to the beginning of our era. A second similar prediction of Halley awaits verification in the year 2255.

In physics, Halley enunciated for spherical lenses and mirrors the correct formula $\frac{1}{f} = \frac{1}{a_1} + \frac{1}{a_2}$ and that for the barometric determination of altitudes. His mathematical work included graphical discussion of the cubic and biquadratic equations, a method of computing logarithms, and an edition of Apollonius from both Greek and Arabic sources. In his paper on An Estimate of the Degrees of the Mortality of Mankind, drawn from curious Tables of the Births and Funerals at the City of Breslau; with an Attempt to ascertain the Price of Annuities upon Lives, he laid the foundations of a new and important branch of applied mathematics. Having in boyhood occupied himself with magnetic experiments, in middle life he travelled in the tropics and made the first magnetic map, published in 1701 under the title "A general chart, showing at one view the variation of the compass." Drawing curves on this chart through points of declination, he invented a graphical method of wide future usefulness. From naval captain he became professor of geometry at Oxford, then astronomer royal till his death in 1742. One of his most notable achievements in astronomy was the discovery of actual changes in the apparent relative positions of the fixed stars, Aldebaran, Arcturus, and Sirius — answering a question centuries old.

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CHAPTER XIV

NATURAL AND PHYSICAL SCIENCE IN THE EIGHTEENTH CENTURY

> The seventeenth and eighteenth centuries mark the period in which, owing to the use of the several vernacular languages of Europe in the place of the medieval Latin, thought became nationalized. Thus it was that . . . people could make journeys of exploration in the region of thought from one country to another, bringing home with them new and fresh ideas. Such journeys . . . were those of Voltaire to England in 1726 . . . of Adam Smith in 1765 to France.
>
> — *Merz.*

In the preface to one of his volumes of essays, Lord Morley speaks of the eighteenth century as the scientific Renaissance. Such it undoubtedly was, for it was in this century and especially in its latter half, that chemistry, geology, botany, zoölogy, and physics, began to make deep impression on the learned world, while astronomy and mathematics ventured upon bolder and more far-reaching generalizations than they had ever before made. Science as a special discipline, or as a branch of learning worthy of the highest consideration, had as yet scarcely begun to make itself felt, but the names of Newton and Descartes were frequently heard in the salons of Paris and keen observers like Voltaire perceived the rising of a new tide in the affairs of men. A growth of popular interest might naturally have been expected after the great discoveries of the sixteenth and seventeenth centuries. What was not looked for was the concurrence of those political and social upheavals ever since rightly known as revolutions; viz. the French Revolution, the American Revolution and, probably most important of all, the Industrial Revolution.

CHEMISTRY: DECLINE OF THE PHLOGISTON THEORY. — We have already touched upon the work of Boerhaave and Hales in the field of organic chemistry, so-called, and may now pass on to the studies of Black, Bergmann and others on the *gas sylvestre* (carbonic acid) of Van Helmont. Dr. Joseph Black of Edinburgh, a physician of note and, as we shall see, one of the first to put the science of heat on a sure foundation, seeking to explain the phenomena accompanying the making and the slaking of "quick" lime, — phenomena now familar to every beginner in chemistry but in 1750 puzzling to all, — remembered that Hales had found that "air" could be driven off from certain substances by heating, and suspected that in the burning of limestone to make quicklime, something might be driven off, the loss of which would make it lighter. This something he tried to obtain by causing acid to act upon limestone (in the ordinary laboratory fashion of to-day) and collecting the gas evolved by the aid of Hales' pneumatic trough. He next weighed the gas and the remaining limestone and found that the weight of the former agreed with the loss of weight of the latter. He then reversed the experiment, causing "fixed air" (as he called it) to bubble through a solution of lime, whereupon, as he had anticipated, a white, chalk-like powder appeared and fell to the bottom. This simple experiment proved extremely fruitful, and we can now see that in its use of analysis and synthesis, in its partly quantitative character, and in the chemical reasoning employed, it was also highly instructive. Best of all, it did not require any hypothetical, immaterial or mystical "phlogiston" for satisfactory explanation of all the hitherto puzzling phenomena involved. Black invented for the gas thus driven off by heat or acid the term "fixed air," because it was evidently "fixed" in the limestone or chalky precipitate, and because any gas or vapor not obviously something else, was still supposed to be "air," — the true nature and chemical composition of the atmosphere being still (in 1750) quite unknown.

At this point Bergmann, a Swedish chemist of distinction, by the use of litmus (which Boyle had recommended as a test for acids) and other means, discovered that the "fixed air" of Black is an *acid*, and accordingly named it "aërial acid." Bergmann also weighed the new gas, finding it heavier than air, and discovered that it is very soluble in water.

To sum up: It was now known that there exists an invisible, odorless gas, resembling air but heavier than air and more soluble in water; that it is acid, and capable of attaching itself to lime, making a kind of chalk; that it will not support life, yet is present in the human breath, as well as in some mineral waters; and that it is given off during fermentations. It only remained for Lavoisier to discover (in 1779) that this gas is compounded of two very common elements — carbon and oxygen — tightly bound together, and may therefore be called, as it often is today, "carbonic acid." But before this could happen other investigations had to prepare the way, and especially the discovery of the new "element," oxygen.

A New Chemistry. Priestley and Lavoisier. — We have now reached a period of remarkable activity and rapid progress in chemical research. While Black was hard at work upon chemical problems in Scotland, and Bergmann in Sweden, Cavendish was similarly engaged in England, and in 1766 reported to the Royal Society his discovery of a new kind of gas to which, for the reason that it took fire whenever flame was applied to it, and also because he believed it to be the cause of the occasional explosions in mines, he gave the name "inflammable air." Cavendish obtained this gas by treating iron, tin, zinc, or other metals with sulphuric acid, very much as Black had obtained fixed air by treating limestone with acids. Inflammable air was, however, obviously quite unlike fixed air, since it was lighter than air — not heavier — and was readily burned. It resembled it, nevertheless, in that a lighted candle plunged into it went out, and animals died in it just as they did in fixed air. It had another peculiar property; viz. that of forming with air an explosive mixture. This new gas as we now know was hydrogen.

Not long after, in 1772, other new gases were separated and studied; viz. nitrogen by Rutherford, and nitric oxide by Priestley. It was on August 1, 1774, however, that Priestley made his most important discovery, and one that proved to be the very corner-stone of the splendid edifice in which modern chemistry now dwells, namely, the discovery of *oxygen*. Joseph Priestley, fearless reformer, Unitarian clergyman, and tireless experimenter in natural philosophy, had already made important and interesting discoveries when, in 1774, as stated above, he decomposed by heat the reddish powder obtained by calcining mercury, and collected and examined the gas given off. Candles and glowing coals burned in this gas with extraordinary energy, and mice lived in it under a bell glass even longer than in ordinary air. And, since it was derived from a burnt, *i. e.* dephlogisticated, metal and yet was colorless and odorless like ordinary air, Priestley named it "dephlogisticated air." The following is his own account of his work: —

> There are, I believe, very few maxims in philosophy that have laid firmer hold upon the mind than that air, meaning atmospheric air, is a simple elementary substance, indestructible and unalterable at least as much so as water is supposed to be. In the course of my inquiries I was, however, soon satisfied that atmospheric air is not an unalterable thing; for that, according to my first hypothesis, the phlogiston with which it becomes loaded from bodies burning in it, and the animals breathing it, and various other chemical processes, so far alters and depraves it as to render it altogether unfit for inflammation, respiration, and other purposes to which it is subservient; and I had discovered that agitation in the water, the process of vegetation, and probably other natural processes, restore it to its original purity. . . .
>
> Having procured a lens of twelve inches diameter and twenty inches focal distance, I proceeded with the greatest alacrity, by the help of it, to discover what kind of air a great variety of substances would yield, putting them into the vessel, which I filled with quicksilver, and kept inverted in a basin of the same. . . . With this apparatus, after a variety of experiments . . . on the 1st of August, 1774, I endeavored to extract air from *mercurius calcinatus per se;* and I presently found that, by means of this lens, air was expelled from it very readily. Having got about three or four times as much as the bulk of my materials, I admitted water to it, and found that it was not imbibed by it. But what surprised me more than I can express was that a candle burned in this air with a remarkably vigorous

> flame, very much like that enlarged flame with which a candle burns in nitrous oxide, exposed to iron or liver of sulphur; but as I had got nothing like this remarkable appearance from any kind of air besides this particular modification of vitreous air, and I knew no vitreous acid was used in the preparation of *mercurius calcinatus*, I was utterly at a loss to account for it. . . .
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> The flame of the candle, besides being larger, burned with more splendor and heat than in that species of nitrous air; and a piece of red-hot wood sparkled in it, exactly like paper dipped in a solution of nitre, and it consumed very fast; an experiment that I had never thought of trying with dephlogisticated nitrous air.
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> . . . I had so little suspicion of the air from the *mercurius calcinatus*, etc., being wholesome, that I had not even thought of applying it to the test of nitrous air; but thinking (as my reader must imagine I frequently must have done) on the candle burning in it after long agitation in water, it occurred to me at last to make the experiment; and, putting one measure of nitrous air to two measures of this air, I found not only that it was diminished, but that it was diminished quite as much as common air, and that the redness of the mixture was likewise equal to a similar mixture of nitrous and common air. . . . The next day I was more surprised than ever I had been before with finding that, after the above-mentioned mixture of nitrous air and the air from *mercurius calcinatus* had stood all night, . . . a candle burned in it, even better than in common air.

At almost the same time (1775) Scheele, a Swedish chemist, independently discovered the same gas. "Scheele remained a poor apothecary all his life, yet was really one of the first chemists of Europe." His name for oxygen was "empyreal air."

But if Black and Cavendish and Priestley and Scheele and others laid the foundations of modern chemistry, it was the yet more famous Lavoisier, who, building upon the results of his predecessors, began the erection of the present lofty superstructure. Lavoisier soon dismissed forever the long-standing, mystical theory of phlogiston through his unremitting use of *the balance*, for the introduction and use of which in analysis he has been rightly called "the founder of quantitative chemistry." By means of the balance he proved that when metals are burnt in air, the resulting substances weigh more than did the metal; and that if burnt in a closed space the loss in weight of the air equals the gain in weight of the metal. And when, finally, he reversed the experiment, decomposing the red powder of burnt mercury as Priestley had done, and weighing as before, he found that the loss of weight of the red powder at the end was exactly equal to the weight of the "dephlogisticated air" driven off.

Lavoisier named the gas thus derived oxygen (acid producer), because its compounds seemed to him to be chiefly acids. And since Black's "fixed air" was Bergmann's "aërial acid," and could be made by burning charcoal in air, Lavoisier suspected that fixed air contained oxygen as well as carbon. Accordingly, he proceeded to burn charcoal in pure oxygen, and obtained fixed air or aërial acid, which he thereupon analyzed, proving that 100 parts contained 72 of oxygen and 28 of carbon. He therefore named it *carbonic acid.* Afterward, by burning a diamond in pure oxygen and obtaining carbonic acid, he proved that one of the precious "stones" is really a form of carbon. Priestley died clinging to the phlogiston theory, but his dephlogisticated air had now become Lavoisier's oxygen. Lavoisier, one of the most brilliant men of science in any age, continued as long as he lived to do remarkable work. He repeated with more precision the experiment of Cavendish on the synthesis of water from inflammable air and ordinary air (or oxygen), and named the former *hydrogen* (water producer). But, unfortunately for science, Lavoisier, since he had held government office as a farmer-general, found no favor in the eyes of the leaders of the French Revolution and on May 18, 1794, at the age of 51, he was guillotined. Thus was the end of the eighteenth century, otherwise in most respects favorable to science and other learning, disgraced by a foul blot on civilization, as had been the end of the sixteenth by the burning of Giordano Bruno; the latter a victim to misdirected religious conservatism, Lavoisier to equally misdirected political radicalism.

THE SYNTHESIS OF WATER. — In 1784 Cavendish added to his other brilliant discoveries that of the composition of water. He had himself discovered inflammable air or hydrogen, and now he found that by exploding a mixture of this gas with oxygen, water was produced.

> By experiments with the globe it appeared, says Cavendish, that when inflammable and common air are exploded in a proper proportion, almost all the inflammable air, and near one-fifth the common air, lose their elasticity and are condensed into dew. And by this experiment it appears that this dew is plain water, and consequently that almost all the inflammable air is turned into pure water.
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> In order to examine the nature of the matter condensed on firing a mixture of dephlogisticated and inflammable air, I took a glass globe, holding 8800 grain measures, furnished with a brass cock and an apparatus for firing by electricity. This globe was well exhausted by an air-pump, and then filled with a mixture of inflammable and dephlogisticated air by shutting the cock, fastening the bent glass tube into its mouth, and letting up the end of it into a glass jar inverted into water and containing a mixture of 19,500 grain measures of dephlogisticated air, and 37,000 of inflammable air; so that, upon opening the cock, some of this mixed air rushed through the bent tube and filled the globe. The cock was then shut and the included air fired by electricity, by means of which almost all of it lost its elasticity (was condensed into water vapors). The cock was then again opened so as to let in more of the same air to supply the place of that destroyed by the explosion, which was again fired, and the operation continued till almost the whole of the mixture was let into the globe and exploded. By this means, though the globe held not more than a sixth part of the mixture, almost the whole of it was exploded therein without any fresh exhaustion of the globe.

BEGINNINGS OF MODERN IDEAS OF SOUND. — We have seen above that the Greeks were deeply interested in sound, as well as in music. The invention of the monochord with the discovery by Pythagoras of the relation between the length of a vibrating string and the sound which it produces was the first step in the right direction, although any mystical relation between sound and number such as the Pythagoreans inferred has no basis.

From Pythagoras to Galileo little or no progress was made. Galileo recognized that sound is due to vibrations in the air falling upon the ear-drum, and in one of his dialogues explains concord and dissonance by concurrence or conflict of such vibrations. He shows how vibrations causing sound may be made visible, and how to measure the relative length of sound waves by scraping a brass plate with a chisel, thereby making dust on the plate take up positions in parallel lines. That air is really the intermediary was proved in 1705 by Hawksbee's experiment of placing a clock in a vacuum.

After Galileo, the studies, mathematical and experimental, of Newton, Euler, and Sauveur (1653–1715) brought acoustics to the point where it was taken up and given much of its present form by Chladni (1756–1827) — "the Father of Modern Acoustics." Sauveur, eminent physicist and musician, deserves more than passing notice from the fact that he "had neither voice nor ear" (being half deaf and dumb) and yet achieved distinction for his original researches in both sound and music. Intended by his parents for the church, he early manifested delight in mechanical contrivances and in arithmetic, but the course of his life was determined by a copy of Euclid which accidentally came to his notice. He thereupon abandoned an ecclesiastical career and, having in consequence lost the support of his relatives, obtained a livelihood by teaching mathematics, becoming a professor of that subject in 1686. During the remainder of his life the study of acoustics, and particularly the scientific theory of music (in which he was the first to draw attention to overtones or harmonics) occupied much of his attention. Many of his papers were published in the Memoirs of the French Academy, 1700–1714. The work of Chladni at the beginning of the nineteenth century laid broad and deep the foundations of acoustics as we know that science to-day, and upon this foundation Helmholtz and Tyndall in the middle of that century reared a large part of the modern superstructure. Chladni carried much further the experiments of Galileo on vibrating plates, substituting a violin bow for the chisel, and sand for dust on the plates, obtaining thereby that wonderful variety of figures which is nowadays demonstrated to beginners by every teacher of natural philosophy. He also devised a simple method of counting the number of vibrations corresponding with each note.

THE BEGINNINGS OF MODERN IDEAS OF HEAT: LATENT AND SPECIFIC HEAT; CALORIMETRY. — The earlier ideas of the nature of heat are not unlike those of light. Theories of emission were at first preferred for both, and present ideas of vibration or undulation have appeared only recently. Heat was even regarded as an imponderable yet material substance, "caloric," emitted by hot bodies and absorbed by cold ones. High temperature meant the presence, and low temperature, the absence, of caloric. The rise of mercury in the thermometer tube was explained as due to the expansion of the mercury not, as to-day, by increase of distance between molecules, but by the addition of caloric and the consequent increase of total material. Francis Bacon remarked on the problem of heat and in an interesting passage on the proper method of its study shows that he knew many of its phenomena, including its development "in bodies heated by rubbing." But it remained for Black, of Edinburgh, whose work on fixed air or carbonic acid we have dwelt upon above, to make the fundamental researches which paved the way both for the study of the theory of heat by Rumford at the end of the eighteenth century, and for its industrial uses by Watt in steam engineering in the middle of that century. Black, while experimenting on heating and cooling, discovered that heat may be applied to boiling water, or to water containing melting ice, without raising the temperature. Obviously, such applied heat must either be lost or somehow become latent. Further experiments showed that once the ice is all melted, or once the escape of steam is stopped by covering the water in a closed vessel, the temperature begins to rise; the heat is no longer concealed, lost, or absorbed, but produces obvious effects. Apparently the lost heat was somehow used in melting the ice and in making the steam. These and other experiments by Black proved suggestive to Watt, who at this time was trying to improve upon the air-and-steam engine of Newcomen, in which atmospheric pressure was used to push a piston in a cylinder filled with and then emptied of steam, and it was largely by the aid of Black's studies on latent heat that Watt was enabled and encouraged to persevere and push to completion his own epoch-making discoveries in steam engineering.

Black was also the first to recognize and investigate what we know today as "specific" heat and, by means of a cavity in a block of ice into which various heated bodies were brought, to weigh the water each would produce while cooling from the same temperature, — in other words to invent and use the *calorimeter*.

EIGHTEENTH CENTURY RESEARCHES ON LIGHT. — These start from the great work of Newton, and especially of Huygens, in the previous century, and continue with the fruitful inventions of the achromatic telescope by Hall in 1733, and the work of Dollond, an English optician, upon achromatic lenses, leading up to the construction in 1758 of achromatic telescope objectives. The achromatic telescope now became a serviceable instrument, but the compound microscope had to wait more than half a century longer for correspondingly serviceable achromatic objectives. It was not until the opening of the nineteenth century that much further progress was made in our knowledge of light. It is rather for progress in sound, in heat, and in electricity, that eighteenth century physical science is chiefly notable.

BEGINNINGS OF MODERN IDEAS OF ELECTRICITY AND MAGNETISM. — The seventeenth century had witnessed no great progress in these subjects, and the sixteenth century work of William Gilbert stood as almost the only important contribution to our knowledge of them until about 1730. Von Guericke, following suggestions of Gilbert, had, it is true, made a rude electrical machine which he described in a work published in 1672, and had observed the electric spark, which with his machine was so small as to be seen only in the dark and to be heard with difficulty. Much more important were the observations of Francis Hawksbee (or Hauksbee), "one of the most active experimental philosophers of his age," and one of the first to study capillary action, who in 1705 communicated to the Royal Society several curious experiments on what he called "the mercurial phosphorus," showing that light could be produced by passing common air through mercury placed in a well-exhausted receiver. These phenomena, which had been observed before Hawksbee's time and had been variously explained, were attributed by him to electricity, for he remarked their resemblance to lightning. Like Newton he used a revolving glass globe, rubbed by the hand, to generate electricity. These and other results he published in 1707–1709.

Further investigations by Wall, Gray and Wheeler, Desaguliers, Dufay, and many others prepared the way for Watson, Franklin, Galvani, and Volta, whose investigations in the latter half of the eighteenth century, added to those just referred to, would alone make that century forever famous in the history of science. On these brilliant discoveries we can only briefly touch. Wall (1708) compared the electric spark and its crackling to lightning and thunder. Gray and Wheeler (1729) and later Dufay created what has been called an epoch in the history of electricity by discovering that different bodies differ in electrical conductivity, while Desaguliers confirmed and extended their results. Dufay (1699–1739) repeated these and other experiments and discovered that there are *two kinds of electricity,* positive and negative, or, as he called them from their source of origin, *vitreous* and *resinous.* Watson undertook with the aid of a party of friends from the Royal Society to determine the velocity of the electric current and found "that through the whole length of a wire 12,276 feet long, the velocity of electricity was instantaneous." Many others had also made important, if minor, contributions to the new science of the electric "fluid" — for like heat (caloric) electricity was regarded as material though imponderable — before that science was further developed and widely popularized by Franklin.

Benjamin Franklin (1706–1790) was not the first American to do good work for science, but he was the first to gain wide renown in it together with an international reputation. Even before 1750, Franklin had argued that all the known phenomena of electricity had their counterpart in lightning, but it was not until

June, 1752, that he made his famous kite experiment, which showed that lightning is really an electrical phenomenon, since a Leyden jar can be charged from the skies, and at the same time proved that atmospheric and machine-made (frictional) electricity are one and the same thing. This daring experiment was performed also in Europe at about the same time by others, and marks one of the greatest triumphs of science in any age, for it simplified and to a great extent explained one of the oldest and most awe-inspiring phenomena of nature. Furthermore, by correlating the thunderbolts of Zeus and the shocks of a Leyden jar with the sparks of an electrical machine, and even with those from a cat's back, it tended mightily to inspire confidence in natural philosophy and to lessen correspondingly the universal dread of unseen, mysterious, and supposedly supernatural, powers or influences. Other important work in electricity was done in this century by Beccaria in Italy, Canton and Symmer in England, and many others.

THE BEGINNINGS OF MODERN IDEAS OF THE EARTH. — The eighteenth century saw important progress also in biological subjects, — although the word biology was not yet born, and zoölogy and botany, even, were still undifferentiated and closely associated with geological knowledge under the broad and hospitable Aristotelian term "natural history." Physiology meanwhile retained its close connection with its parent medical science, its logical relations to zoölogy and botany being generally unrecognized. Inquiries were, however, on foot destined, as we can now see, to bring about changes, namely, to differentiate natural history into geology, botany, and zoölogy and, finally, to integrate the two latter sciences into one greater than either, viz. biology.

It must have occurred to the thoughtful reader of the foregoing pages that the four elements of the Greeks and their followers have by this time lost their primitive character, and become highly complex compounds or combinations of phenomena. Air, for example, by the end of the eighteenth century was proved to be a mixture of various elements and compounds, to possess weight and to exert pressure, to have no part in abhorrence of vacua, and to be the seat of marvellous aqueous and electrical phenomena.

Fire, long a mystery, was by this time regarded, not as an "element," but as a luminous centre of intense chemical change. Water, above all, — abundant, important, susceptible of metamorphism into ice, snow, dew, fog, and steam, — had surrendered the secrets of its very being, having been split apart into hydrogen and oxygen gases, and created or restored as a liquid by bringing these two gases together at high temperature. Here also, as in Franklin's kite experiment, mystery, if not dispelled, was at least driven back; and the suggestion became natural and reasonable, — If only enough were known, might not many other mysteries in nature be lessened, if not altogether done away?

But, while these more comprehensible and more rational ideas of air, fire, and water were now the common property of natural philosophy, natural history still held unsolved most of its ancient problems. The earth, for example, while probably no longer regarded as one of the four elements, was as yet a standing puzzle in respect to its origin, both as a whole and as to its parts. Astronomy had proved the planetary character of the earth but had not yet suggested for it or for its fellows any natural, as opposed to supernatural, origin, and was entirely silent as to the sources and history of the earth's crust, so rich in minerals, metals, volcanoes, earthquakes, soils, craters, gases and particularly fossils, — those mute remains which could no longer be disposed of as freaks of nature, but must be looked upon as indefeasible witnesses to a prehistoric past. Leonardo in the fifteenth and Palissy in the sixteenth century revived the ideas of Pythagoras and Xenophanes as to the true nature of fossils, but no further progress of note was made for upwards of a hundred years, when about 1670 Steno, a Dane, and Scilla, an Italian, published studies on petrifactions, illustrated with drawings. Hooke, already referred to (p. 268), Ray, the naturalist, and later (1695) Woodward, made collections of chalk, gravel, coal, and marble, and gravely discussed their meaning in terms of the Flood of Noah. In this unsatisfactory position matters stood at the end of the seventeenth century, and it was not until nearly the middle of the eighteenth, viz. in 1740, that much further progress was made. At that time Lazzaro Moro put forward the view that the rocks must have been in process of formation when fossils were included in them, and that the earth's crust evidently consists of strata, superimposed one upon another. He also reasoned from the character of the included fossils, back to the character of the rocks containing them, — arguing that some must have been laid down in fresh water, others in salt water, and hence some in rivers or lakes, and some in the sea.

In 1765 the first school of mines of which we have record was established at Freiberg, in Saxony, and here appeared in 1775 a student of the natural history of minerals and of the earth, viz. Abraham Werner, son of an Inspector of Mines at Freiberg, and eventually a popular teacher there of mining and geology. Werner's name is associated with a special school — the Neptunists — who, following him, held that the crust of the earth had been laid down in water. In opposition, another school — the Vulcanists — arose, holding that it has come rather by fire, volcanoes, and the like.

Towards the end of the eighteenth century, Dr. Hutton of Edinburgh, and William Smith, an English surveyor, made patient, accurate, and detailed studies of fossils and their distribution, and of erosion and other work of water, over a considerable area, and published, the former a Theory of the Earth (1788), the latter a geological map of England (1815). These formed a solid basis for that epoch-making work by Lyell, in 1830, to which we shall refer hereafter. Hutton deserves to be especially remembered with honor for his insistence that the best interpreter of the past is the present; that if we would know how rocks were formed ages ago, we have only to observe how they are being formed today. This simple doctrine of "uniformitarianism" was not only an inspiration to Lyell, but, largely through Lyell, prepared the way for Darwinism and other evolutionary ideas requiring time with slow change, in the scientific revolution of the nineteenth century.

EIGHTEENTH CENTURY PROGRESS IN BOTANY, ZOÖLOGY, ETC. — The great world of plant and animal life, even at the middle of the eighteenth century, was still almost unexplored and unclassified. The early work of Aristotle in zoölogy and of Theophrastus in botany, as well as that of Gesner in the sixteenth century and Ray and Grew and Malpighi and Willughby in the seventeenth have been referred to above, but until we come to Buffon (1707–1785), the French naturalist, and Linnæus (Carl von Linné) (1707–1778), the Swedish, we meet with no other great name, and find no important researches on record within the field of natural history. Buffon's special contribution to science was a fine work on "Natural History," and an infectious enthusiasm which so popularized him that 20,000 people are said to have mourned at his funeral.

Linnæus was also an immensely popular writer and teacher of natural history, who at the same time advanced the science of botany by introducing an order and system into the classification of plants which facilitated their comparative study. It is not too much to say that Linnæus established botany as a science. He also did much work upon animals and minerals, but his famous dictum, "stones grow, plants grow and live, and animals grow, live, and feel," while emphasizing an important and fundamental similarity in natural objects, has long since lost whatever standing it may once have had. It is not so much in their properties as in their substance that stones, plants, and animals agree, and the greatest service done by Linnæus for science was his insistence on the importance of the careful observation of likeness and difference, and of clear and accurate description. To this end he introduced a binomial system, so that closer and more accurate classification in natural history was facilitated and ever after employed. His first great work, *Systema naturæ,* was published in 1735. His collection of plants, insects, books, etc. now forms the nucleus of the Linnæan Society library in London, founded in 1788. The special procedures adopted by him proved to be artificial, and were soon replaced by a more natural basis of classification introduced by de Jussieu; but the scientific names applied to many animals (including man himself, *Homo sapiens*) and many plants, are still in common use throughout the scientific world.

> In the time of Aristotle man took his place naturally at the head of the other animals. . . . But the influence of religion and philosophy did not long permit of this association. Man came to be regarded as the *chef-d'œuvre* of creation, a thing apart 'a little lower than the angels.' In the eighteenth century came a startling change, man was wrenched from this detached and isolated attitude and linked on once more to the beasts of the field. This was the work of Linnæus. . . .
>
> Buffon did not classify, he described . . . the genius of Linnæus lay in classification. Order and method were with him a passion. In his *Systema naturæ* he fixed the place of man in nature, arranging *Homo sapiens* as a distinct species in the order *Primates,* together with the apes, the lemurs and the bats. — *Haddon.*

Progress in Comparative Anatomy and Physiology. —The seventeenth century was peculiarly rich in physiological and anatomical discoveries, largely because it was endowed with a man of genius in physiology, Harvey, and with a new and valuable instrument, — the compound microscope. It is true that this last did not fulfil its promise, because of mechanical defects, but it was good enough to enable Malpighi to clinch with positive proof Harvey's theory of the circulation of the blood, besides revealing certain anatomical features of spleen and kidney, hitherto unknown. In 1743 Haller (1707–1777) of Berne proved that the muscles do not depend for their contractility, as had been supposed, upon "vital spirits" sent in through the nerves, but possess independent and intrinsic powers of contraction even when separated from the nervous system or from the body itself. This important discovery, together with much excellent anatomical work, was made by Haller at Göttingen, where he was professor "of anatomy, surgery, and botany" and whither he soon drew large numbers of enthusiastic pupils. Haller will long be remembered, not only for his great work in physiology and in teaching, but also as one of the founders of comparative anatomy. In this subject, however, he was soon left far behind by the famous John Hunter (1728–1793) and William Hunter, his brother.

We must not omit to observe that with comparative anatomy, comparative botany, and comparative geology and mineralogy, eighteenth century science was now laying solid foundations for the great generalizations of the nineteenth century. Comparative physiology, even, was making a beginning, with the experiments of Bonnet (1720–1793) upon the reproduction of lost parts in the lower animals, and of Spallanzani (1729–1799) upon spontaneous generation. To consideration of the labors of these last we shall return.

THE INDUSTRIAL REVOLUTION. INVENTIONS. POWER.—Far-reaching in their consequences as were the French Revolution of 1793 and the American Revolution of 1776, it is the Industrial Revolution, especially after 1770, with which the student of the history of science has chiefly to deal. Before the Industrial Revolution, *i.e.* almost everywhere before 1760 or even 1770, whatever machinery existed was run mostly by hand or foot, and was hence easily operated in the separate homes of the workers. But within the next thirty years the factory system had come, with coöperative labor in or about some central power-plant, and with machinery driven by water power or steam. With this change, which increased the output of the individual, and took work and workers out of the home, a revolution began which is still affecting every country and has modified the very structure of human society.

The change was probably imminent in any event, for the use of water power had begun before the introduction of steam; but the perfection of the steam-engine by Watt, who as we have seen, was powerfully aided by the scientific studies of his fellow countryman, Black, on heat, steam, evaporation, and calorimetry, greatly hastened, and soon made almost universal, the mighty change. Henceforth machinery was to become the handmaid of toil, and to bring with it not only factory industry in place of home industry but, before long, improved means of transportation, effecting a virtual shrinkage of the world and a far closer contact of mankind.

Almost coinciding with the introduction of water power and steam power, came a great burst of invention. The spinning "jenny" and the "water frame" came almost hand in hand with the "mule" and the "power loom;" while, as if to meet these on the cotton field, the cotton "gin" (engine), was invented (by Eli Whitney of Connecticut) to replace the slow and tedious process of separating the cotton fibre or staple from its seed — hitherto laboriously done by hand. Applied chemistry also began to appear, *e.g.* in the manufacture of illuminating gas by Murdoch at Salford, England, in 1792, while the discoveries of Galvani and Volta at the very end of the century opened up that new era of electricity in the midst of which we dwell to-day.

THE INFLUENCE OF SCIENCE UPON THE SPIRIT OF THE EIGHTEENTH CENTURY. — Writers on the literature of the eighteenth century, after condemning it because of its comparative barrenness in great works of art or literature, are apt to find the reason in one or another aspect of the growth of science. Professor Dowden, for example, in his essay on Goethe, remarks that

> Rousseau's emancipation of the heart, was felt in the eighteenth century to be a blessed deliverance from the eager, yet too arid, speculation of the age,

although he admits that: —

> Humanity, as Voltaire said, had lost its title-deeds, and the task of the eighteenth century was to recover them.

Dowden's unusually charitable judgment of the century is more or less typical of literary opinion generally.

For the scientist, on the other hand, few centuries in all history are more important, for the eighteenth was not only rich in scientific performance but still more pregnant with promise. And even in art — if in that term music be included — and literature, a century which produced a Haydn, a Mozart and a Beethoven, with a Burns, a Voltaire, a Wordsworth and a Goethe, need not fear to hold up its head.

We have mentioned above the first School of Mines: viz. that at Freiberg, in Saxony (1765). The first School of Civil Engineering was established in Paris (1747). In this century also were established new universities, *e.g.*, Yale (1701), Göttingen (1737), Princeton (1746), Bonn (1777) and Brussels (1781).

The "physiocrats" and the "encyclopædists" of the French school of practical philosophers also deserve notice, for they were professedly inspired by science and seeking to apply it to human society. Even in so humble a pursuit as the attempt to overcome in time of famine the prejudices of the populace against the potato, Turgot and his fellows did good work for applied science. Nor should we forget the service to social science of Count Rumford, who for the first time grappled boldly with problems as far apart as the control of mendicity, of smoky chimneys, and of poverty. Much of Rumford's best work, though done in the nineteenth century, had its origin in the scientific spirit and achievements of the eighteenth.

As the century drew to its close, an English physician, Edward Jenner, by the use of the basic methods of inductive scientific research — accurate observation, skillful experimentation, careful generalization and thorough verification—created a new science, preventive medicine, and conferred upon mankind the priceless blessings of vaccination. (See Appendix G.)

The nebular hypothesis of Laplace, through its central idea of natural development rather than sudden and special (artificial) creation of the solar system, was an important preparation of men's minds for theories of transformation or evolution. Hutton's Theory of the Earth enforced the same idea for the familiar earth, while the metamorphoses of the parts of the flower, pointed out by Goethe, helped to pave the way for acceptance of the idea of gradual modification of organs and even of organisms into others. To these matters we shall return in our discussion of Evolution in the final chapter.

REFERENCES FOR READING

(See page 461.)

CHAPTER XV

MODERN TENDENCIES IN MATHEMATICAL SCIENCE

Mathematics is the queen of the sciences and arithmetic the queen of mathematics. She often condescends to render service to astronomy and other natural sciences, but in all relations she is entitled to the first rank. — *Gauss.*

Thought-economy is most highly developed in mathematics, that science which has reached the highest formal development, and on which natural science so frequently calls for assistance. Strange as it may seem, the strength of mathematics lies in the avoidance of all unnecessary thoughts, in the utmost economy of thought-operations. The symbols of order, which we call numbers, form already a system of wonderful simplicity and economy. When in the multiplication of a number with several digits we employ the multiplication table and thus make use of previously accomplished results rather than repeat them each time, when by the use of tables of logarithms we avoid new numerical calculations by replacing them by others long since performed, when we employ determinants instead of carrying through from the beginning the solution of a system of equations, when we decompose new integral expressions into others that are familiar, — we see in all this but a faint reflection of the intellectual activity of a Lagrange or Cauchy, who with the keen discernment of a military commander marshals a whole troop of completed operations in the execution of a new one. — *Mach.*

The iron labor of conscious logical reasoning demands great perseverance and great caution; it moves on but slowly, and is rarely illuminated by brilliant flashes of genius. It knows little of that facility with which the most varied instances come thronging into the memory of the philologist or historian. Rather is it an essential condition of the methodical progress of mathematical reasoning that the mind should remain concentrated on a single point, undisturbed alike by collateral ideas on the one hand, and by wishes and hopes on the other, and moving on steadily in the direction it has deliberately chosen. — *Helmholtz.*

Nature herself exhibits to us measurable and observable quantities in definite mathematical dependence; the conception of a function is suggested by all the processes of nature where we observe natural phenomena varying according to distance or to time. Nearly all the "known" functions have presented themselves in the attempt to solve geometrical, mechanical, or physical problems. — *Merz.*

We have now reached a period of maturity in the evolution of mathematical science beyond which any attempt to follow its details would involve technical discussions outside the range of this work. The present chapter will be devoted to a general survey of modern tendencies in pure and applied mathematics, in mechanics, in mathematical physics and in astronomy. The most notable single fact in the centuries under discussion is the increasing specialization resulting from the great expansion of scientific knowledge. It is no longer possible for the individual scholar to command the range at once of philosophy, mathematics, physics, chemistry, and the natural sciences. It has even become more and more difficult to have a general knowledge of any one of these broad fields.

MATHEMATICS AND MECHANICS IN THE EIGHTEENTH CENTURY. — The invention of the infinitesimal calculus by Newton and Leibnitz was comparable in its relations and consequences with the discovery of a new world by Columbus two centuries earlier. As in that case the discovery was not an absolutely sudden one; other explorers had hoped or imagined, but only genius of that highest order which we call inspired, gained the complete revelation. The years next following the great discovery were naturally a period of eager and wide-ranging exploration, of optimistic self-confident pioneering. Such was the power of the new method, that one might rashly hope no secret of nature could long resist its attack. As circumnavigation of the globe was not long in following the discovery of America, so the cycle of mathematical knowledge might be completed. The parallel has failed. The calculus grew out of the insistent grappling by mathematicians with problems which had defied the feebler tools of the earlier mathematics. One obstacle after another has been gradually surmounted by the invention of new and more powerful methods of ever increasing generality, just as increasingly powerful telescopes have revealed unnumbered new suns; and no boundary or limit to this evolutionary progress can be foreseen or imagined. On the other hand, as the new world has been gradually settled, civilized, and cultivated, so the fields of mathematics which were opened up in the eighteenth century have been critically examined in the nineteenth, with much revision of fundamentals.

The main features of eighteenth century mathematics were: — the working out of the differential and integral calculus into substantially the form they have ever since retained; the beginnings of differential equations as a natural outgrowth of integral calculus, and the beginnings of the calculus of variations; the systematic application of the new ideas to mechanics, and in particular to celestial mechanics. The century was also notable for important discoveries in astronomy and physics, including for example that of the aberration of light; a vigorous attack on "the problem of three bodies"; and the earlier telescopic work of the Herschels, culminating in the discovery of a new planet, Uranus.

Among the leading mathematicians of the period were Maclaurin of Scotland, various members of the Swiss Bernoulli family, Euler also a native of Switzerland, Lagrange of Italy, and in France, Clairaut, d'Alembert, and Laplace. In spite of the unique supremacy of Newton, the absence of Britons from this list is notable. The bitter personal controversy between Newton's adherents and those of Leibnitz produced or aggravated an unfortunate division between the English and the continental mathematicians. For the former, persistence in Newton's inferior notation became a matter of national pride, and progress was correspondingly retarded. Of the mathematicians named above, the Bernoullis and Euler on the continent and Maclaurin in Scotland bore a leading part in the systematization of the calculus, while Lagrange and Laplace were preëminent in the development of analytical and celestial mechanics respectively.

Maclaurin's (1698–1746) Treatise of Fluxions (1742) was "the first logical and systematic exposition of the method of fluxions," and the applications to problems contained in it were characterized by Lagrange as the "masterpiece of geometry, comparable with the finest and most ingenious work of Archimedes." Maclaurin's point of view may be illustrated by the following passage: —

> Magnitudes were supposed to be generated by motion; and, by comparing the increments that were generated in any equal successive parts of the time, it was first determined whether the motion was uniform, accelerated, or retarded. . . . When the motion was accelerated, this increment was resolved into two parts; that which alone would have been generated if the motion had not been accelerated, but had continued uniform from the beginning of the time, and that which was generated in consequence of the continual acceleration of the motion during that time. The latter part was rejected, and the former only retained for measuring the motion at the beginning of the time. And in like manner, when the motion was retarded, . . . so that the motion at the time proposed was accurately measured, and the *ratio* of the fluxions always accurately represented. In the method of infinitesimals, the element, by which any quantity increases or decreases, is supposed to be infinitely small, and is generally expressed by two or more terms, some of which are infinitely less than the rest, which being neglected as of no importance, the remaining terms form what is called the *difference* of the proposed quantity. The terms that are neglected in this manner, as infinitely less than the other terms of the element, are the very same which arise in consequence of the acceleration, or retardation, of the generating motion, during the infinitely small time in which the element is generated. . . . The conclusions are accurately true, without even an infinitely small error. . . .

Daniel Bernoulli (1700–1782) made such good use of the new mathematical methods in attacking previously unsolved problems of mechanics, that he has been called the founder of mathematical physics. He recognized the importance of the principle of the conservation of force anticipated in part by Huygens.

Euler (1707–1783), while Swiss by birth, spent most of his life at the courts of St. Petersburg and Berlin. In spite of partial and ultimately complete blindness, his scientific productivity was enormous, and one of the most ambitious scientific undertakings of our own time is the publication of his complete works in 45 volumes, by international coöperation. Our college mathematics — algebra, analytic geometry, and the calculus — owes its present shape largely to his works.

Euler's Complete Introduction to Algebra was one of the most influential books on algebra in the eighteenth century, and not the least because it is written with extraordinary clearness and in easily intelligible form. Euler was at that time already totally blind. He picked out a young man whom he had brought with him from Berlin as an attendant and who could reckon tolerably, but who otherwise had no understanding of mathematics. He was a tailor by trade and of moderate intellectual capacity. To him Euler dictated this book, and the amanuensis not only understood everything well but in a short time acquired the power to carry out difficult algebraic processes by himself with much facility. It was this book which completing the development begun by Vieta made algebra an international mathematical shorthand.

Euler formulated the idea of function which has proved so fundamental in modern mathematics, both pure and applied. His work also contains the first systematic treatment of the calculus of variations, which is defined as "the method of finding the change caused in an expression containing any number of variables when one lets all or any of the variables change" or more geometrically "a method of finding curves having a particular property in the highest or the lowest degree."

In other fields Euler "was the first to treat the vibrations of light analytically and to deduce the equation of the curve of vibration as dependent upon elasticity and density. . . . He deduced the law of refraction analytically and explained that the rays of greater wavelength must suffer the least deviation. . . . He studied dispersion in the search for a corrective for chromatic aberration, which Newton had declared unattainable. . . . It was this investigation that induced Dolland to construct his achromatic lenses. . . . Euler was thus the only physicist of the eighteenth century who advanced the undulatory theory." — *Bull. Amer. Math. Soc., Dec.,* 1907.

Euler gained a share of the prize of £20,000 offered by the British parliament for a method of determining longitude at sea, half of the same prize falling to Harrison the maker of a ship's chronometer sufficiently accurate for the same purpose.

He was probably the most versatile as well as the most prolific of mathematicians of all time. There is scarcely any branch of modern analysis to which he was not a large contributor, and his extraordinary powers of devising and applying methods of calculation were employed by him with great success in each of the existing branches of applied mathematics; problems of abstract dynamics, of optics, of the motion of fluids, and of astronomy were all in turn subjected to his analysis and solved. — *Berry.*

It is the invaluable merit of the great Basle mathematician Leonhard Euler, to have freed the analytical calculus from all geometrical bonds, and thus to have established *analysis* as an independent science, which from his time on has maintained an unchallenged leadership in the field of mathematics. — *Hankel.*

Progress in Theoretical Mechanics. The rapid development of mechanics in the eighteenth century culminated in the great classical treatises of d'Alembert (1717–1783) — *Traité de dynamique* — and Lagrange (1736–1813) — *Mécanique analytique,* systematizing and coördinating the theories and results thus far obtained. D'Alembert, working out ideas based on Huygens' theory of the centre of oscillation, formulated a very general dynamical principle since known under his name: —

On a system of points M, M', M'' ... connected with one another in any way, the forces P, P', P'' ... are impressed. These forces would impart to the *free* points of the system certain determinate motions. To the *connected* points, however, *different* motions are usually imparted — motions which could be produced by the forces W, W', W'' ... These last are the motions which we shall study.

Conceive the force P resolved into W and V, the force P' into W' and V', and the force P'' into W'' and V'', and so on. Since, owing to the connections, only the components W, W', W'' ... are effective, therefore the forces V, V', V'' ... must be *equilibrated* by the connections. We will call the forces P, P', P'' ... the *impressed* forces, the forces W, W', W'' ..., which produce the actual motions, the *effective* forces, and the forces V, V', V'' ... the forces *gained and lost,* or the *equilibrated* forces. We perceive, thus, that if we resolve the impressed forces into the effective forces and the equilibrated forces, the latter form a system balanced by the connections. — *Mach.*

To d'Alembert is attributed the celebrated epigram concerning Benjamin Franklin, "He snatched the thunderbolt from heaven, the sceptre from tyrants" (*Eripuit coelo fulmen sceptrumque tyrannis*).

J. L. Lagrange (1736–1813), a native of Turin, also spent many years in Berlin and his later life in Paris, where he became professor at the newly established *École polytechnique.* At the age of 25 he was pronounced the greatest mathematician living. His chief work, the *Mécanique analytique,* is a masterly discussion of the whole subject, showing by the aid of the new mathematical methods its dependence on a few fundamental principles. On the death of his royal patron, Frederick the Great, in 1787, he was invited from Berlin not only to Paris, but to Spain and to Naples, accepting the first-named opportunity. Lagrange's works include also very important contributions to differential equations and the calculus of variations, of which any detailed account would be too technical for our purpose. The significance and importance of Lagrange's *Mécanique analytique* are within its field comparable with those of Newton's *Principia.*

Lagrange like Newton has possessed in the highest degree the fortunate art of discovering the universal principles which constitute the essence of science. This art he understands how to unite with a rare elegance in the development of the most abstruse theories. — *Laplace.*

In contrast with the predominantly geometrical and synthetic methods of Newton, Lagrange's methods are mainly analytical.

Generality of points of view and of methods, precision and elegance in presentation, have become, since Lagrange, the common

> property of all who would lay claim to the rank of scientific mathematicians. — *Hankel.*

> When we have grasped the spirit of the infinitesimal method, and have verified the exactness of its results either by the geometrical method of prime and ultimate ratios, or by the analytical method of derived functions, we may employ infinitely small quantities as a sure and valuable means of shortening and simplifying our proofs.
>
> — *Lagrange.*

Lagrange also applied his great powers of analysis to problems in astronomy and in cartography.

CELESTIAL MECHANICS. — Pierre Simon Laplace (1749–1827) was of humble Norman antecedents which he in later life somewhat disdained, and played a great part in the scientific activity under Napoleon. In the five volumes of his *Mécanique céleste,* he produced a permanent monument to his own genius. It was his lofty ambition

> to offer a complete solution of the great mechanical problem presented by the solar system, and bring theory to coincide so closely with observation that empirical equations should no longer find a place in astronomical tables.

> He regarded analysis merely as a means of attacking physical problems, though the ability with which he invented the necessary analysis is almost phenomenal. As long as his results were true he took very little trouble to explain the steps by which he arrived at them; he never studied elegance or symmetry in his processes, and it was sufficient for him if he could by any means solve the particular question he was discussing. — *Ball.*

Bowditch, the American translator of his great work, remarks significantly: —

> I never come across one of Laplace's '*Thus it plainly appears*' without feeling sure that I have hours of hard work before me to fill up the chasm and find out and show *how* it plainly appears.

In the words of the historian Todhunter,

> a complete evolution of the history will restore the reputation of Laplace to its just eminence. The advance of mathematical science is

on the whole remarkably gradual, for with the single exception of Newton there is very little exhibition of great and sudden developments; but the possessions of one generation are received, augmented and transmitted by the next. It may be confidently maintained that no single person has contributed more to the general stock than Laplace.

THE PERTURBATION PROBLEM. — Newton had worked out the theory of a single planet or satellite revolving about its primary. The consequent discrepancies were held by some to indicate inexactness in his hypothetical laws. Laplace occupied himself with a thorough study of the great problem of three bodies,[1] and without fully solving it, accounted to a great extent for the discrepancies in question. In particular he maintained the stability of the solar system. His *Mécanique céleste* has been characterized as an infinitely extended and enriched edition of Newton's *Principia.*

In his confidence in the extending range of mathematical methods Laplace says: —

> Given for one instant an intelligence which could comprehend all the forces by which nature is animated and the respective positions of the beings which compose it, if moreover this intelligence were vast enough to submit these data to analysis, it would embrace in the same formula both the movements of the largest bodies in the universe and those of the lightest atom: to it nothing would be uncertain, and the future as the past would be present to its eyes. The human mind offers a feeble outline of that intelligence, in the perfection which it has given to astronomy. Its discoveries in mechanics and in geometry, joined to that of universal gravity, have enabled it to comprehend in the same analytical expressions the past and future states of the world system.

THE NEBULAR HYPOTHESIS. — In his *Exposition du système du monde,* "one of the most perfect and charmingly written popular treatises on astronomy ever published, in which the great mathematician never uses either an algebraical formula or a geometrical diagram", Laplace presents the arguments for his nebular hypothesis along the following general lines: —

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[1] Given at any time the positions and motions of three mutually gravitating bodies, to determine their positions and motions at any other time — a particular case of the actual more general problem: Given the 18 known bodies of the solar system, and their positions and motions at any time, to deduce from their mutual gravitation by a process of mathematical calculation their positions and motions at any other time; and to show that these agree with those actually observed.

In spite of the separation of the planets they bear certain remarkable relations to each other;

All the planets travel about the sun in the same direction and almost in the same plane;

The satellites also travel about their planets in this same direction and almost in the same plane;

Finally, sun, planets and satellites revolve in the same sense about their own axes and this rotation is approximately in the orbital plane.

These agreements cannot be accidental. Laplace seeks the cause in the existence of an original vast nebulous mass forming a sort of atmosphere about the sun and extending beyond the outermost planet. Initial or acquired rotation of the nebula attended by gradual cooling and contraction has caused the centrifugal separation of masses analogous to Saturn's rings, out of which planets have gradually condensed, throwing off their own satellites in the process. This hypothesis had already been proposed in substance by Kant in 1755. Its later history will be touched on in a following chapter.

Laplace was also deeply interested in the theory of probability, as may be illustrated by the following passages: —

> The most important questions of life are, for the most part, really only problems of probability. Strictly speaking one may even say that nearly all our knowledge is problematical; and in the small number of things which we are able to know with certainty, even in the mathematical sciences themselves, induction and analogy, the principal means for discovering truth, are based on probabilities, so that the entire system of human knowledge is connected with this theory.
>
> It is remarkable that a science (probabilities) which began with the consideration of games of chance, should have become the most important object of human knowledge.

The theory of probabilities is at bottom nothing but common sense reduced to calculus; it enables us to appreciate with exactness that which accurate minds feel with a sort of instinct for which ofttimes they are unable to account. If we consider the analytical methods to which this theory has given birth, the truth of the principles on which it is based, the fine and delicate logic which their employment in the solution of problems requires, the public utilities whose establishment rests upon it, the extension which it has received and which it may still receive through its application to the most important problems of natural philosophy and the moral sciences; if again we observe that, even in matters which cannot be submitted to the calculus, it gives us the surest suggestions for the guidance of our judgments, and that it teaches us to avoid the illusions which often mislead us, then we shall see that there is no science more worthy of our contemplations nor a more useful one for admission to our system of public education.

MODERN ASTRONOMY. TELESCOPIC DISCOVERIES. — The immense impetus given to astronomy by the revolutionary discoveries of Copernicus, Tycho Brahe, Galileo, Kepler, and Newton, followed in the eighteenth century by the complete working out of the mathematical consequences of the gravitation theory by Laplace and others, placed the science in advance of all its rivals and seemed to make it a model for their imitation.

A different and most far-reaching tendency appears with the work of the Herschels. Friedrich Wilhelm Herschel (1738–1822), a poor German musician emigrating to England and devoting his spare time unremittingly to astronomy, with the help of his capable sister laid the foundations of modern physical astronomy. In 1781 he amazed himself as well as the scientific world by discovering beyond Saturn a new planet, Uranus, — taking it at first for a comet. Constructing more and more powerful telescopes he discovered several satellites of Uranus and two of Saturn. He also determined a motion of the solar system as a whole, towards a point in the constellation Hercules. He catalogued more than 800 double stars and more than 2000 nebulas, recognizing among the latter, as he believed, different stages of the evolution of other planetary systems. He observes:

> This method of viewing the heavens seems to throw them into a new kind of light. They are now seen to resemble a luxuriant garden, which contains the greatest variety of productions, in different flourishing beds; and one advantage we may at least reap from it is, that we can, as it were, extend the range of our experience to an immense duration. For, to continue the simile I have borrowed from the vegetable kingdom, is it not almost the same thing, whether we live successively to witness the germination, blooming, foliage, fecundity, fading, withering, and corruption of a plant, or whether a vast number of specimens selected from every stage through which the plant passes in the course of its existence, be brought at once to our view?

With a reminiscence of Descartes, he says: —

> I determined to accept nothing on faith, but to see with my own eyes what others had seen before me. . . . When I had carefully and thoroughly perfected the great instrument in all its parts I made systematic use of it in my observations of the heavens, first forming a determination never to pass by any, the smallest, portion of them without due investigation.

To the eighteenth century also belong elaborate and costly expeditions — including one organized by the American Philosophical Society of Philadelphia — to observe transits of Venus, as a means for determining the distance from the sun to the earth.

MATHEMATICAL PROGRESS AND PHYSICAL SCIENCE. — Besides the extension of mathematical ideas and methods to mechanics, astronomy, optics and other branches of physics, chemistry was now also becoming a quantitative science. So Scheele begins a work published in 1777: —

> To resolve bodies skilfully into their components, to discover their properties and to combine them in different ways, is the chief purpose of chemistry.

Richter, in his Stoichiometry (1792–1802), even speaks of chemistry as a branch of applied mathematics. Already the pioneer Robert Boyle had written: —

> I confess, that after I began . . . to discern how useful mathematicks may be made to physicks, I have often wished that I had

> employed about the speculative part of geometry, and the cultivation of the specious Algebra I had been taught very young, a good part of that time and industry that I had spent about surveying and fortification (of which I remember I once wrote an entire treatise) and other parts of practick mathematicks.
>
> Mathematicks may help the naturalists, both to frame hypotheses, and to judge of those that are proposed to them, especially such as relate to mathematical subjects in conjunction with others.

Even in natural science Stephen Hales says in 1727: —

> And since we are assured that the all-wise Creator has observed the most exact proportions, *of number, weight and measure,* in the make of all things; the most likely way therefore to get any insight into the nature of these parts of the creation, must in all reason be to number, weigh and measure. And we have much encouragement to pursue this method, of searching into the nature of things, from the great success that has attended any attempts of this kind.

Summing up these tendencies, a recent writer remarks: —

> In the eighteenth century mathematics was regarded by many scholars as the ideal, the completeness and exactness of whose methods should be arrived at by other less highly developed branches. So Laplace's popularized version of his celestial mechanics met an eager need, and even Voltaire undertook the championship of the Newtonian philosophy. Logic and even ethics were drawn into the mathematical retinue. For Maupertuis the good is a positive quantity, the bad a negative. Joys and griefs make up human life according to the laws of algebraic addition, and it is the business of statesmen to see that the positive balance is as large as possible. The great Buffon adds to his natural history a supplement on moral arithmetic. Mathematics aims at the leadership both in natural science and in human affairs.

In spite of this perhaps exaggerated predilection in learned and polite society, educational curricula remained weak and conservative. Powerful progressive tendencies growing out of the French Revolution found expression in the founding of the *École polytechnique,* — under the leadership of Monge, — which has ever since been an important centre of mathematical activity.

Its curriculum included, in the first year, analytic geometry of space and descriptive geometry, in the second, mechanics of solids and liquids, in the third, theory of mechanics.

Reviewing eighteenth century mathematics, a recent German writer says: —

> In the science itself there showed itself with the close of the eighteenth century a certain exhaustion. 'The mine is, it seems to me, too deep,' wrote Lagrange in the year 1781 to d'Alembert, 'and unless new veins are discovered it must sooner or later be abandoned.' In the nineteenth century men have dug deeper and struck noble ores, but serious obstacles opposed the progress. It appeared that the men of genius of the illustrious period had to some extent practised bad building and the whole framework threatened to cave in unless the passages were newly supported and the oncoming floods of doubt conducted away. For two generations a considerable share of the efforts of mathematics must be applied to the hard work of security and safety, a labor from which even the greatest . . . have not held back.

NINETEENTH CENTURY MATHEMATICS. — As in the century following Newton France became the great centre of mathematical activity, so in the nineteenth century the leadership passed to Germany, under the inspiration of Gauss and Riemann of Göttingen, Jacobi of Königsberg, Weierstrass of Berlin, — to mention but a few of those no longer living. Outside of Germany conspicuous names are Cauchy, Galois, Hermite, Legendre, and Poincaré in France, Cayley and Sylvester in England, Abel in Sweden, and Lobatchewski in Russia.

Characteristic of this period are: the development of a general theory of functions based on unifying coördinating principles, compensating the powerful specializing tendencies, and a profound critical revision of the previously accepted axioms, leading for example to the development of a non-Euclidean geometry. In the science generally there is systematic development of instruction and research, notably in the German universities; of publication, by the establishment of mathematical journals, and the preparation of encyclopedias; numerous national societies are formed, and international congresses held. These tendencies are naturally not confined to the mathematical sciences. In some respects mathematics has merely enjoyed its share in the prosperity of a more scientific age, in some it has perhaps suffered, at any rate relatively, from the powerful stimulus given the natural sciences by the working out of evolutionary theories. From a position of acknowledged primacy among a small number of recognized sciences, it has come to be regarded as but one of many.

It is impossible here even to enumerate the different branches of mathematical science developed during this period. Certain typical features may however be touched upon.

NON-EUCLIDEAN GEOMETRY. — Each century takes over as a heritage from its predecessors a number of problems whose solution previous generations of mathematicians have arduously but vainly sought. It is a signal achievement of the nineteenth century to have triumphed over some of the most celebrated of these problems.

The most ancient of them is the quadrature of the circle, which already appears in our oldest mathematical document, the Papyrus Rhind, B.C. 2000. Its impossibility was finally shown by Lindemann, 1882.

But of all problems which have come down from the past, by far the most celebrated and important relates to Euclid's parallel axiom. Its solution has profoundly affected our views of space and given rise to questions even deeper and more far-reaching, which embrace the entire foundation of geometry and our space conception.— *Pierpont* (1904).

I am convinced more and more that the necessary truth of our geometry cannot be demonstrated, at least not *by* the *human* intellect *to* the human understanding. Perhaps in another world we may gain other insights into the nature of space which at present are unattainable to us. Until then we must consider geometry as of equal rank not with arithmetic, which is purely *a priori*, but with mechanics.

— *Gauss* (1817).

There is no doubt that it can be rigorously established that the sum of the angles of a rectilinear triangle cannot exceed 180°. But it is otherwise with the statement that the sum of the angles cannot be less than 180°; this is the real Gordian knot, the rocks which cause

> the wreck of all. . . . I have been occupied with the problem over thirty years and I doubt if anyone has given it more serious attention, though I have never published anything concerning it.— *Gauss* (1824).
>
> I will add that I have recently received from Hungary a little paper on Non-Euclidean geometry, in which I rediscover all *my own ideas* and *results* worked out with great elegance. . . . The writer is a very young Austrian officer, the son of one of my early friends, with whom I often discussed the subject in 1798, although my ideas were at that time far removed from the development and maturity which they have received through the original reflections of this young man. I consider the young geometer von Bolyai a genius of the first rank. — *Gauss* (1832).

The gradual adoption of new and revolutionary ideas on this subject may be further illustrated by the following passages: —

> The characteristic features of our space are not necessities of thought, and the truth of Euclid's axioms, in so far as they specially differentiate our space from other conceivable spaces, must be established by experience and by experience only. — *R. S. Ball.*
>
> If the Euclidean assumptions are true, the constitution of those parts of space which are at an infinite distance from us, geometry upon the plane at infinity, is just as well known as the geometry of any portion of this room. In this infinite and thoroughly well-known space the Universe is situated during at least some portion of an infinite and thoroughly well-known time. So that there we have real knowledge of something at least that concerns the Cosmos; something that is true throughout the Immensities and the Eternities. That something Lobatchewski and his successors have taken away. The geometer of today knows nothing about the nature of the actually existing space at an infinite distance; he knows nothing about the properties of this present space in a past or future eternity. He knows, indeed, that the laws assumed by Euclid are true with an accuracy that no direct experiment can approach, not only in this place where we are, but in places at a distance from us that no astronomer has conceived; but he knows this as of Here and Now; beyond this range is a There and Then of which he knows nothing at present, but may ultimately come to know more. — *Clifford.*
>
> Everything in physical science, from the law of gravitation to the building of bridges, from the spectroscope to the art of navigation,

would be profoundly modified by any considerable inaccuracy in the hypothesis that our actual space is Euclidean. The observed truth of physical science, therefore, constitutes overwhelming empirical evidence that this hypothesis is very approximately correct, even if not rigidly true. — *Russell.*

The most suggestive and notable achievement of the last century is the discovery of Non-Euclidean geometry. — *Hilbert.*

What Vesalius was to Galen, what Copernicus was to Ptolemy, that was Lobatchewski to Euclid. There is, indeed, a somewhat instructive parallel between the last two cases. Copernicus and Lobatchewski were both of Slavic origin. Each of them has brought about a revolution in scientific ideas so great that it can only be compared with that wrought by the other. And the reason of the transcendent importance of these two changes is that they are changes in the conception of the Cosmos. . . . And in virtue of these two revolutions the idea of the Universe, the Macrocosm, the All, as subject of human knowledge, and therefore of human interest, has fallen to pieces. — *Clifford.*

Geometrical axioms are neither synthetic *a priori* conclusions nor experimental facts. They are conventions: our choice, amongst all possible conventions, is guided by experimental facts; but it remains free, and is only limited by the necessity of avoiding all contradiction. . . . In other words, axioms of geometry are only definitions in disguise. That being so what ought one to think of this question: Is the Euclidean Geometry true? The question is nonsense. One might as well ask whether the metric system is true and the old measures false; whether Cartesian co-ordinates are true and polar co-ordinates false. — *Poincaré.*

To make non-Euclidean geometry intelligible to laymen the following illustration has been given by Helmholtz: —

Think of the image of the world in a convex mirror. . . . A well-made convex mirror of moderate aperture represents the objects in front of it as apparently solid and in fixed positions behind its surface. But the images of the distant horizon and of the sun in the sky lie behind the mirror at a limited distance, equal to its focal length. Between these and the surface of the mirror are found the images of all the other objects before it, but the images are diminished and flattened in proportion to the distance of their objects from the mirror. . . . Yet every straight line or plane in the outer world is represented by a straight [?] line or plane in the image. The image of a man measuring with a rule a straight line from the mirror, would contract more and more the farther he went, but with his shrunken rule the man in the image would count out exactly the same number of centimeters as the real man. And, in general, all geometrical measurements of lines and angles made with regularly varying images of real instruments would yield exactly the same results as in the outer world, all lines of sight in the mirror would be represented by straight lines of sight in the mirror. In short, I do not see how men in the mirror are to discover that their bodies are not rigid solids and their experiences good examples of the correctness of Euclidean axioms. But if they could look out upon our world as we look into theirs without overstepping the boundary, they must declare it to be a picture in a spherical mirror, and would speak of us just as we speak of them; and if two inhabitants of the different worlds could communicate with one another, neither, as far as I can see, would be able to convince the other that he had the true, the other the distorted, relation. Indeed I cannot see that such a question would have any meaning at all, so long as mechanical considerations are not mixed up with it.

IMAGINARY NUMBERS. — The solution of algebraic equations had always been hampered by the seeming impossibility of performing the inverse processes involved. The equation $x + 5 = 0$ could not be solved before negative numbers were known; and the equations $2x = 5$ and $x^2 = 2$ would be equally insoluble without fractions and irrational numbers. Such equations as $x^2 + 1 = 0$ still remained a stumbling-block at the beginning of the nineteenth century. Gauss first pierced the mystery and released algebra from its traditional restriction, proving that an equation of any degree has a corresponding number of roots of the form $a + b\sqrt{-1}$ — a discovery of far-reaching importance not merely for higher mathematics but even for electrical engineering.

That this subject [of imaginary magnitudes] has hitherto been considered from the wrong point of view and surrounded by a mysterious obscurity, is to be attributed largely to an ill-adapted notation.

If for instance, $+1$, -1, $\sqrt{-1}$ had been called direct, inverse, and lateral units, instead of positive, negative, and imaginary (or even impossible) such an obscurity would have been out of the question.
— *Gauss.*

Concluding an address on the history of mathematics in the nineteenth century, a recent writer says: —

What strikes us at once in our survey of mathematics in the last century is its colossal proportions and rapid growth in nearly all directions, the great variety of its branches, the generality and complexity of its methods, an inexhaustible creative imagination, the fearless introduction and employment of ideal elements, and an appreciation for a refined and logical development of all its parts.— *Pierpont.*

Probably no other department of knowledge plays a larger part outside its own narrower domain than mathematics. Some of its more elementary conceptions and methods have become part of the common heritage of our civilization, interwoven in the every-day life of the people. Perhaps the greatest labor-saving invention that the world has seen belongs to the formal side of mathematics; I allude to our system of numerical notation. . . . Without taking too literally the celebrated dictum of the great philosopher Kant that the amount of real science to be found in any special subject is the amount of mathematics contained therein, it must be admitted that each branch of science which is concerned with natural phenomena, when it has reached a certain stage of development, becomes accessible to, and has need of, mathematical methods and language; this stage has, for example, been reached in our time by parts of the science of chemistry. — *Hobson.*

I often say that when you can measure what you are speaking about and express it in numbers, you know something about it; but when you cannot measure it, when you cannot express it in numbers, your knowledge is of a meager and unsatisfactory kind; it may be the beginning of knowledge, but you have scarcely in your thoughts advanced to the stage of science. — *Kelvin.*

THE DISCOVERY OF NEPTUNE. — In a century filled with remarkable scientific achievement, no single triumph has been more conspicuous, or in some respects more dramatic, than the discovery of the planet Neptune by Adams and Leverrier. From the time of Newton the perturbations of the planets had been the subject of continual observation and study. Improved telescopes demanded — and at the same time facilitated — more extended and refined computations. Discrepancies between computed and observed positions indicated disturbing forces of known or in some cases unknown origin. In particular, irregularities — never exceeding two minutes of arc — in the motion of the most recently discovered planet Uranus, led the young Cambridge graduate John Couch Adams (1819–1892) and the eminent French astronomer Leverrier (1811–1877) to independent attacks on the formidable problem of determining the mass and position of a hypothetical new planet which could cause the observed effects on Uranus. Unfortunately for Adams the necessary coöperation on the part of the observatories was not promptly available, so that the actual discovery connected itself with the somewhat later work of Leverrier. The discovery was naturally accepted as an extraordinary illustration of the power of mathematical astronomy and a convincing proof of the Newtonian theory of gravitation.

> The discovery of this planet [Neptune] is justly reckoned as the greatest triumph of mathematical astronomy. Uranus failed to move precisely in the path which the computers predicted for it, and was misguided by some unknown influence to an extent which a keen eye might almost see without telescopic aid. . . . These minute discrepancies constituted the data which were found sufficient for calculating the position of a hitherto unknown planet, and bringing it to light. Leverrier wrote to Galle, in substance: *Direct your telescope to a point on the ecliptic in the constellation of Aquarius, in longitude 326°, and you will find within a degree of that place a new planet, looking like a star of about the ninth magnitude, and having a perceptible disc.* The planet was found at Berlin on the night of Sept. 26, 1846, in exact accordance with this prediction, within half an hour after the astronomers began looking for it, and only about 52′ distant from the precise point that Leverrier had indicated.
>
> — *Young.*

While the telescope serves as a means of penetrating space, and of bringing its remotest regions nearer us, mathematics, by inductive

> reasoning, has led us onwards to the remotest regions of heaven, and brought a portion of them within the range of our possibilities; nay, in our own times — so propitious to the extension of knowledge — the application of all the elements yielded by the present conditions of astronomy has even revealed to the intellectual eyes a heavenly body, and assigned to it its place, orbit, mass, before a single telescope has been directed towards it. — *Humboldt.*

COSMIC EVOLUTION. — Reference has been made to the nebular hypothesis included by Laplace in his extended discussion of the solar system. During the nineteenth century this theory has been subjected to searching scrutiny from many points of view and much doubt has been cast on its validity.

The following summary of present opinion is given by Hale in his Stellar Evolution: —

> The nebular hypothesis of Laplace still remains as the most serious attempt to exhibit the development of the solar system. Attacked on many grounds, and showing signs of weakness that seem to demand radical modification of Laplace's original ideas, it nevertheless presents a picture of the solar system which has served to connect in a general way a mass of individual phenomena, and to give significance to apparently isolated facts that offer little of interest without the illumination of this governing principle.
>
> We are now in a position to regard the study of evolution as that of a single great problem, beginning with the origin of the stars in the nebulæ and culminating in those difficult and complex sciences that endeavor to account, not merely for the phenomena of life, but for the laws which control a society composed of human beings.

As a complement to the preceding may be added the following from another specialist in planetary evolution: —

> It is to the glory of astronomy that in it were initiated the two most fundamental intellectual movements in the history of mankind, viz. the establishment of the possibility of science and of the doctrine of evolution. Our intellectual ancestors in the valleys of the Euphrates and the Nile and on the hills of Greece looked up into the sky at night and saw order there and not chaos. By painstaking obser-

vations and calculations they discovered the relatively simple laws of the motions of the heavenly bodies, whose invariable and exact fulfilment led to the belief that the whole universe in all its parts is orderly and that science is possible. In the modern world this conclusion is so commonplace that its immense value is apt to be overlooked, but a study of the superstitions and the hopeless stagnation of those portions of mankind which have not yet made the discovery gives us some measure of its worth. The modern supplement to the conception that the universe is not a chaos is that not only is it an orderly universe at any instant, but that it changes from one state to another in a continuous and orderly fashion. This doctrine that science is extensive in time, as well as in space, is the fundamental element in the theory of evolution and the completion of the conception of science itself. The ideas of evolution in a scientific form were first applied to the relatively simple celestial phenomena. More than a century before the appearance of Darwin's 'Origin of Species,' and the philosophical writings of Spencer, another Englishman, Thomas Wright, published a book on the origin of worlds. Laplace's nebular hypothesis gave the geologists a basis for their work, which in turn paved the way for that of Darwin. For half a century now, the doctrine of evolution has been a fundamental factor in the elaboration of all scientific theories, and its influence has spread to every field of intellectual effort. It has been the good fortune of mankind that his skies have sometimes been free of clouds and that he has been able to observe those relatively simple yet majestic and impersonal celestial phenomena which have not only led to so important results as the founding of science and the doctrine of evolution, but have strongly colored his poetry, philosophy and religion, and have stimulated him to the elaboration of some of his most profound mathematical theories. — *Moulton.*

DISTANCE OF THE STARS. — Among other astronomical discoveries bearing a notable relation to the history of mathematical science is that of measurable stellar parallax by Bessel (1784–1846). One of the traditional objections to the Copernican theory had been the fact that no change could be detected in the relative position of the stars, such as would apparently result from revolution of the earth in a vast orbit. Now with more and more powerful instruments it turned out that there were stars near enough to show precisely the displacement discovered. In 1837 Bessel attacked this ancient problem successfully by making extremely accurate observation of the relative positions of a certain double star (71 Cygni) and its celestial neighbors. He obtained for the distance of the double star 657,000 times the mean distance from the earth to the sun. Such inconceivably vast distances have been since conveniently expressed in a unit called the light-year, *i.e.* the distance a ray of light travels in an entire year at 186,000 miles per second.

MATHEMATICAL PHYSICS. — The further progress of applied mathematics in the nineteenth century has been interestingly summarized by Woodward in a presidential address to the American Mathematical Society, from which the following extracts are quoted.

> Next came the splendid contributions of George Green under the modest title of 'An essay on the application of mathematical analysis to the theories of electricity and magnetism.' It is in this essay that the term 'potential function' first occurs. Herein also his remarkable theorem in pure mathematics, since universally known as Green's theorem, and probably the most important instrument of investigation in the whole range of mathematical physics, made its appearance. We are all now able to understand, in a general way at least, the importance of Green's work, and the progress made since the publication of his essay in 1828. But to fully appreciate his work and subsequent progress one needs to know the outlook for the mathematico-physical sciences as it appeared to Green at this time and to realize his refined sensitiveness in promulgating his discoveries. 'It must certainly be regarded as a pleasing prospect to analysts,' he says in his preface, 'that at a time when astronomy, from the state of perfection to which it has attained, leaves little room for further applications of their art, the rest of the physical sciences should show themselves daily more and more willing to submit to it.' . . . 'Should the present essay tend in any way to facilitate the application of analysis to one of the most interesting of the physical sciences, the author will deem himself amply repaid for any labor he may have bestowed upon it; and it is hoped the difficulty of the subject will incline mathematicians to read this work with indulgence, more partic-

ularly when they are informed that it was written by a young man who has been obliged to obtain the little knowledge he possesses, at such intervals and by such means as other indispensable avocations which offer but few opportunities of mental improvement, afforded.' Where in the history of science have we a finer instance of that sort of modesty which springs from a knowledge of things?

Just as the theories of astronomy and geodesy originated in the needs of the surveyor and navigator, so has the theory of elasticity grown out of the needs of the architect and engineer. From such prosaic questions, in fact, as those relating to the stiffness and the strength of beams, has been developed one of the most comprehensive and most delightfully intricate of the mathematico-physical sciences. Although founded by Galileo, Hooke, and Mariotte in the seventeenth century, and cultivated by the Bernoullis and Euler in the last century, it is, in its generality, a peculiar product of the present century. It may be said to be the engineers' contribution of the century to the domain of mathematical physics, since many of its most conspicuous devotees, like Navier, Lamé, Rankine, and Saint-Venant, were distinguished members of the profession of engineering. . . .

The theory of elasticity has for its object the discovery of the laws which govern the elastic and plastic deformation of bodies or media. In the attainment of this object it is essential to pass from the finite and grossly sensible parts of media to the infinitesimal and faintly sensible parts. Thus the theory is sometimes called molecular mechanics, since its range extends to infinitely small particles of matter if not to the ultimate molecules themselves. It is easy, therefore, considering the complexity of matter as we know it in the more elementary sciences, to understand why the theory of elasticity should present difficulties of a formidable character and require a treatment and a nomenclature peculiarly its own. . . .

It is from such elementary dynamical and kinematical considerations as these that this theory has grown to be not only an indispensable aid to the engineer and physicist, but one of the most attractive fields for the pure mathematician. As Pearson has remarked, 'There is scarcely a branch of physical investigation, from the planning of a gigantic bridge to the most delicate fringes of color exhibited by a crystal, wherein it does not play its part.' It is, indeed, fundamental in its relations to the theory of structures, to the theory of hydromechanics, to the elastic solid theory of light, and to the theory of crystalline media.

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CHAPTER XVI

SOME ADVANCES IN PHYSICAL SCIENCE IN THE NINETEENTH CENTURY. ENERGY AND THE CONSERVATION OF ENERGY

About a century after the publication of the *Principia*, which, by propounding the gravitation formula, raised the ancient and indefinite notion of Attraction to the rank of a useful and rigorously defined expression, another favorite theory [Atomism] of the ancient philosophers was similarly elevated to the rank of a leading and useful scientific idea.

The law of gravitation embraced cosmical and some molar phenomena, but led to vagueness when applied to molecular actions. The atomic theory led to a complete systematization of chemical compounds, but afforded no clue to the mysteries of chemical affinity. And the kinetic or mechanical theories of light, of electricity and magnetism, led rather to a new dualism, the division of science into sciences of matter and of the ether. . . . A more general term had to be found under which the different terms could be comprised, which would give a still higher generalization, a more complete unification of knowledge. One of the principal performances of the second half of the nineteenth century has been to find this more general term, and to trace its all-pervading existence on a cosmical, a molar, and a molecular scale . . . this greatest of all exact generalizations — the conception of energy.

Electrified and magnetised bodies attract or repel each other according to laws discovered by men who never doubted that the action took place at a distance, without the intervention of any medium, and who would have regarded the discovery of such a medium as complicating rather than as explaining the undoubted phenomena of attraction. — *Merz.*

Through metaphysics first; then through alchemy and chemistry, through physical and astronomical spectroscopy, lastly through radio-activity, science has slowly groped its way to the atom. — *Soddy.*

There is in nature a certain magnitude of unsubstantial quality, which keeps its value under all alterations of the objects observed, while its manner of appearance changes most variously. — *Mayer.*

I shall lose no time in repeating and extending these experiments, being satisfied that the grand agents of nature are, by the Creator's fiat, *indestructible;* and that whatever mechanical force is expended, an exact equivalent of heat is *always* obtained. — *Joule.*

Heat and work are equivalent. The entropy of the universe tends to a maximum. — *Clausius.*

The later eighteenth and the whole of the nineteenth centuries are characterized by increasingly rapid development of the physical sciences, which become more and more completely differentiated, and more and more important in their influence upon industry and civilization. While it is evidently impossible within our available space to describe all phases of this varied development, we shall attempt to enumerate some of those which are most general in their character and most far-reaching in their consequences. A relatively complete and highly instructive review of the whole subject may be found in Merz's History of European Thought in the Nineteenth Century.

At the beginning of this century mathematics was in a stage of triumphant expansion, in which the related sciences of astronomy and mechanics participated. General physics and chemistry were still in the preliminary stage of collecting and coördinating data, with attempts at quantitative interpretation, while in their train the natural sciences were following somewhat haltingly.

The most notable advance in physical science during the century is the gradual working out of the great fundamental principle of the *conservation of energy,* affecting profoundly the whole range of phenomena. Of equal — or even greater — importance is the gradual realization of progressive development — *evolution* — not only in plant and animal life but even in the inorganic world. Physics is gradually enriched by experimental researches and by the working out of mathematical theories of heat, light, magnetism and electricity. Chemistry, largely hitherto a collection of unrelated facts, becomes more and more coördinated with physics and mathematics by means of the spectroscope, the principle of the conservation of energy, the atomic theory, the kinetic theory of gases, and the study of molecular structure. On the other hand, its relations with the organic world are made more clear through the investigation of the compounds of carbon.

All other sciences, pure and applied, as well as the industries, profit unexpectedly and almost inconceivably by these nineteenth century advances in physics and chemistry. The older observational and mathematical astronomy achieves a marvellous triumph in the discovery of a new planet Neptune, as related in Chapter XV, and even this is soon rivalled by the startling achievements of the new physical and chemical astronomy.

Reserving for the following chapter a sketch of the development of the natural sciences under the ultimately dominant influence of the theory of evolution, we proceed to outline briefly some of the more notable advances in the physical sciences.

MODERN PHYSICS. — Some of the main features in the development of physics in the nineteenth century have been:—the working out of consistent theories of light and radiant heat as wave phenomena of a peculiar hypothetical medium called the "ether"; the extensive investigation of electrical and magnetic phenomena and the development of an electromagnetic theory even so far as to include optics; the working out of a kinetic theory of gases with important relations to chemical as well as physical theory; the elaboration of general theories of matter, force, and energy, all culminating in the crowning discovery of the great unifying principle of the Conservation of Energy.

HEAT, THERMOMETRY: CARNOT, RUMFORD. — The invention of the thermometer has been traced in Chapter XII. To the nineteenth century belongs the determination of an absolute scale[1] as distinguished from the arbitrary one previously employed.

The idea that heat is not a substance but a mode of molecular motion arose in the seventeenth and eighteenth centuries, but was first given a substantial experimental basis by the researches of Benjamin Thompson, Count Rumford (1753–1814), who showed that by friction of two bodies an unlimited amount of heat could be generated. His results were reported to the Royal Society in 1798.

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[1] The absolute scale is based on the indirect determination of a temperature (−273° Centigrade = −459° Fahrenheit) at which the internal activity which constitutes heat is supposed to cease.

Rumford made a cylinder of gun-metal rotate in a box containing water, and by the friction of a revolving borer driven by horse-power the water was heated to boiling in two and a half hours.

Deeply impressed he exclaims:

> What is heat? Is there any such thing as an igneous fluid? . . . Anything which any *insulated* body, or system of bodies, can continue to furnish *without limitation*, cannot possibly be a material substance; and it appears to me to be extremely difficult, if not quite impossible, to form any distinct idea of anything, capable of being excited, and communicated, in the manner the heat was excited and communicated in these experiments, except it be MOTION.

The "mechanical equivalent of heat" — *i.e.* the work required to heat one pound of water one degree — was roughly calculated.

Epoch-making in the theory of heat were the researches of Sadi Carnot (1796–1832), whose words follow:

> Wherever there is a difference of temperature followed by return to equilibrium the generation of power may take place. Water vapor is one means, but not the only one. . . . A solid body, for example a metal bar, gains and loses in length when it is alternately heated and cooled, and thus is able to move bodies fastened to its ends. . . .

The whole process he pictures as a *cycle* in which a certain portion of the heat applied is converted into work, a certain other portion being lost. Thus the new science of thermodynamics was born. The thorough and complete investigation of the "mechanical equivalent of heat" belongs to J. P. Joule (1818–1889) of Manchester, England, a pupil of Dalton the chemist.

LIGHT; WAVE THEORY, VELOCITY: YOUNG, FRESNEL.— As stated in Chapter XIV Huygens had supported a wave theory of light, while Newton accepted an emission theory. That sound was propagated by atmospheric waves was well known. There was a troublesome contrast however in the phenomenon of shadows.

How could wave-propagation be reconciled with sharply defined shadows? Why should not light "go round a corner" as well as sound? These difficulties were met by Thomas Young who revived Huygens' wave theory, which was definitively established by Fresnel's researches on refraction, beginning in 1815.

The determination of the velocity of light by observations of the moons of Jupiter has been mentioned already (page 286). About 1850 this problem was solved by a new method devised by Fizeau. A ray of light passes between the teeth of a wheel to a mirror and back again. During the time required by the ray to pass thus out and back, the gap through which it has passed may have been just replaced by a tooth, in which case the light will be intercepted. By measuring the speed of the wheel when varied in a definite way the speed of the light ray may be determined. The result agreed with that obtained by the astronomical method within about .5 %. At almost the same time Foucault, by an ingenious laboratory device, proved that light travels more slowly in water than in air — a result incompatible with the emission theory.

The sporadic beginnings of a genuine kinetic view of natural phenomena, after having been cultivated . . . by Huygens and Euler, and early in the nineteenth century by Rumford and Young, were united into a consistent physical theory by Fresnel, who has been termed the Newton of optics, and who consistently, and all but completely, worked out one great example of this kind of reasoning. He has the glory of having not only established the undulatory theory of light on a firm foundation, but still more of having impressed natural philosophers with the importance of studying the laws of regular vibratory motion and the phenomena of periodicity in the most general manner.

In astronomy and optics the suggestion of common sense, which regards the earth as stationary and light as an emission travelling in straight lines, had indeed allowed a certain amount of definite knowledge . . . to be accumulated. A real physical theory, however, was impossible until the notions suggested by common sense were completely reversed, and an ideal construction put in the place of a seemingly obvious theory. This was done in astronomy at one stroke by Copernicus; in optics only gradually, tentatively, and hesitatingly.

Newton himself had pronounced the pure emission theory to be insufficient — and only a preliminary formulation.

Young boldly generalized the undulatory theory by maintaining that "a luminiferous ether pervades the universe, rare and elastic in a high degree," that the sensation of different colors depends on the different frequency of vibration excited by light in the retina. . . .

In January, 1817, long before Fresnel had made up his mind to adopt a similar conclusion . . . Young announced in a letter . . . that in the assumption of transverse vibrations, after the manner of the vibrations of a stretched string, lay the possibility of explaining polarization. . . . — *Merz.*

The Spectroscope and Spectrum Analysis. — For closer study of the spectrum of Newton and the "dark lines" observed by Fraunhofer in 1815 (and in 1802 by Wollaston) in the spectrum, Kirchhoff and Bunsen in 1859–60 perfected the "spectroscope." This is essentially no more than a telescope so attached to the prism producing the spectrum from a slit as to facilitate minute scrutiny of the latter. It was by these workers and at this time that spectrum analysis became firmly established as a means of detecting the chemical constituents of celestial bodies. Kirchhoff wrote in 1859: —

I conclude that colored flames in the spectra of which bright lines present themselves, so weaken the rays of the color of these lines, when such rays pour through them, that in place of the bright lines, dark ones appear as soon as there is brought behind the flame a source of light of sufficient intensity in which these lines are otherwise wanting, thus originating two great applications of his principle — the search, through the study of the spectra of distant stellar sources of light, after the ingredients which are present in those distant luminaries, and the search, through the study of the flames of terrestrial substances, for new spectral lines announcing yet undiscovered elements.

In 1862, only three years after Kirchhoff and Bunsen's application of the spectroscope to the study of the sun, Huggins measured the position of the lines in the spectra of about forty stars, with a small slit spectroscope attached to an 8-inch telescope. In 1876 he successfully applied photography to a study of the ultra-violet region of stellar spectra, and in 1879 published his paper On the Photographic Spectra of Stars. The results were arranged and discussed with reference to their bearing on stellar evolution. — *Hale.*

The first application of the spectroscope to the corona of the sun was made in 1868 by Janssen and Lockyer, independently, revealing the chemical composition of the solar prominences as chiefly hydrogen, calcium, and helium.

ELECTRICITY AND MAGNETISM: FARADAY, GREEN, AMPÈRE, MAXWELL. — Seebeck (1770–1831) in his work On the Magnetism of the Galvanic Circuit published a first account of the magnetic field illustrated by magnetized iron filings and later so fruitfully investigated by Faraday. The sciences of electricity and magnetism had originated in the latter part of the eighteenth century with Coulomb's use of the torsion-balance, by means of which he made accurate comparison between the attractive or repulsive forces exercised by electrified and magnetized bodies, and the mechanical forces required to twist wires. Thus he found the first definite units, a process carried much farther by Gauss and Weber.

Ampère (1775–1836) stimulated by Oersted's discovery of the effect of the electric current on magnets, published in 1820 a fundamental discussion of electrodynamics and soon after enunciated his celebrated law: —

> Two parallel and like directed currents attract each other, while two parallel currents of opposite directions repel each other.

He also succeeded in expressing the quantitative relations involved by a mathematical formula.

Faraday, one of the most distinguished investigators in the whole history of physical science, rescued electricity from the mysterious notion of currents acting on each other through empty space, by the fruitful conception of a magnetic field, of which a new and comprehensive mathematical theory was gradually worked out by Maxwell. Faraday's discoveries were so far-reaching that they have even been coupled with the law of the conservation of energy and Darwin's theory of descent as the greatest scientific ideas of the latter half of the century. He observes: —

Atoms and lines of force have become a practical — shall I say a popular? — reality, whereas they were once only the convenient method of a single original mind for gathering together and unifying in thought a bewildering mass of observed phenomena, or at most capable of being utilized for a mathematical description and calculation of actual effects.

Yet Helmholtz says of Faraday:

> It is indeed remarkable in the highest degree to observe how, by a kind of intuition, without using a single formula, he found out a number of comprehensive theorems, which can only be strictly proved by the highest powers of mathematical analysis. . . . I know how often I found myself despairingly staring at his descriptions of lines of force, their number and tension, or looking for the meaning of sentences in which the galvanic current is defined as an axis of force. . . .

Faraday apprehended the principle of the conservation of energy even before it had come to clear expression as common property, saying, for example, in refuting the theory that electricity could be generated by metallic contact alone: —

> But in no case, not even in those of [electric fishes], is there a pure creation or a production of power without a corresponding exhaustion of something to supply it.

Like Young, Dalton, and Joule, Faraday did not belong to the orthodox Cambridge school then dominant in English mathematical and physical science, and recognition of the significance of his ideas was consequently retarded.

> What the atomic theory has done for chemistry, Faraday's lines of force are now doing for electrical and magnetic phenomena. . . . Yet the circumstances under which Faraday's work was done were those of penury.

ELECTROMAGNETIC THEORY OF LIGHT. — In 1845 Faraday writes:

> I . . . . have at last succeeded *in magnetising and electrifying a ray of light, and in illuminating a magnetic line of force.* . . . Employing a ray of light, we can tell, *by the eye,* the direction of the magnetic lines through a body: and by the alteration of the ray and its optical effect on the eye, can see the course of the lines just as we can see the course of a thread of glass.

> I have deduced the relation between the statical and dynamical measures of electricity, and have shown by a comparison of the electro-magnetic experiments of Kohlrausch and Weber with the velocity of light as found by Fizeau, that the elasticity of the magnetic medium in air is the same as that of the luminiferous medium, if these two coexistent, coextensive and equally elastic media are not rather one medium. . . . We can scarcely avoid the inference that light consists in the transverse undulations of the same medium which is the cause of electric and magnetic phenomena. — *Maxwell.*
>
> We must not listen to any suggestion that we may look upon the luminiferous ether as an ideal way of putting the thing. A real matter between us and the remoter stars I believe there is, and that light consists of real motions of that matter, motions just such as are described by Fresnel and Young, motions in the way of transverse vibrations.
>
> — *Kelvin,* Baltimore lecture.

Hertz, a pupil of Helmholtz, first proved in 1887 the existence of those undulations which now bear his name, showing also that these travel with the rapidity of light, and that they are, like light and heat waves, susceptible of reflection, refraction, and polarization, and until he measured their length and velocity, no great progress was made in verifying those relations experimentally. Such more recent applications of Hertz's ideas as radio-telegraphy and radio-telephony testify to their immense practical as well as theoretical importance.

With the establishment of the electromagnetic theory of light, what we may call the undulatory series became complete. Sound had long been known to be due to waves or "undulations" and the wave theory of heat and of light was accepted, so that it had only remained to prove the existence of electrical and magnetic undulations, and to show that such waves moved with the velocity and other characteristics of light. This it was which was done mathematically by Maxwell and experimentally by Hertz.

> The velocity of propagation of an electro-magnetic disturbance in air . . . does not differ more from the velocity of light in air . . than the several calculated values of these quantities differ among each other. — *Maxwell.*

> KINETIC THEORY OF GASES. CLAUSIUS. The modern theory of gases was born . . . when Joule in 1857 actually calculated the velocity with which a particle of hydrogen . . . must be moving, assuming that the atmospheric pressure is equilibrated by the rectilinear motion and impact of the supposed particles of the gas on each other and the walls of the containing vessel. This meant taking the atomic view of matter in real earnest, not merely symbolically, as chemists had done. — *Merz.*

The theory owes its full development, however, to the researches of Maxwell, Clausius and Boltzmann.

> The great turning-point, indeed, lay in the kinetic theory of gases, which . . . had introduced quite novel considerations by showing how the dead pressure of gases and vapors could be explained on the hypothesis of a very rapid but disorderly translational movement of the smallest particles in every possible direction.

THE CONCEPTION OF ENERGY. — Newton's *Principia* contains by implication the modern notion of energy — but the first clear and consistent fixing of the modern terminology is found in Poncelet's *Mécanique industrielle,* 1829. The idea of *work* was thus developed from the standpoint of the engineer — notably under the influence of Rankine; while on the other hand, it is a not less remarkable fact that Black, Young, Mayer and Helmholtz all came to their scientific work through another form of applied science — medicine.

A considerable step toward the general idea of the conservation of energy was taken by Rumford in his determination of the mechanical equivalent of heat, but the final achievement is due mainly to Joule in England and Mayer and Helmholtz in Germany. In 1847 Helmholtz read before the Physical Society of Berlin one of the most remarkable papers of the century (*Die Erhaltung der Kraft*), in which he says with full justice: —

> I think in the foregoing I have proved that the above mentioned law does not go against any hitherto known facts of natural science, but is supported by a large number of them in a striking manner. I have tried to enumerate as completely as possible what consequences result from the combination of other known laws of nature, and how

they require to be confirmed by further experiments. The aim of this investigation, and what must excuse me likewise for its hypothetical sections, was to explain to natural philosophers the theoretical and practical importance of the law, the complete verification of which may well be looked upon as one of the main problems of physical science in the near future.

Fifteen years later Helmholtz spoke of the principle as follows: —

'The last decades of scientific development have led us to the recognition of a new universal law of all natural phenomena, which, from its extraordinarily extended range, and from the connection which it constitutes between natural phenomena of all kinds, even of the remotest times and the most distant places, is especially fitted to give us an idea of what I have described as the character of the natural sciences, which I have chosen as the subject of this lecture. This law is the Law of the Conservation of Force, a term the meaning of which I must first explain. It is not absolutely new; for individual domains of natural phenomena it was enunciated by Newton and Daniel Bernoulli; and Rumford and Humphry Davy have recognised distinct features of its presence in the laws of heat. The possibility that it was of universal application was first stated by Mayer in 1842, while almost simultaneously with, and independently of him, Joule made a series of important and difficult experiments on the relation of heat to mechanical force, which supplied the chief points in which the comparison of the new theory with experience was still wanting. The law in question asserts, that the *quantity of force which can be brought into action in the whole of Nature is unchangeable*, and can neither be increased nor diminished.

This doctrine is now so fundamental and so familiar as to require no further comment. The indestructibility of matter had already become axiomatic. Henceforth, energy also was to be considered constant and indestructible.

DISSIPATION OF ENERGY. — It remained for William Thomson (Lord Kelvin), applying the principle of the conservation of energy to the thermodynamic laws of Carnot, to deduce the other great principle of the Dissipation of Energy, which recognizes that while *total* energy is constant, *useful* energy is diminishing by the continual degeneration of other forms into non-useful or dissipated heat. All workers in this field, from Carnot to Thomson, had appreciated the impossibility of "perpetual motion." Helmholtz expresses his appreciation of Thomson's contribution to the theory in a striking passage:

> We must admire the acumen of Thomson, who could read between the letters of a mathematical equation, for some time known, which spoke only of heat, volume and pressure of bodies, conclusions which threaten the universe, though indeed only in infinite time, with eternal death.

Thomson, more than any other thinker, put the problem into common-sense language. . . . He saw at once, when adopting Joule's doctrine of the convertibility of heat and mechanical work, that, if all processes in the world be reduced to those of a perfect mechanism, they will have this property of a perfect machine, namely, that it can work backward as well as forward. It is against all reason and common sense to carry out this idea in its integrity and completeness. If then, the motion of every particle of matter in the universe were precisely reversed at any instant, the course of nature would be simply reversed forever after. The bursting bubble of foam at the foot of a waterfall would reunite and descend into the water; the thermal motions would reconcentrate their energy and throw the mass up the fall in drops, re-forming into a close column of ascending water. Heat which had been generated by the friction of solids and dissipated by conduction and radiation with absorption, would come again to the place of contact and throw the moving body back against the force to which it had previously yielded. Boulders would recover from the mud the materials required to rebuild them into their previous jagged forms, and would become re-united to the mountain-peak from which they had formerly broken away. And also, if the materialistic hypothesis of life were true, living creatures would grow backwards with conscious knowledge of the future, but with no memory of the past, and would become again unborn. But the real phenomena of life infinitely transcend human science; and speculation regarding consequences of their imagined reversal is utterly unprofitable. Far otherwise, however, is it in respect to the reversal of the motions of matter uninfluenced by life, a very elementary consideration of which leads to the full explanation of the theory of dissipation of energy.

MODERN CHEMISTRY. — Main features in nineteenth century chemistry are: — the discovery of the fundamental quantitative relations of chemical reactions; the development of a consistent and definite theory of atoms, molecules and valence; the synthesis of organic substances; the discovery of periodic relations and characteristics; the development of ideas of chemical structure; the development of electro-chemistry; the foundation of physical chemistry.

With Lavoisier, "the father of modern chemistry," the science, heretofore descriptive and empirical, had become quantitative and productive, seeking like the older sciences of astronomy and physics to make itself mathematical — an exact science. Postulating the existence of indestructible elementary substances, Lavoisier controlled and interpreted chemical reactions by careful weighing. Until he entered the field there was no generalization wide enough to entitle chemistry to be called a science.

CHEMICAL LABORATORIES: LIEBIG. — In the nineteenth century chemical studies received a powerful impetus through the establishment of teaching laboratories at the universities — in which Liebig at Giessen in 1826 was a pioneer. He writes:

> At Giessen all were concentrated in the work, and this was a passionate enjoyment. . . . The necessity of an institute where the pupil could instruct himself in the chemical art, . . . was then in the air, and so it came about that on the opening of my laboratory . . . pupils came to me from all sides. . . . I saw very soon that all progress in organic chemistry depended on its simplification. . . . The first years of my residence at Giessen were almost exclusively devoted to the improvement of organic analysis, and with the first successes there began at the small university an activity such as the world had not yet seen. . . . A kindly fate had brought together in Giessen the most talented youths from all countries of Europe. . . . Every one was obliged to find his own way for himself. . . . We worked from dawn to the fall of night.

To investigate the essence of a natural phenomenon, three conditions are necessary. We must first study and know the phenomenon itself, from all sides; we must then determine in what relation it stands to other natural phenomena; and lastly, when we have ascertained all these relations, we have to solve the problem of measuring these relations and the laws of mutual dependence — that is, of expressing them in numbers. In the first period of chemistry, all the powers of men's minds were devoted to acquiring a knowledge of the properties of bodies. . . . This is the alchemistical period. The second period embraces the determination of the mutual relations or connections of these properties; this is the period of phlogistic chemistry. In the third . . . we ascertain by weight and measure and express in numbers the degree in which the properties of bodies are mutually dependent. The inductive sciences begin with the substance itself, then come just ideas, and lastly, mathematics are called in, and, with the aid of numbers, complete the work.

QUANTITATIVE RELATIONS: ATOMS; MOLECULES; VALENCE. —

It took . . . nearly a century . . . before the rule of definite proportions was generally established, becoming a guide for chemical analysis. . . .

The vaguer terms of chemical affinity and elective attraction, of chemical action, of adhesion and elasticity . . . gradually disappeared, when by the aid of the chemical balance each simple substance and each definite compound began to be characterized and labelled with a fixed number. — *Merz.*

Proust, analyzing various metallic oxides and sulphides, obtained constant percentage results, from which however no obvious inferences could be drawn by him. Dalton (1766–1844) had the happy inspiration to interpret these figures in relation to weights of the combined oxygen, making the lightest element, hydrogen, the unit or measure of his system. His hypothesis that elements combine in weights proportional to small whole numbers — the "law of multiple proportions," has since been verified by innumerable analyses.

It has recently been shown that Dalton was in the habit of regarding all physical phenomena as the result of the interaction of small particles. He was thus naturally led to the conception of definite atomic weights to be determined by experiment. In the words of Dalton:

We can as well undertake to incorporate a new planet in the solar system or to annihilate one there as to create or destroy an atom of hydrogen. All the changes we can effect consist in the separation of atoms bound together before and in the union of those previously separated.

The atomic theory while highly serviceable has always been subjected to severe criticism. In 1840, for example, Dumas declared that it did not deserve the confidence placed in it, and that if he could he would banish the word "atom," convinced that science should confine itself to what could be known by experience. As late as 1852 Frankland says:

> I had not proceeded far, in the investigation of the organo-metallic compounds before the facts brought to light began to impress upon me the existence of a fixity in the maximum combining value or capacity of saturation in the metallic elements which had not before been suspected. . . . It was evident that the atoms of zinc, tin, arsenic . . . had only room, . . . for the attachment of a fixed and definite number of the atoms of other elements.
>
> Independent researches have, in combination with the older chemical theories, introduced so much definiteness into this line of thought that 'the Newtonian theory of gravitation is not surer to us now than is the atomic or molecular theory in chemistry and physics — so far, at all events, as its assertion of heterogeneousness in the minute structure of matter, apparently homogeneous to our senses, and to our most delicate direct instrumental tests.' — *Kelvin,* 1886.

The three main criticisms of the atomic theory are: —

(1) that it is based on inference, not on direct observation; and is therefore only a provisional hypothesis; (2) that it takes no account of chemical forces — "affinity"; (3) that it over-emphasizes analysis.

The idea of "atomicity" and "valency" . . . was not possible without the clear notion of the "molecule" as distinct from the "atom." This idea had lain dormant in the now celebrated but long forgotten law of Avogadro, which was established in 1811 almost immediately after the appearance of Dalton's atomic theory.

> It had been known since . . . Boyle and Mariotte that equal volumes of different gases under equal pressure change their volumes equally if the pressure is varied equally, and it was also known . . .

that equal volumes of different gases under equal pressure change their volumes equally with equal rise of temperature. These facts suggested to Avogadro, and almost simultaneously to Ampère, the very simple assumption that this is owing to the fact that equal volumes of different gases contain an equal number of the smallest independent particles of matter. This is Avogadro's celebrated hypothesis. It was the first step in the direct physical verification of the atomic view of matter. — *Merz.*

SYNTHESIS OF ORGANIC SUBSTANCES. — Until the middle of the nineteenth century there was an apparently fundamental separation between organic and inorganic nature. Since then they have been brought together by the general laws of energy and to some extent by the principles of evolution, as will appear in the following chapter. In 1828 Wöhler (of Göttingen) had indeed succeeded in preparing urea out of inorganic materials, a discovery which disproved such difference as was hitherto considered to exist between organic and inorganic bodies.

A PERIODIC LAW AMONG THE ELEMENTS. — With gradually increasing knowledge of the fundamental constants of chemistry — the atomic weights — attempts were naturally made to connect these with the chemical and physical properties of the corresponding elements: valence, affinity, specific gravity, specific heat, etc. In 1869–71 Mendeléjeff, a Russian chemist, succeeded in establishing remarkable relations between these data, and on tabulating them enunciated his Periodic Law, which has resulted in the discovery of several new and hitherto unsuspected elements. As the existence of the planet Neptune (page 341) had been predicted to fill an apparent gap in a system, so Mendeléjeff under the periodic law was able to predict the existence of other and missing elements in the series of chemical elements. And just as the prediction of Adams and Leverrier was fulfilled by the actual discovery of Neptune, so the prophecy of Mendeléjeff was justified by the discovery of gallium in 1871, scandium in 1879, and germanium in 1886. Furthermore, the periodic law enabled Mendeléjeff to question the correctness of certain accepted atomic weights, and here, also, he was justified by subsequent redeterminations.

It may be questioned whether the celebrated periodic law of Newlands, Lothar Meyer and Mendeléjeff, which has brought some order into the atomic and other numbers referring to the different elements, and has even made it possible to predict the existence of unknown elements with definite properties, stands really in a firmer position than the once well-known but now forgotten law of Bode, according to which the gap in the series which gives the distances of the planets from the sun indicated the existence of a planet between Mars and Jupiter.

CHEMICAL STRUCTURE. — Crystallography — a science of the nineteenth century — established an important connection between chemistry and geometry. Haüy made mineralogy "as precise and methodical as astronomy. . . . He was to Werner and Romé de l'Isle, his predecessors, what Newton had been to Kepler and Copernicus."

In the early years of the atomic theory Wollaston had predicted that philosophers would seek a geometrical conception of the distribution of the elementary particles in space — a prophecy first practically fulfilled by Van't Hoff's Chemistry in Space (1875).

> The chemical character is dependent primarily upon the arrangement and number of the atoms, and in a lesser degree upon their chemical nature (V. Meyer). The atomic view first became a scientific instrument, when arithmetical relations of a definite and unalterable kind were proved to exist; it became a yet more useful instrument, when to the arithmetical there were added geometrical conceptions.
> — *Merz.*

PHYSICAL CHEMISTRY: ELECTROLYTIC AND THERMODYNAMIC DEVELOPMENTS OF CHEMISTRY. — In the latter part of the nineteenth century much light was thrown on a wide range of physical and chemical phenomena by the study of solutions and their electrolytic behavior. Much had already been accomplished by Davy in the decomposition of substances by the electric current, leading for example to the first isolation of the elements, sodium and potassium. Faraday showed that for a given substance the amount decomposed is dependent solely on the quantity of electricity passed through and that for different substances the amounts set free at the electrodes are proportional to their chemical equivalents. To him the name electrolysis is due. A closer study of the phenomena of electrolysis led Clausius to the hypothesis that the molecules of salts, acids, and bases, previously regarded as disintegrated only by the passage of the electric current, are already dissociated in ordinary solutions. To these electrically charged part-molecules Faraday gave the name *ions*. Arrhenius proved that salts in dilute solution are dissociated into their ions almost completely, instead of only very slightly as Clausius supposed. This theory of Arrhenius, known as the Theory of Electrolytic Dissociation, of which an account would be too technical for the present purpose, coordinates and correlates heterogeneous masses of chemical facts, which apparently bore little or no relation to one another, and refers them to a common cause.

During the latter part of the nineteenth century a study of the rate and equilibrium conditions of chemical reactions led by degrees to the formulation of the so-called law of mass action and to many important thermodynamic relations. Chemistry thus came to share with physics the possibility of utilizing the calculus, becoming thereby more fully a quantitative science.

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CHAPTER XVII

SOME ADVANCES IN NATURAL SCIENCE IN THE NINETEENTH CENTURY. COSMOGONY AND EVOLUTION

What the classical renaissance was to men of the fifteenth and sixteenth centuries, the scientific movement is to us. It has given a new trend to education. It has changed the outlook of the mind. It has given a new intellectual background to life. — *Sadler.*

The rapid increase of natural knowledge, which is the chief characteristic of our age, is effected in various ways. The main army of science moves to the conquest of new worlds slowly and surely, nor ever cedes an inch of the territory gained. But the advance is covered and facilitated by the ceaseless activity of clouds of light troops provided with a weapon — always efficient, if not always an arm of precision — the scientific imagination. It is the business of these *enfants perdus* of science to make raids into the realm of ignorance wherever they see, or think they see, a chance; and cheerfully to accept defeat, or it may be annihilation, as the reward of error. Unfortunately the public, which watches the progress of the campaign, too often mistakes a dashing incursion . . . for a forward movement of the main body; fondly imagining that the strategic movement to the rear, which occasionally follows, indicates a battle lost by science. — *Huxley.*

Influence of Eighteenth Century Revolutions. — If the French Revolution had done no more than to upset as it did the social equilibrium of the centuries, its effect in stimulating inquiry and generating doubt in almost every direction could not have failed to further scientific studies and promote wholesome investigation into the fundamental relations of man and nature. But even before that revolution, some of the ablest minds in France, keenly alive to the teachings of Descartes and Newton and the lessons of seventeenth century science, had rejected the current cosmogony of Moses, although they had nothing with which to replace it. In particular, the eighteenth century questioned all custom and authority, and the theory of special creation possessed no other basis.

The American Revolution was likewise an uprising against long established custom and authority, and accordingly contributed to the doubts and questionings of the time, while the Industrial Revolution, by fundamental and world-wide changes, such as the introduction of machinery and the factory system, and by its tendency to concentrate and urbanize populations previously rural and segregated, facilitated intellectual contact, promoted discussion, and aroused and excited inquiry and investigation.

THE SCIENTIFIC REVOLUTION. — The most brilliant single achievement of nineteenth century science was the detection by Adams and Leverrier of the presence in our solar system of Neptune, a new and hitherto unknown planet. But the most revolutionary achievement, and probably the most far-reaching, was the assembling and formulation of convincing evidence in favor of organic evolution, *i.e.* of the gradual development, rather than the sudden creation, of living things. It is difficult to-day to realize the commotion into which the intellectual world was thrown at the middle of the nineteenth century when a new and promising solution of the long-standing problem of the origin of the different kinds (species) of plants and animals by means of natural rather than supernatural law, was propounded by Darwin and Wallace. And while the last half of the eighteenth century was the period of great political and social revolutions, — the French, the American, and the Industrial, — the last half of the nineteenth century experienced, in its acceptance of a new cosmogony, a fourth, even more profound and momentous, the Scientific Revolution. The discovery of Neptune was a triumph of mathematics and astronomy, the establishment of the theory of organic evolution, a triumph of biology. The discovery of Adams and Leverrier was immediately accepted and everywhere applauded, but the ideas broached by Darwin and his collaborators encountered widespread and powerful opposition, and were accepted only tardily and reluctantly.

INFLUENCE OF THE RAPID INCREASE OF KNOWLEDGE. — The invention of printing, the discovery of the new world, and the works of such intellectual giants as Galileo, Kepler, and Newton, followed as these were by the rapid increase of knowledge, both of nature and of man, in the seventeenth and eighteenth centuries, had placed within the reach of all a vast amount of new facts touching the familiar heavens and the familiar earth. Moreover, these facts were mostly capable of some sort of rational interpretation, *i.e.* of assignment to a place in some category of facts or phenomena already understood and regarded as natural rather than supernatural. In short, in the eighteenth and nineteenth centuries the stock of human knowledge had been not only rapidly and immensely enlarged and enriched, but at the same time more or less successfully correlated with knowledge previously possessed and valued. Some of this new knowledge, moreover, was so different from the old as to seem like a fresh revelation.

GRADUAL APPRECIATION OF THE PERMANENCE AND SCOPE OF NATURAL LAW. — While it had been easy hitherto to assume the occasional suspension of, or interference with, the ordinary course of events by supernatural or other unseen or unknown influences, it gradually became clear that no such suspension or interference could happen without upsetting what seemed to be the natural and orderly sequence of events, — what we now call "the order of nature." Hence doubt arose in many minds whether such suspensions or interferences do in fact occur, and whether fixed and changeless law is not a fundamental phenomenon of nature. The vastness and variety also of the heavens, no less than the order conspicuous in a mighty system so nicely balanced and so perfectly correlated as must be the cosmos explored and described by Copernicus, Galileo, Kepler, Newton and their successors, gradually dawned upon the human intellect, and profoundly impressed mankind.

Moreover, if the thoughtful turned from the contemplation of the macrocosm — the heavens — and the revelations of the telescope, to the microcosm — man, — the labors of Vesalius and the Italian anatomists, and of Harvey and the microscopists, served to show that here also law and order and a kind of mechanical regularity and perfection held sway, while plants and the lower animals had long been observed to be strictly obedient to and dependent upon the laws of nature in respect to climate, season, food, reproduction, etc.

NATURAL THEOLOGY AND AN AGE OF REASON. — At the end of the seventeenth century John Ray, an English zoölogist, drew attention to the remarkable adaptations everywhere discoverable in nature and especially in plants and animals, and suggested that these adaptations were sufficient to prove the existence of "design" in the universe, — a powerful argument in favor of the Mosaic cosmogony. The same idea was urged more at length by others in the eighteenth century as an offset to the growing scepticism of the age, and especially by Butler in his great work on the Analogy of Religion, Natural and Revealed, to the Course and Constitution of Nature (1736), and by Paley in his famous Natural Theology (1802). More popular and more radical influences were simultaneously at work in the opposite direction, as for example, Paine's Rights of Man and Age of Reason, while Gibbon with prodigious learning, and Hume with searching philosophical criticism, added to the increasing dissatisfaction of the thoughtful with the current cosmogony, — a dissatisfaction which had been rapidly growing under the doctrine of the necessity of doubt emphasized by Descartes.

Between 1842 and 1846 appeared a revolutionary work entitled Vestiges of Creation, by an anonymous author, which aroused intense interest in scientific circles and a storm of criticism from those who held to the old cosmogony. It is now known to have been written by Robert Chambers, an Edinburgh publisher who preferred to remain unknown from fear of injuring his partners by bringing down upon them the wrath of critics for his heterodoxy. Chambers was an amateur geologist and in his "Vestiges" undertakes to treat the genesis of the earth on more rational and more natural principles than was possible by following the orthodox theory of special creation.

The publication of Darwin's Origin of Species in 1859 resulted, after a period of earnest and sometimes acrimonious discussion, in the establishment of what is now known as the theory or doctrine of organic evolution, and in the displacement of the prevailing theory of special creation, — a forward step which removed the principal stumbling-block in the way of acceptance of the theory of general evolution, inorganic as well as organic, telluric as well as stellar. In other words, Darwin's work at one blow cleared the way for a new cosmogony.

NATURAL PHILOSOPHY AND NATURAL HISTORY. DIFFERENTIATION AND HYBRIDIZING OF THE SCIENCES. — Mathematics, always recognized as a principal branch of the tree of learning, at the beginning of the nineteenth century still held its place as the mother of the sciences. Astronomy, also, often called the queen of the sciences, still occupied its ancient and honorable position, having by this time lost all traces of discreditable affiliation with astrology. With the physical and natural sciences, as we now know them, the case was different. These still existed in a comparatively amorphous and largely undifferentiated condition as "natural philosophy" and "natural history," — the former the lineal descendant of the Ionian nature philosophy, now promoted to a high place in public esteem by the work of Newton, whose great *Principia* were *philosophiae naturalis*. Gradually, as time went on, chemistry was more and more differentiated from natural philosophy, until about 1875 it became customary to drop the term natural philosophy, using instead two terms, chemistry and physics. By the end of the century the present practice was fully established.

The primitive condition of the natural history sciences at the beginning of the century may be inferred from the remark of an eminent geologist (Geikie) that at that time geology and biology were not yet inductive sciences. By 1880, however, natural history had budded off geology, botany, zoölogy, and physiology as independent sciences, and the parent term, though still employed, was rapidly falling into disuse, having become much too broad for any single science.

Meantime, hybridizing as well as differentiation has become common, *e.g.* of physics with chemistry (physical chemistry), and of mathematics with physics (mathematical physics) and with many other sciences. We now have also fertile hybrids between chemistry and biology (physiological chemistry, bio-chemistry, chemical biology, etc.), and between physics and geology (geophysics), between electricity and chemistry (electro-chemistry), between physics and mineralogy, etc. Hybridizing of this kind is one of the most characteristic as well as one of the most fruitful phenomena of the nineteenth century.

Botany, zoölogy and geology, daughters of natural history, were the children of its old age, for the term "natural history" is as ancient as Aristotle, while geology was not fully born until the publication of Lyell's Principles in 1830, and biology not until the era of the great generalists of the Victorian Age — Darwin, Spencer, Huxley, etc. — soon after the middle of what one of them, Wallace, well qualified by his own great work to speak with authority, has called "the wonderful century." Botany and zoölogy as such arose about the beginning of the century. Geology, dealing with the natural history of the earth and its lifeless contents, and biology, dealing with the world of life, have both now very numerous progeny, *e.g.* from geology, stratigraphy, mineralogy, petrography, petrology, palæontology, etc., and from biology, zoölogy and botany, with their numerous subdivisions, — morphology, physiology, cytology, anthropology, bacteriology, parasitology, etc. Here also highly prolific crossing has occurred both among members of each minor group and also between members of the two major groups, natural philosophy and natural history; as, for example, in palæontology, which may be said to be half geology and half biology, in physiological optics, in bio-metrics, etc. To the very beginning of the century belongs the first appearance of the term "biology," introduced by Treviranus (1776–1837) a German naturalist and professor of mathematics in Bremen who in 1802–1805 published a work entitled *Biologie, oder Philosophie der lebenden Natur.*

The greatest achievement of natural history, not only of the nineteenth century but of all time, was the bringing about of the general acceptance of *a new cosmogony* — the theory of evolution — on the presentation by the naturalist Darwin of convincing evidence in favor of organic evolution together with a plausible explanation of the mechanism (natural selection) of its operation. To this we shall return. In a century so rich and so varied in its achievements in natural science as was the nineteenth, we can obviously only touch — and that very briefly — upon a few of the more important and fundamental.

Of the remarkable progress of all the sciences during the Victorian era, Huxley has given the best brief and general account in his essay entitled Advance of Science in the Last Half Century (1887), prepared in celebration of the first fifty years of the reign of Queen Victoria.

PROGRESS IN ZOÖLOGY. — The work of Buffon and Linnæus in the field of biology and of Werner and Hutton and Smith in that of geology has been referred to in Chapter XIV. Of these only Werner (d. 1817) and Smith (d. 1839) lived on into the nineteenth century.

The return of the astronomers and the geologists to ancient ideas of gradual development or, as this is now called, evolution, for the lifeless earth, was foreshadowed for the living world with Bonnet (1720–1793) who in 1764, in his *Contemplation de la nature,* advanced the theory that living things form a gradual and natural "scale" (ladder), rising from lowest to highest without any break in continuity. Buffon, in his great work on natural history, which was published between 1749–1804 in 44 quarto volumes, had dealt with the animal world very much as Linnæus had dealt with plants, Buffon excelling in description, Linnæus both in description and in classification and holding firmly to the idea of the fixity as well as the definite demarcation of species.

Meantime, epoch-making work in zoölogy was being done by three investigators — Lamarck, Cuvier, and St. Hilaire — at the Museum of Natural History in Paris. In 1778 Lamarck (1744–1829) published a small book on botany. In 1801 appeared his great work On the Organization of Living Bodies, which is now a landmark in the history of biology and of the doctrine of organic evolution. In this work and in his *Philosophie zoölogique,* Lamarck boldly proposes to substitute for special creation —

the current theory of cosmogony — the idea of gradual development or evolution, an ancient idea thenceforward made the keynote of his speculations. Systematic zoölogy and comparative anatomy, the latter already well begun by Hunter in the eighteenth century, were immensely advanced by Cuvier (1769–1832), who however clung tenaciously to the theory of special creation; while Geoffroy St. Hilaire (1772–1844) — also a comparative anatomist, but one whose interests lay rather in the functional than in the anatomical resemblances of the parts of animals, and who is therefore regarded as "the father of homology" — on the whole opposed Cuvier's and favored Lamarck's ideas. His *Philosophie anatomique* appeared in 1818–1822.

> Nature, said St. Hilaire, has formed all living beings on one plan, essentially the same in principle, but varied in a thousand ways in all the minor parts; all the differences are only a complication and modification of the same organs.
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> This similarity of structure, or *homology* as it is called, which runs through all animals, was thus first clearly stated by St. Hilaire, and has now been carefully worked out and confirmed. . . . Yet Cuvier opposed it to the last, for his mind was full of another idea which is equally true; namely, how perfectly each part of an animal is made to fit all the other parts; and it seemed to him impossible that this could be, unless each part was created expressly for the work it had to do.
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> The discussion between the two friends became so animated that all Europe was excited by it. It is said that Goethe, then an old man of eighty-one, meeting a friend, exclaimed, 'Well, what do you think of this great event? the volcano has burst forth, all is in flames.' His friend thought he spoke of the French Revolution of July, 1830, which had just occurred, and he answered accordingly. 'You do not understand me,' said Goethe, 'I speak of the discussion between Cuvier and St. Hilaire: the matter is of the highest importance. The method of looking at nature which St. Hilaire has introduced can now never be lost sight of.' — *Arabella Buckley Fisher.*

The history of zoölogy in the first half of the nineteenth century is chiefly that of the work of Cuvier, St. Hilaire, Lamarck, Agassiz and their disciples.

Progress in Botany. — The Linnæan system of classification for the higher plants was a purely empirical system, based largely upon the number of stamens and not involving any ideas of relationship through descent. B. de Jussieu (1699–1767), a friend and pupil of Linnæus, had proposed a different system of classification in which weight is given to the totality of resemblances of whatever kind, and this, which almost inevitably led to the close association of *related* forms, was an important step toward a natural classification, *i.e.* one based avowedly upon relationship, common ancestry, or descent. De Jussieu's nephew Antoine de Jussieu continued and extended his uncle's work. A. de Candolle (1778–1841) later adopted and extended de Jussieu's system which, with his own, now forms the basis of our present natural system. It is noteworthy that this change of opinion in regard to the relationship of the species of plants was ultimately effected without theological protest.[1]

The discovery by Goethe of the homologies of the parts, and by Linnæus of the organs of sex, of the flower, were important steps toward the modern theory of the evolution of plant life.

Progress in Microscopy. The Achromatic Objective. — Compound microscopes, *i.e.* microscopes consisting of two lenses, an objective and an eyepiece, were probably invented at about the same time as telescopes, — which likewise consist of two lenses or systems of lenses. But because of their imperfections, in respect especially to spherical and chromatic aberration, such microscopes were often inferior for use to the best simple microscopes. It was not until about 1835 that the compound microscope, though invented in the seventeenth century, became the superior instrument that it is to-day, through the accumulated improvements of a number of workers, — especially Amici, Lilly, Lister, and Chevalier — resulting in the achromatic objective, free from both spherical and chromatic aberration.

Almost immediately thereafter, with the new microscopy, began a rich harvest of discoveries, in what Pasteur has called the world of the infinitely little, similar to that reaped after the introduction of the telescope by Galileo, and his exploration of the stars, in the world of the infinitely great. The cell theory of Schleiden and Schwann appeared in 1839 and yeast was rediscovered (see p. 378) in 1837. The first contagious disease (*Muscardine*) traced to a fungus parasite was worked out by Bassi in 1837, the first contagious disease (*Favus*) of man due to a fungus, by Schoenlein in 1839. Protoplasm was first described in 1846. Ehrenberg, in 1838, made numerous and important studies on microscopic plants and animals.

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[1] For an account of Linnæus' attitude to the doctrine of the fixity of species see A. D. White, *Warfare of Science*, Vol. I, pp. 47, 59, 60.

EMBRYOLOGY. — If in 1828 one sharp boundary which had always been supposed to stand between the organic and the inorganic world was broken down by Wöhler's discovery that urea, a substance hitherto exclusively of animal origin, could be obtained in the laboratory by heating an inorganic substance, ammonium cyanate, another well-defined boundary believed to exist between the higher and the lower animals had been broken down a year earlier, when a Russian zoölogist, Karl Ernst von Baer (1792–1876), announced that mammals, including man himself, reproduce by eggs, precisely as do the lower animals. In 1828 von Baer published our first important work on comparative embryology, — of which science he thus became the founder.

The discovery by von Baer of the human ovum overthrew completely the "animalculists" who for centuries had contended that within the earliest embryo of man the future offspring existed completely formed, but only in miniature. This theory, because it assumed for development a mere unfolding, was known as embryologic *evolution*. Harvey, on the other hand, had propounded a theory of *epigenesis*, *i.e.* development comprising growth and differentiation out of an originally minute, simple, and undifferentiated body. This "body" — the human ovum — was now described by von Baer as $\frac{1}{200}$ inch in diameter and nowise different in appearance from other animal eggs in their earliest stages. Comparative anatomy had already shown that Linnæus was right in placing man among the animals, and now embryology confirmed and strengthened this view of man's place in nature.

PROGRESS IN PHYSIOLOGY. JOHANNES MÜLLER. CLAUDE BERNARD. — To the work upon physiology of Harvey in the seventeenth century and of Haller and Bichat and others in the eighteenth was now added that of Johannes Müller (1801–1858) whose "Elements of Physiology," appearing between 1837 and 1840, put the whole subject on a fresh and thoroughly scientific basis. Müller has been called the founder of modern physiology. Fortunate in his pupils — DuBois Reymond, Helmholtz, Ludwig, Volkmann, and Vierordt — these were no less fortunate in their master, for Müller was a great teacher, and for the rest of the century the teachings of Johannes Müller and his disciples furnished a powerful stimulus and a safe guide to physiological research, especially in Germany.

In France, also, physiology won renown and recognition through the researches of Claude Bernard (1813–1878), a pupil of Magendie, whose assistant he became in 1841 and whom he succeeded as assistant professor in 1847 and as professor in 1855. Bernard was the first occupant of the newly established chair of physiology at the Sorbonne. The laboratory was attached to his professorship until 1864. On his death in 1878 he was accorded by the State the honor of a public funeral, — the first ever bestowed by France upon a man of science and only 84 years after the public guillotining of Lavoisier. By his discovery of the significance of the pancreatic secretion and especially of the glycogenic function of the liver Bernard opened up the vast field of "internal secretions," the study of which has yielded, and is still yielding, some of the most fruitful results of physiological research hitherto obtained. Before Bernard, each organ seemed to have one function and only one, but since his time this simple, mechanical concept has given way to a realization of correlations and complexities within the animal mechanism such as had not then been dreamt of.

A great step forward in this dark field was taken in 1843 by the French physiologist, Claude Bernard, a man whose name should be remembered for his striking discoveries, ingenious and skillful experiments, his clear thoughts, lofty imagination and the beautiful, simple and luminous style in which his books and papers were written. — *Mathews.*

In England and America the newer physiology made but scant progress until Foster published (in 1876) in England an epoch-making treatise embodying in a fascinating form the methods and results of continental physiology.

Pathology before Pasteur. — Before the nineteenth century disease was regarded as an inscrutable mystery. Epidemics, plagues, and pestilences came and went, without apparent reason. The most fatal and therefore most famous of these was the Black Death of the fourteenth century. Others had been the Plague of Athens, the Sweating Sickness, the Dancing Mania, and Leprosy. One of the worst and commonest was Scurvy, which attacked chiefly sailors, soldiers, prisoners and the poor.

Attempts to explain disease were manifold. Primitive man naturally attributed it to the power of evil spirits (dæmons or devils) and sought prevention and cure in exorcism and the casting out of devils. Hippocrates looked for the sources of disease in abnormal mixtures of four great juices or "humors" of the body, — blood, yellow bile, phlegm, and black bile; and his theory had the merit of being based upon natural rather than supernatural ideas, for which reason probably it survived until the time of Sydenham in the seventeenth century. But the theory of Hippocrates failed to account for epidemics, for which the cause had to be sought in meteorological disturbances, such as storms, or in astronomical phenomena, such as comets or eclipses, or in unusual terrestrial happenings, such as earthquakes, volcanic eruptions, the flight of birds, the appearance of insects, vermin and what not. With the increase of knowledge in the sixteenth and seventeenth centuries ideas of this primitive kind no longer sufficed, and Sydenham urged that disease must have an independent material basis, a *materies morbi.* Not much progress was made, however, even by Sydenham, and the eighteenth century left behind it no important contributions to the theory of disease, — the work of Hahnemann, for example, bearing upon therapeutics rather than pathology.

The nineteenth century began as a period of agnosticism in pathology. The older theories were discredited, but beyond a general belief in the material basis of the agencies of disease almost nothing was known. Scurvy, indeed, had been shown to be due to lack of certain kinds of food, and smallpox had been proved to be preventable both by inoculation and vaccination. Boyle, in the seventeenth century, had ventured the guess that diseases might be "fermentations," but as fermentations were not yet understood the suggestion had little value. Light finally came from two sources, viz., from parasitology and from zymology, — the science of fermentations. It had long been recognized that the mistletoe was a parasite causing serious disease in trees, and that tapeworms might cause disease in animals which they infested. It was not, however, until the microscope came into use that the itch, long known as a contagious disease, was found to be due to a parasitic insect, while flies, fleas, bedbugs and lice were still thought to be annoying rather than dangerous. The discovery in 1837 that "honeycomb" of the scalp (*Favus*), an infectious disease in which yellow crusts appear on the head, is due to a vegetable parasite related to the moulds was a surprise, as was the demonstration in 1839 that an infectious disease (*Muscardine*) of silkworms is likewise due to a mould.

The microscope also served to reveal what have been called "the footprints of disease" within the cells and tissues, making possible the work in cellular pathology by Virchow.

THE GERM THEORY OF FERMENTATION, PUTREFACTION AND DISEASE. PASTEUR. — At about this time the achromatic compound microscope was coming into use, and by its aid the alcoholic fermentation, hitherto regarded as a purely chemical phenomenon, was found to be intimately associated with, if not actually caused by, a living, growing microörganism, yeast, observed and figured by Leeuwenhoek in 1680, but in the early nineteenth century regarded rather as potent organic matter in some peculiar catalytic state or condition (Liebig) than as a living thing. Between the rediscovery of yeast in 1837 and Pasteur's epoch-making studies upon it in 1859, fermentation was studied by several workers of eminence — among whom was Helmholtz — but it was chiefly Pasteur who in a memorable series of researches finally proved that yeast is the one and only cause of the alcoholic fermentation, — a biological or "germ" theory of fermentation, thus displacing Liebig's chemical or catalytic theory, — the germ in this case being yeast. By the use of the microscope, combined with new and ingenious methods of cultivation of yeast and other microbes, Pasteur, between 1859 and 1865, proved beyond doubt that yeast is the agent of the alcoholic fermentation and that other microbes are the agents of other familiar fermentations, such as the butyric and acetic. His work was marked by remarkable precision and refinement.

From a germ theory of normal fermentations, putrefactions, and decay it was a short step to a germ theory of undesirable or abnormal fermentations, such as often occur in brewing and wine making. In these last, the microscope revealed to Pasteur the presence of strange forms foreign to the ordinary fermentation, and evidently wild yeast, moulds, or other extraneous microbes which by interfering with or supplanting the normal forms, produced disagreeable, or abnormal, *i.e.* "diseased", beer or wine.

Similarly, it was only a second step from the diseases of wine and beer to those of animals and man. A disastrous epidemic disease affecting silkworms in the south of France at this time brought to Pasteur an urgent request that he should make an investigation. "But," said Pasteur, "I have never handled a silkworm." "So much the better," said Dumas, the chemist, who insisted that he should thus patriotically enter the field of animal pathology. Pasteur yielded and spent three years in studies upon the silkworm disease, with results invaluable to science and especially to pathology.

ANTISEPTIC AND ASEPTIC SURGERY. LISTER. — Meantime Lister, an English surgeon resident in Edinburgh, led on by Pasteur's researches, introduced a new and scientific treatment of open wounds, based on the germ theory. Open wounds whether made by accident or in surgery ordinarily suppurate, *i.e.* become red, swollen and inflamed and eventually produce pus. Lister surmised that this suppuration is due to germs from the air, from the surgeon's hands, from instruments, etc., and acting on this theory proposed to prevent such wound diseases by destroying the germs in the air and upon the wound by some "antiseptic," *i.e.* some substance that should prevent sepsis (putrefaction) or suppuration. Carbolic acid (phenol) had recently been introduced into commerce and was highly recommended as a deodorant. This Lister used, and with results so satisfactory that his antiseptic surgery soon became famous. It has since given way to aseptic surgery, which differs from it simply in preventing wound-infection rather than intreating it after it has occurred. In battle-fields antiseptic surgery must still be used, since the work of the surgeon is done only after the wound has been made. Antiseptic and aseptic surgery are among the most priceless blessings of the race and among the greatest triumphs of nineteenth century science.

One serious objection stood in the way of the establishment and acceptance of the germ theory; viz., the possibility that germs were the consequence and not the cause of fermentation, putrefaction, or disease; and this objection was frequently urged. In 1876, however, it was met and overcome by Robert Koch, a district physician of Wollstein, in Prussia. By the use of the methods of Pasteur, Koch succeeded in making a series of successive cultivations of the microbes of anthrax (splenic fever, *charbon,* or malignant pustule) in such a way that at the end of his experiment he had a pure culture of the microbes in question. Obviously, if with these he could *produce* the disease by infecting a susceptible animal or a wound, they must be its cause and not its consequence. In this he was completely successful, thereby establishing beyond all possible peradventure the truth of the germ theory.

Rise of Bacteriology and Parasitology. — The labors of Pasteur, Lister, Koch and others soon led to the birth of a new science, — Bacteriology — of which the first fruit was the discovery in rapid succession by Koch and his pupils of the hitherto unknown germs of some of the worst and most mysterious diseases afflicting the human race; the bacillus of typhoid fever in

1879, — more fully worked out in 1884; the bacillus of tuberculosis in 1882; the vibrio of Asiatic cholera in 1883; the bacilli of lockjaw and of diphtheria in 1884; the bacillus of bubonic plague in 1894; and about the same time by others of the microörganisms of malaria, sleeping sickness, and several other diseases.

In some cases such as smallpox and yellow fever no germs have yet been observed; but this seems at present to be because they are too small to be seen with the microscope or to be held back, as most germs are, by pipe-clay filters. If, nevertheless, we review the list just given of those plagues in which the causative microorganism was detected and cultivated between 1879 and 1889, we cannot avoid the conclusion that the ninth decade of the nineteenth century was the most important hitherto in the history of pathology. When we go further, and compare these rich discoveries and the fruit they have since borne, in preventive medicine, preventive sanitation and preventive hygiene, with our previous ignorance of the nature of disease and of its control, we realize that since that decade the world has possessed not only a new pathology but also a new science.

Besides bacteriology another science, parasitology, has also become prominent since the decade of the great pathological discoveries. The parasitic character of the mistletoe, the tapeworm, the flea, the louse, the mosquito and other visible pests was long ago evident, but it was only after the discoveries of Pasteur and Koch and their disciples were fully comprehended that the germ theory of disease was seen to be at bottom a theory of parasitism. Thereupon *parasitology* assumed a new place and a new significance as a branch of pathology.

BIOGENESIS *versus* SPONTANEOUS GENERATION. — The question of the origin and beginnings of life on the earth has always been obscure and perplexing to mankind, and up to the middle of the nineteenth century the account attributed to Moses, — the so-called theory of special creation, — was still predominant though about to give way to the theory of evolution. A similar obscurity veiled the beginnings of individual life. *Omne vivum ex ovo* (all life from the egg) was the motto of those who thought only of the higher animals. *Omne vivum ex vivo* was that of those who held that living things come only from antecedent life, even if not from eggs. Both of these groups were biogenesists, since they maintained that living things come only from other living things. Opposed to them were the abiogenesists who disputed these ideas and believed in "spontaneous" generation (abiogenesis), *i.e.* in the origin of living things from lifeless or non-living matter.

The dispute was very old, Aristotle, for example, having favored the idea of spontaneous generation. In the eighteenth century Spallanzani for biogenesis and Needham for abiogenesis had fought over again the ancient battle. Lamarck, at the beginning of the nineteenth century, looked with favor upon abiogenesis. The improved microscope of that century seemed at first to strengthen the evidence for spontaneous generation by revealing almost everywhere the presence of microbic life, and the idea of an apparently easy generation of new life was welcomed by some interested in evolution, as accounting naturally rather than supernaturally for the origin of life in general. Meantime, the discovery of the mammalian ovum by von Baer in 1827 had somewhat improved the position of the biogenesists, but the whole question remained open and unsettled at the middle of the century and until it was attacked by Pasteur, who was thoroughly equipped with the most exact scientific methods of the day. For the details of the struggle in which Pasteur battled for biogenesis we must refer the reader to the Life of Pasteur, by Radot, and to Tyndall's Floating Matter of the Air. The upshot was that all the evidence advanced by the advocates of spontaneous generation was shown by Pasteur, — ably seconded by Tyndall, — to be due to defective technique; for when such defects were corrected no evidence remained of the generation of life by lifeless matter. Thus was triumphantly closed, at least for the time, one of the most ancient of controversies.

Obviously, the question of a possible spontaneous generation of living matter from lifeless under special conditions such as may have existed during the early history of our globe remains open. All that modern science has done is to controvert such evidence as has been advanced of its ordinary and frequent occurrence under such natural conditions as prevail to-day.

PROGRESS OF GEOLOGICAL SCIENCE. — In 1785 Hutton, to whom we have already briefly referred, presented to the Royal Society of Edinburgh a paper entitled Theory of the Earth, or an Investigation of the Laws Observable in the Composition, Dissolution and Restoration of Land upon the Globe (p. 317).

> In this remarkable work the doctrine is expounded that geology is not cosmogony, but must confine itself to the study of the materials of the earth; that everywhere evidence may be seen that the present rocks of the earth's surface have been formed out of the waste of older rocks . . . that every portion of the upraised land is subject to decay; and that this decay must tend to advance until the whole of the land has been worn away. . . . In some of these broad generalizations Hutton was anticipated by the Italian geologists; but to him belongs the credit of having first perceived their mutual relations and combined them in a luminous coherent theory everywhere based upon observation. . . . It is by his *Theory of the Earth* that Hutton will be remembered with reverence while geology continues to be cultivated. — *Geikie.*

In the early part of the nineteenth century it was, nevertheless, firmly held that the earth had undergone various "revolutions," "catastrophes" and the like which, taken together with the Flood of Noah, were sufficient to explain its present surface features, such as mountains, valleys, plains, boulders, caves, deserts, sea-coasts, etc. These views were summed up in the term Catastrophism, *i.e.* that

> at a number of successive epochs — of which the age of Noah was the latest — great revolutions had taken place on the earth's surface; that during each of these cataclysms all living things were destroyed; and that, after an interval, the world was restocked with fresh assemblages of plants and animals, to be destroyed in turn and entombed in the strata at the next revolution. — *Judd.*

Moreover, at the beginning of the century most considerations of the earth and of the living world were dominated by two preconceived ideas: first, that the universe, including the earth and its belongings, had originated as described in the first chapter of

Genesis, and second, that the present features of the earth are to be explained chiefly by the more recent Flood of Noah described in the seventh chapter.

> Before geology had attained to the position of an inductive science, it was customary to begin all investigations into the history of the earth by propounding or adopting some more or less fanciful hypothesis in explanation of the origin of our planet, or even of the universe. . . . To the illustrious James Hutton (1785) geologists are indebted for strenuously upholding the doctrine that it is no part of the province of geology to discuss the origin of things. He taught them that in the materials from which geological evidence is to be compiled there can be found 'no traces of a beginning, no prospect of an end.' — *Geikie.*

> The vast deposits of sand, gravel and clay, with the embedded remains of contemporaneous animal and vegetable life with which they (glacial torrents) everywhere covered the plains, were viewed till recently solely in relation to the Mosaic narrative of a universal deluge, and were referred implicitly to that source. — *Wilson.*

As late as 1823 Buckland, a distinguished English geologist, published a work on extinct animals from a Yorkshire cavern entitled *Reliquæ Diluvianæ.* As for plants and animals, the almost universal opinion was that these, like the earth, had been specially created and had remained ever since substantially unchanged.

As for the crust of the earth, composed, as this is often seen to be in section, of unlike layers, it was a long time before the current idea of sudden creation could be replaced by one so different as that of slow and steady deposition. This change, however, was finally though only gradually effected, largely through actual observation and measurement of the slow deposits of rivers, and other geological phenomena of to-day, combined with Lyell's thesis that those of the past were essentially similar. The *time element,* in brief, began to be recognized as a new and an important factor in the making of the earth's crust.

Reference has been made above to Lyell's revolutionary treatise, The Principles of Geology, published in 1830. This great work which adopted, emphasized, and extended the works of

Hutton and Smith, eventually overthrew catastrophism and established uniformitarianism in its place. (See Appendix H.)

GLACIERS AND GLACIAL THEORIES. — The occurrence on the earth's surface, and even on mountain tops, of boulders evidently of distant rather than local origin, had long been a puzzle even to geologists. The primitive hypothesis of their deposit during the Flood — the so-called diluvial theory — no longer sufficed to satisfy inquiring minds, and equally inadequate was the idea of von Buch that boulders had been thrown up like cannon shot by volcanoes and had fallen where found. In 1837 Louis Agassiz (1807–1873) advanced the present doctrine: viz. that boulders have been deposited after the melting of masses of ice by which they were slowly brought from a distance. Agassiz supported this ice or "glacial" theory by personal observations and studies of the glaciers of the Alps, and eventually propounded that general theory of glaciation or ice caps at the earth's poles which is now universally accepted. Few theories have ever proved more satisfactory, scientifically speaking, more popular, in the best sense, or more productive of simple explanations of widespread and diverse phenomena. We have only to set over against the glacial the diluvial theory, with its fatal weakness in requiring the transportation by water of huge masses of rock over long distances, or the projectile theory, with its requirement of showers of flying boulders falling almost anywhere, to realize the simplicity and adequacy of the glacial theory. The word "boulder," nevertheless, remains as a reminder of the diluvial theory, since it is derived from words signifying "the noise of a stone in a stream."

RISE OF PALÆONTOLOGY. — Before the nineteenth century precise knowledge of extinct animals and plants was almost wholly wanting. Fossils had indeed been observed from the earliest times but although occasionally correctly interpreted, as for example by Pythagoras and Xenophanes, Leonardo da Vinci and Palissy. Hooke (p. 268) at the end of the seventeenth century first made the important suggestion that fossils might serve as indicators of phases in the earth's history and as proof of the existence at one time of a tropical climate in England. In the eighteenth century these ideas were developed by Woodward and others, while J. Gesner introduced into geology the suggestion of great age for the earth by estimating the time required for the elevation of certain fossiliferous strata in the Apennines at 80,000 years. Buffon in France speculated upon the successive emergence and depression of the continents, and Werner in Saxony noted in successive formations the gradual approach of extinct forms of life towards existing forms.

In the early part of the nineteenth century palæontology began to take on its modern form. Pallas had, indeed, discovered vast deposits of extinct mammoths and rhinoceroses in Siberia in 1768–1774, — Blumenbach had distinguished between the fossil mammoth and the living elephant in 1780, — and in 1793 the American mastodon was recognized as different from both fossil mammoth and living elephant. In 1793 Lamarck recapitulated and emphasized the methods and results of his predecessors and sought to account for the phenomena, partly by changes in the habits and partly by changes in habitat of extinct forms, modifications from whatever source being held to be conserved and accumulated by inheritance, and in 1800 Cuvier published an important paper on fossil and living elephants. Not long after, remains of huge extinct reptiles were discovered: of the ichthyosaurus and plesiosaurus in 1821; of the mososaurus in 1822; of fossil crocodiles in France in 1831; of the iguanodon in 1848. These "finds" opened up a new world of buried ancient life almost beneath our feet scarcely inferior in interest to the starry world far overhead, which had so long excited the curiosity and wonder of mankind.

In 1854 in the caves of Belgium were found remains of lions and other animals (including man) which were obviously unlike the same species to-day, and have ever since been spoken of as the "cave" lion, the "cave" tiger, etc. With the human remains were discovered prehistoric implements testifying both to the antiquity of man and to the superiority of the cave men to other cave animals. Fossil ferns and other plants were also found, and even fossil insects, — the latter often in a remarkably good state of preservation. Here, also, recognition of the time element became inevitable and subversive of the idea of sudden and special creation. Hitherto belief in the antiquity of man was exceptional, and proofs of such antiquity were almost wholly wanting. This discovery, therefore, was profoundly important and of far-reaching significance not only in palæontology but also in the foundation of what is to-day known as anthropology.

ANCIENT AND MODERN THEORIES OF COSMOGONY. — When the wonder and curiosity of primitive man developed into speculation touching his origin, or the origin of things about him, or the origin of the visible universe, cosmogony began. Until the middle of the nineteenth century the Jewish or Mosaic cosmogony embodied in the first chapter of the Hebrew Scriptures, and accounting for the origin of the cosmos, both organic and inorganic, by a sudden and special creation, was almost universally accepted throughout Christendom. Recent investigations indicate that this theory was really pre-Jewish in origin and probably Babylonian. If so, a cosmogony long antedating Greek theories prevailed throughout Christian Europe and America to the middle of the nineteenth century. In the seventeenth century even Galileo, Kepler, and Newton raised no question of its essential validity.

> Of the two great minds of the seventeenth century, Newton and Leibnitz, both profoundly religious as well as philosophical, one produced the theory of gravitation [and] the other objected to that theory as subversive of natural religion. — *Asa Gray.*

The eighteenth century was an age of doubt. Descartes, who "doubted whatever could be doubted," had been succeeded by Voltaire and the encyclopædists in France, and by Hume and Gibbon in England, and incredulity, not to say scepticism, was in the air. Yet no new theory of cosmogony appeared, and merely to doubt an old hypothesis is neither to destroy nor to supplant it. Observations, ideas, and discoveries, however, had long been accumulating and were now multiplying, which were destined to undermine the Mosaic theory and establish something very different, and more resembling Greek cosmogonies, in its place.

A complete cosmogony should, in theory at least, attempt to account both for the origin of the cosmos and for its present aspects. This the theory of special creation failed to do. It described the origin of the cosmos at a period evidently remote, — inasmuch as it was stated elsewhere in the Scriptures that many generations had come and gone since the Creation, — but was silent as to any essential progress or modifications in the meantime. Hence, for those accepting special creation the inference naturally was that no great changes either in the heavens or in the earth had, in fact, taken place since the initial act of creation, and that the present aspect of the cosmos is substantially its primitive aspect. On this theory mankind and other living things had not developed, but rather stood still or even — as in the case of "the fall of man" — actually retrograded from a more perfect type. In complete contrast with this ancient, Oriental theory the modern theory of Evolution, making no pretence to solve the problem of the origin of the cosmos, attempts only to explain some of its present aspects.

Relationship of the Heavens and the Earth. — It had for centuries been taken for granted, as a part of the geocentric theory, that the heavens and the earth had little if anything in common — the earth being the centre of things and of first importance. Copernicus, however, had shown that the earth is inferior, and tributary to the sun, while his great successors Galileo, Kepler, and Newton had proved both earth and sun to be no more than members of a huge system of heavenly bodies strictly correlated by gravitation. Hence, when Franklin drew down lightning from the terrestrial heavens and the spectroscopists not long after proved a substantial chemical identity between the earth and celestial bodies, the older cosmogony began to seem both primitive and parochial.

Above all, the ideas of Kant and Laplace, which seemed to indicate not merely a structural and material kinship between the heavens and the earth, — such as that later revealed by the spectroscope, — or a functional similarity, — such as that discovered by Franklin, — but, more remarkable than either, a family relationship through a common ancestry, weighed heavily against the theory of special and separate creation by an extraneous will in remote time, with no provision for change to meet changing conditions.

THE SCALE OF LIFE AND THE PHASES OF LIFE. — It had often been commented upon that in the world of life there is always present and requiring explanation what Bonnet called the "scale of life," *i.e.* the fact that plants and animals not only differ greatly in structure and complexity, but that both may be arranged in a kind of ascending or descending natural scale (ladder) with highly complex forms at the top and relatively simple ones at the bottom. On the doctrine of special creation it was difficult to find any reason for or advantage in the existence of such a scale, while the suggestion was obvious that the higher forms had somehow come from or passed through lower forms. A rapid modification of living forms, such as this suggestion required, is obvious in everyday life being exemplified by the so-called phases of life, — in which infancy makes way for youth, youth for maturity, and maturity for age. To the attentive observer living matter appears to be thus forever changing, and on the whole progressing or advancing from simplicity to complexity.

In this respect the stellar universe at first sight appears very different, for here permanence rather than change seems to prevail. As for the earth, changes, indeed, do often occur and sometimes progressively, as in erosion, glacial action, and the work of the tides, so that the earth seems to stand in this respect somewhere between changing and advancing organic life and the unchanging heavens. When, therefore, the idea broached by Hutton and Smith at the end of the eighteenth century that the surface of the earth has been made and is still being made what it is to-day, not suddenly and once for all by special creation centuries ago, but gradually, and by forces and processes similar to those now acting, a new and revolutionary notion of the genesis of the earth's crust arose, and one contrary to the idea of special creation. The same idea carried further and developed by Lyell in 1830 became thenceforward all important: —

Amid all the revolutions of the globe the economy of Nature has been uniform, and her laws are the only things that have resisted the general movement. The rivers and the rocks, the seas and the continents have been changed in all their parts; but the laws which direct those changes, and the rules to which they are subject, have remained invariably the same. — *Lyell's Vol. I, Title Page Motto.*

GENERAL RESEMBLANCE OF MAN TO THE LOWER ANIMALS. — At the end of the eighteenth century the increase of knowledge nowhere led to more startling revelations than in comparative anatomy, for a very moderate amount of dissection of the various types of vertebrates suffices to show that all of these, — including man himself, — are built upon the same general plan. Similarity extends even into minute details, as in the lungs, aortic arches, teeth, eyes and ears, and the complex musculature of the limbs. Similarity in the organs and processes of reproduction among the higher animals had been recognized ever since the time of Aristotle, and the discovery of the mammalian ovum by von Baer in 1827 simply added another link to the long chain of resemblances between man and other higher vertebrates and the lower, — such as reptiles, frogs, and fishes. Embryology now strengthened this chain by showing that the embryos of these various animals are more alike than are the adults into which they develop — thus suggesting that all were originally similar or even identical, but had afterwards become differentiated. Malthus in his startling work on the Principle of Population had proved a tendency in mankind to multiply, like other animals, without reason and beyond the means of subsistence. The antiquity of man, long suspected, was established by the finding, in 1854, by Boucher de Perthes, of human remains along with those of extinct animals in the caves of northern France. Archæology and linguistic studies had already contributed to disprove the conventional chronology (which held that the creation of the world occurred 4004 B.C.) and thus, indirectly, the current cosmogony, by discovery of the remains of prehistoric culture, and by showing that the modern European languages and arts are evidently direct and related descendants of earlier and sometimes extinct forms.

ANATOMICAL AND MICROSCOPICAL SIMILARITY OF ANIMALS AND PLANTS. ORGANS, TISSUES, CELLS, AND PROTOPLASMS. — The old cosmology emphasized differences rather than resemblances between animals and plants for, barring the one fact of life, there is at first sight little in common between them. It was always plain that both are provided with organs, and that in respect to functions, such as growth, differentiation, the phases of life and reproduction, there is great similarity. The improved microscope of 1830–1850 now revealed a further and striking similarity, by showing that the organs in both plants and animals are made up of tissues, and these in turn of cells; and when about 1845 it was found that both plant and animal cells contain a slimy, colorless substance, apparently similar, if not actually identical, in all living things, it was not long before the same name "protoplasm" (first life) was given to this fundamental substance, whether it occurred in plant or in animal cells, — in leaves or in muscles. It was of this same fundamental protoplasm, common to both plants and animals, that Huxley wrote his famous essay entitled The Physical Basis of Life.

FUNDAMENTAL UNITY OF NATURE. ORGANIC *versus* INORGANIC WORLD. — Between things organic and inorganic until the nineteenth century there was supposed to be a great gap. When, therefore, in 1828 Wöhler produced in the laboratory urea, a typical organic substance hitherto unknown except as an animal excretion, by merely heating ammonium cyanate, it became evident that in chemistry the term "organic" had lost its former meaning. Since that time many compounds once believed to be capable of production only by living things, have been made in the laboratory, with the result that the organic and the inorganic worlds have been drawn nearer together. The further fact that living matter is composed largely of four common chemical elements, carbon, hydrogen, oxygen, and nitrogen, and yields on analysis no peculiar or mystical substance or element, tended to show that life itself might be merely a property of various chemical elements in peculiar combination.

No less surprising than the revelation of the chemical similarity of living and lifeless matter was that by spectrum analysis of the chemical composition of the stars, in which various elements common on the earth were readily detected. This astounding result, taken together with the clearer and more convincing ideas of the conservation of matter and energy, and of the similarity in nature of heat, light, and sound as undulations, served to demonstrate and to emphasize the scope and immanence of natural law, as well as the fine adjustment, balance, and economy of nature, and to cause interference or governance by anything supernatural to seem gratuitous and unwarranted.

TREVIRANUS' BIOLOGY AND LAMARCK'S ZOÖLOGICAL PHILOSOPHY. — Very early in the nineteenth century the two works here mentioned and already referred to appeared, the former introducing into science for the first time the word biology, and thereby formally recognizing a world of life in contradistinction to a world of lifelessness. Both works virtually ignored the old cosmogony as applied to plants and animals, and both sought after some new and less supernatural theory. Lamarck in particular was bold enough to suggest that the elongated neck of the giraffe was not specially created, but had been gradually developed by constant effort to obtain food beyond its ordinary reach. Both authors deserve special mention because with them biology began consciously and frankly to part company with the older cosmogony. Lamarck, for example, after frankly accepting the possibility of spontaneous generation for the origin of living matter sought to explain its present variety and differentiation by four laws, which may be stated as follows: —

1. Life naturally tends to increase and enlarge up to a certain self-determined limit.
2. New organs arise in response to new and reiterated wants and to the changes produced by these wants or by efforts to meet them.
3. The development of organs and their functions is determined by the use of such organs.
4. All changes in organization are conserved by generation and transmitted to offspring.

To these ideas Lamarck clung in spite of criticism, for in the Introduction to his Natural History of Invertebrates, a much later work than his Zoölogical Philosophy, he again affirms: —

> I conceive that a gasteropod mollusk [*e.g.* a snail], which, as it crawls along, finds the need of touching the bodies in front of it, makes efforts to touch those bodies with some of the foremost parts of its head, and sends to these every time quantities of nervous fluids as well as other liquids. I conceive, I say, that it must result from this reiterated afflux towards the points in question that the nerves which abut at these points will by slow degrees be extended. Now, as in the same circumstances, other fluids of the animal flow also to the same places, and especially nourishing fluids, it must follow that two or more tentacles will appear and develop insensibly in those circumstances at the points referred to.

The principal significance of these views to-day is in the attempt which they embody to explain on natural principles phenomena then explained only as supernatural.

VOYAGES AND EXPLORATIONS OF NATURALISTS. — In the nineteenth century voyages, expeditions, and explorations for the first time were undertaken for the sole and specific purpose of the improvement of natural knowledge (to use a phrase associated with the origin of the Royal Society). Of these the first were those of Alexander von Humboldt who, beginning in 1799, made numerous and extensive journeys and observations by land and sea which enabled him many years later to publish his monumental Kosmos, a work replete with observations and reflections on natural philosophy and natural history, which eventually gave him a place, in this century, among the learned men of Germany second only to that occupied by Goethe.

In 1801, Robert Brown, a British botanist, worthy of remembrance also in connection with the so-called Brownian movement of particles under the microscope, accompanied an expedition to Australia and brought back representatives of some 4000 new species of plants. Most fruitful of all was Darwin's famous voyage of the *Beagle* to the Pacific (1831–1836), while in 1838 Karl Ernst von Baer, the eminent founder of comparative embryology, accompanied an exploring expedition to Nova Zembla; Dana sailed to the Pacific in 1838–1842; Huxley in the *Rattlesnake* in 1846–1850; and Alfred Russel Wallace visited the Malay Archipelago a little later.

The result of these scientific explorations was to throw a flood of new light upon the infinite wealth, variety, creative resources and capacity of this earth and its inhabitants, plants as well as animals, and to reveal adaptations of organisms to climate, soil, and other environmental conditions countless in number and marvellous in character, all of which raised again the ancient and vexing questions: How did these adaptations arise? And what was the origin of species?

DARWIN'S ORIGIN OF SPECIES (1859). — For this, the most influential scientific work of the nineteenth century, and one of the most important ever written, the way had now been prepared by the publications of Malthus, Treviranus, Lamarck, Lyell, and Chambers. In particular, as Huxley pointed out, Lyell's work, by showing that the earth is very old and has been long ages in the making had, since 1830, shaken the confidence of men of science in the old cosmogony and so paved the way for Darwin; and when in 1854 remains of prehistoric man were found with those of extinct animals, such as the cave bear and the cave lion, even popular confidence in the Mosaic theory began to be undermined.

Darwin's great work appeared in 1859 and aroused world-wide criticism and controversy. In his autobiography Darwin acknowledges his constant obligation to Lyell and also to Malthus, whose emphasis on the struggle for food revealed to Darwin the fuller meaning of what in the Origin he termed the struggle for existence, a struggle in which by natural selection there must be progressively a survival of the fittest. In the long and fierce battle which now broke out between the defenders of the old cosmogony and the new, and which at first went no further than proposing to account for the origin of species of living things by natural selection instead of by special creation, Darwin was ably supported by his friends the naturalists, — Huxley, Hooker, and Lyell in England and Asa Gray in America.

Darwin's principle of natural selection is stated in his own words as follows: —

> As many more individuals of each species are born than can possibly survive; and as, consequently, there is a frequently recurring struggle for existence, it follows that any being, if it vary however slightly in any manner profitable to itself, under the complex and sometimes varying conditions of life, will have a better chance of surviving, and thus be *naturally selected.* From the strong principle of inheritance, any selected variety will tend to propagate its new and modified form.

THE DESCENT OF MAN. — The last refuge of the defenders of the old cosmogony was man himself, and not a few, among whom was Alfred Russel Wallace, co-discoverer with Darwin of the principle of natural selection, refused to align man and especially man's mentality, with lower animals and their inferior mental powers. Here at least, they affirmed, we see no evidence of evolution. But as time has gone on, and studies have been extended in craniology and in psychology it has become increasingly evident that there is little if any more difference in brain weight and mental power between the highest apes and the lowest men, than between these last and the highest men. Meantime, the doctrine of parsimony, in default of any other than a supernatural hypothesis, naturally awarded the field to the Darwinian theory.

DECLINE OF THE THEORY OF SPECIAL CREATION. — Darwin's theory of the method of origin of the various kinds (species) of plants and animals by natural processes under natural law soon became known as the theory of organic evolution, and before a score of years had passed was almost universally accepted among naturalists.

It is the supreme merit of Darwin to have thus pointed out a method by which a process of gradual development, or evolution, — already accepted for language, art, music, and (after 1830) for the earth's crust, and many years before urged by Laplace for the solar system, — could be made no less applicable to, and acceptable for, plants and animals, and especially for man. Until this was done, there could be no place for the new cosmogony in the general mind.

The theory of organic evolution by natural selection as maintained by Darwin was naturally subjected forthwith to the severest scrutiny, and some of its details have been successfully and destructively criticised. In particular, his explanation of the mechanism of heredity appears to be untenable, as does also his theory that small but incessant variations are gradually accumulated into departures from the original, ultimately sufficient to amount to new species. The studies of Mendel and of Weismann upon inheritance and of De Vries and others upon variation, have supplemented and to some extent supplanted much of Darwin's work upon those subjects. But apart from these and other readjustments of details, the Darwinian theory stands secure and at present affords the most reasonable explanation hitherto proposed of the origin of man and other animals and of plants; an explanation, moreover, in harmony with the general law of evolution now accepted for the origin of existing forms of language, literature, and art; of chemical compounds; of the earth; of the solar system; and of the stars. The law of evolution, long ago foreshadowed by the Greeks, is clearly the law of the lifeless world, and there seems to be no reason why it should not be equally applicable to the world of life,— including man.

INFLUENCE OF AN AGE OF INVENTION AND INDUSTRY. — Acceptance of the theory of evolution was quickened by an increase in popular intelligence and a general openmindedness due in part to the broadening of education, in part to the ease of travel, and in part to the appearance, one after another, of revolutionary inventions, such as the steam engine and cotton machinery with the factory system at the end of the eighteenth century, and the locomotive, the steamboat, the telegraph, the sewing machine, the friction match, and many more, in the first half of the nineteenth century. All these marvels had been wrought within the brief space of a single century, while improved printing-presses, cheaper paper, cheaper newspapers, cheaper books, and cheaper illustrations, as well as more and better schools and schooling had contributed immensely to popular education, mental receptivity, closer contact, industrial coöperation and general intelligence.

Science in the Dawn of the Twentieth Century. — At the beginning of the nineteenth century, science as such had no existence either as a branch of learning or as a special discipline, — still less as a preparation for practical life. Mathematics, highly esteemed largely because of its ancient origin and associations, and natural philosophy, had a limited recognition; but the term science meant as yet hardly more than knowledge or learning. The eighteenth century had, however, sowed broadcast the seeds of science and the nineteenth soon began to reap the harvest. Before 1850 scientific schools as distinct from others had been founded both within and without the older colleges and universities. New associations and academies for the advancement or promotion of science soon sprang up; science courses appeared in some of the public schools; funds for scientific research began to be provided; and thousands of eager and enthusiastic students began to prefer science, and especially applied science, to the older "classical" learning. Meantime, the marvellous achievements of invention and of industry had caught and fixed public interest and attention, so that by the opening of the twentieth century, no branch of learning stood in higher favor than science, either for its own sake or as a preparation for useful service in contemporary life.

The master keys of science, now everywhere employed for unlocking the problems of the cosmos, are: first, *the principles of mathematics*, which admit mankind into the mysteries of the relations of number and space — the abstract skeleton of science, — and second, *the principles of evolution and of energy*, which reveal some, at least, of the secrets of form and of function, not only of the earth and of plants and animals, but of the heavens; something of the prodigious forces of the universe and their orderly behavior; something of that apparently infinite and eternal energy which, while forever changing, is never lost; something, though as yet but little, of the nature and the processes of life.

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APPENDICES

A. THE OATH OF HIPPOCRATES (About 400 B.C.)

[This, as Gomperz observes, is a monument in the history of civilization. It is no less a monument in the history of science, since it proceeds from the Father of Medicine more than two thousand years ago and, if we except mathematics, medicine is the oldest of the sciences. There are many translations of the "Oath," some more literal and some, like the following, more free.]

I swear by Apollo the physician [*Healer*], and Æsculapius, and Health [*Hygeia*], and All-heal [*Panaceia*], and all the gods and goddesses, that according to my ability and judgment I will keep this oath and stipulation: to reckon him who taught me this art equally dear to me as my parents, to share my substance with him and relieve his necessities if required; to regard his offspring as on the same footing with my own brothers, and to teach them this art if they should wish to learn it, without fee or stipulation, and that by precept, lecture and every other mode of instruction, I will impart a knowledge of the art to my own sons, and to those of my teachers, and to disciples bound by a stipulation and oath, according to the law of medicine, but to none others.

I will follow that method of treatment which according to my ability and judgment I consider for the benefit of my patients, and abstain from whatever is deleterious and mischievous. I will give no deadly medicine to anyone if asked, nor suggest any such counsel; furthermore, I will not give to a woman an instrument to produce abortion.

With purity and with holiness I will pass my life and practice my art. I will not cut a person who is suffering with a stone, but will leave this to be done by practitioners of this work. Into whatever houses I enter I will go into them for the benefit of the sick, and will abstain from every voluntary act of mischief and corruption; and further from the seduction of females or males, bond or free.

Whatever, in connection with my professional practice or not in connection with it, I may see or hear in the lives of men, which ought not to be spoken abroad, I will not divulge, as reckoning that all such should be kept secret.

While I continue to keep this oath unviolated, may it be granted to me to enjoy life and the practice of the art, respected by all men at all times; but should I trespass and violate this oath, may the reverse be my lot.

B. THE *OPUS MAJUS* OF ROGER BACON (1267 A.D.)

[AN ANALYSIS OF THE SIXTH PART. BY J. H. BRIDGES.]

Of all the parts of the Opus Majus, the sixth is the most important. It treats of experimental science, domina omnium scientiarum et finis totius speculationis. *Without experience, as Bacon constantly repeats, nothing can be known with certainty. Even the conclusions of mathematical physics, reached by argument from certain principles, must be verified, before the mind can rest satisfied. To this great science all the others are subsidiary; they are to it* ancillæ *or handmaids, an expression that curiously reminds one of Francis Bacon. The reasoning in favour of experience is well worth quoting at length:* —

"*There are two modes in which we acquire knowledge, argument and experiment. Argument shuts up the question, and makes us shut it up too; but it gives no proof, nor does it remove doubt and cause the mind to rest in the conscious possession of truth, unless the truth is discovered by way of experience, e.g. if any man who had never seen fire were to prove by satisfactory argument that fire burns and destroys things, the hearer's mind would not rest satisfied, nor would he avoid fire; until by putting his hand or some combustible thing into it, he proved by actual experiment what the argument laid down; but after the experiment had been made, his mind receives certainty and rests in the possession of truth, which could not be given by argument but only by experience. And this is the case even in mathematics, where there is the strongest demonstration. For let any one have the clearest demonstration about an equilateral triangle without experience of it, his mind will never lay hold of the problem until he has actually before him the intersecting circles and the lines drawn from the point of section to the extremities of a straight line. He will then accept the conclusion with all satisfaction.*" (Op. Maj., p. *445* [*ed. Bridges, ii. 167*].)

This important passage, it seems to me, marks a distinct advance in the philosophy of science. The science of that time proceeded wholly per argumentum; *verification was unknown. Not only, however, does Bacon recognize the necessity for experiment, for observation at first-hand, but he has a clear appreciation of the true nature of scientific verification. He has already expounded his ideal of physical science, the application of mathematics to determine the laws of force and to deduce conclusions from these laws; but he is perfectly aware that these* *general conclusions must be tested by comparison with things, must be verified. The function of experimental science is, in a word, Verification.*

"This Science," says Bacon, "has three great prerogatives in respect to all other sciences. The first is — that it investigates their conclusions by experience. For the principles of the other sciences may be known by experience, but the conclusions are drawn from these principles by way of argument. If they require particular and complete knowledge of those conclusions, the aid of this science must be called in. It is true that mathematics possesses useful experience with regard to its own problems of figure and number, which apply to all the sciences and to experience itself, for no science can be known without mathematics. But if we wish to have complete and thoroughly verified knowledge, we must proceed by the methods of experimental science." (Op. Maj., *p. 448* [*ed. Bridges, ii. 172–3*].)

As an example of his method Bacon analyses the phenomena of the rainbow in a thoroughly scientific manner.

The second and third prerogatives (though not of such importance) may also be mentioned. The second is — that Experimental Science attains to a knowledge of truth which could not be reached by the special sciences; the third — that Experimental Science, using and combining the results of the other sciences, is able to investigate the secret operations of Nature, to predict what the course of events will be, and to invent instruments or machines of wonderful power.

— *Adamson* (quoted by A. G. Little).

PART VI. EXPERIMENTAL SCIENCE

CHAPTER I

Having laid down the general principles of wisdom so far as they are found in language, in mathematics, and in optics, I pass to the subject of experimental science. . . .

When Aristotle speaks of knowledge of the cause as a higher kind of knowledge than that gained by experience, he is speaking of mere empiric knowledge of a fact; I am speaking of experimental knowledge of its cause. There are numerous beliefs commonly held in the absence of experiment, and wholly false, such as that adamant can be broken by goats' blood, that the beaver when chased throws away his testicles, that a vessel of hot water freezes more rapidly than one of cold, and so on. Experience is of two kinds: (1) that in which we use our bodily senses aided by instruments, and by evidence of trustworthy witnesses; and (2) internal experience of things spiritual, which comes of grace, and which often leads to knowledge of earthly things. The mind stained with vice is like a rusty or uneven mirror, in which things seem other than they are. Without virtue a man may repeat words like a parrot, and imitate other men's wisdom like an ape, and all to no purpose. The intellectual effect of a stainless life is well illustrated in the young man who is the bearer of this treatise. The degrees of spiritual experience are seven. (1) Spiritual illumination; (2) virtue; (3) the gift of the Holy Spirit described by Isaiah; (4) the Beatitudes; (5) spiritual sensibility; (6) Fruits, such as the peace of God which passes understanding; (7) states of Rapture. —

CHAPTER II

It is solely by the aid of this science that we shall be able to disabuse men of the fraudulent tricks by which magicians have imposed on them. As compared with other sciences, this science has three characteristics ("praerogatives"). Of these the first is, that it constitutes a test to which all the conclusions of other sciences are to be subjected. In other sciences the principles are discovered by experiment, but the conclusion by reasoning. An instance of this is afforded by the rainbow, and by other phenomena of a similar kind, as haloes, etc. The natural philosopher forms a judgment on these things: the experimenter proceeds to test the judgment. He seeks for visible objects in which the colours of the rainbow appear in the same order. He finds this the case with Irish hexagonal crystals when held in the sun's rays. This property, he discovers, is not peculiar to these crystals, but is common to all transparent substances of similar shape, similarly placed. He finds these colours again on the surface of crystals when slightly roughened. He finds them in the drops that fall from the rower's oar, when the sun's rays strike them, or from a water-wheel, or in the morning-dew on the grass. They may be seen again in sunshine when the eye is half opened, and in many other cases.

CHAPTER III

The shape in which the colours are disposed will vary. Sometimes it is rectangular, sometimes circular.

CHAPTER IV

Armed with these terrestrial facts, the experimenter proceeds to examine the celestial phenomenon. He finds, on examining the sun's altitude and that of the summit of the bow, that the two vary inversely. The bow is always opposite the sun. A line may be drawn from the centre of the sun through the eye of the observer and the centre of the circle of which the bow is an arc to the sun's nadir. As one extremity of this line is depressed, the other is elevated. It becomes thus possible to compute the altitude of the sun beyond which no rainbow is possible, and also the maximum altitude of the bow. It will be found both by calculation and experience that this altitude in the latitude of Paris is forty-two degrees.

Chapter V

Still further investigating the shape of the iris, and the portion of it that can be seen, the experimenter conceives a cone of which the apex is the eye, the base is the circle of the iris, the axis being the line already described drawn from the sun's centre through the eye to the sun's nadir. In cases where this cone is very short, the whole of the base may be above the horizon, as may often be seen in the spray of a waterfall. In the sky, however, the cone is too elongated to admit of this: the base is bisected in various proportions by the plane of the horizon. The arcs visible are not portions of the same circle. When the sun is high, and a small arc is visible, it belongs to a larger circle than the arc seen when the sun is rising or setting. A bow can be seen when the sun is just below the horizon; but owing to terrestrial vapours, only the crown of the arch is usually seen.

Chapter VI

In some latitudes there can be no rainbow at noon even in the winter solstice. When the latitude (*i.e.* the distance from the zenith to the equator) is 24° 25′, the sun's altitude at noon in the winter solstice will be 42°, therefore there can be no bow. Passing north from this latitude, there can always be a noon rainbow till we come to latitude 66° 25′, when at the winter solstice there is no sun. Similar calculations can be made for other latitudes.

Chapter VII

We have now to inquire whether the iris comes from incident, reflected, or refracted rays. Is the bow an image of the sun? Are the colours on the clouds real? Why is the iris of circular form? Here we call experiment to our aid. We find on trial that if we move in a direction parallel to the rainbow it follows us with a velocity exactly equal to our own. The same phenomenon occurs with respect to the sun. We have seen that the sun is always opposite the rainbow; the line between the centre of the bow and the centre of the sun passing through the eye of the observer. If the sun were apparently stationary, this would involve the bow moving much faster than the observer, the latter moving through the same angle, but at less distance from the apex. But this is not so. Therefore there is an apparent motion of the sun concurrently with that of the bow. The case is analogous to what happens when a hundred men are ranged in line facing the sun. Each sees the sun in front of him. Their shadows seem parallel, though we know in reality they must diverge, yet owing to the vast distance of the sun this divergence is imperceptible. We are thus brought to the conclusion that, supposing a rainbow to occur, each of the hundred men, facing backwards, would see a different rainbow, to the centre of which his own shadow would point. The rays causing the iris are therefore not incident rays, otherwise the colour would appear fixed in the cloud. And for the same reason they are not refracted rays, for in refraction the image does not follow the change of place of the observer, as is the case here. One condition of the phenomenon is that the atmosphere shall be more illuminated at the standpoint of the observer, and less at the position of the cloud. The movement of the sun from east to west during the appearance of the rainbow may be left out of account.

CHAPTER VIII

The colours in the bow arise from an ocular deception. They are analogous to those which appear when the eyes are weak or half-shut. They are not due to the same cause as the colours produced when light shines through a crystal, since these do not, like the colours of the rainbow, shift with the position of the observer.

CHAPTER IX

Each drop of rain in the cloud is to be regarded as a spherical mirror; these being small and close together, the effect is that of a continuous image rather than of a multitude of images. The colour is due to the distortion of the image caused by the sphericity of the mirror.

CHAPTER X

The diversity of colours has been attributed to varieties in the texture of the cloud, the denser parts producing violet and blue, the lighter parts red and orange. But we see the same colours in the dew drops, where there can be no such differences of density; similarly in the crystal. Aristotle has been wrongly translated and interpreted in this matter. Another erroneous belief is that lunar rainbows occur only once in fifty years. They may occur at any full moon under suitable atmospheric conditions.

CHAPTER XI

The shape of the bow is a difficulty. It cannot be explained by refraction. It is to be observed that the same colour is continued all round the circle in each ring. All parts of the ring therefore preserve the same relation of the solar ray to the eye. This implies circularity of form. It is asked why the whole space contained by the circle is not occupied with colour. Because from the points in this central area rays equal to the angle of incidence are not reflected to the eye.

CHAPTER XII

The cloud therefore is not coloured; the appearance of colour, for it is only an appearance, is given by rays reflected from the raindrops. Of colours there are five, white, blue, red, green, black; though Aristotle, dividing blue and green into other shades, speaks of seven. These colours appear to have some relation to the various structures of the eye. In addition to the problem of the rainbow, there is the problem of haloes and coronal. On this I give the best explanation that as yet occurs to me. I do not however, pretend that it is satisfactory. Far more careful experiments, made with properly constructed instruments, are needed before an adequate explanation can be given.

THE SECOND PREROGATIVE OF EXPERIMENTAL SCIENCE

In all sciences Experiment is able to reveal truths quite unconnected with the discussion of principles, and with regard to which it is useless in the first instance to assign a reason. The initial state of mind should be readiness to believe; this should be followed by experiment: reasoning should come last. I subjoin examples of my meaning.

1. The astronomer constructs his spherical astrolabe, by which he can observe the precise longitude and latitude of heavenly bodies at different times. But it is not inconceivable that experiment may devise means of bringing this instrument into such relation with the

revolution of the heavens that it should follow their course. The motion of the tides, the periodic changes in certain diseases, the diurnal opening and closing of flowers, are facts tending to belief that such a discovery is possible. If effected it would supersede all other astronomical instruments. 2. My next example relates to the act of prolonging human life. As yet we have nothing to rely on but ordinary rules of health. These are observed but by few, and usually not till the close of life, when it is too late. If a suitable regimen were observed by all, no doubt life would be much prolonged. But there are special remedies unknown as yet to medicine, but to be found by experiment, which may extend the period of life much further. Observation of the habits of certain animals may guide us to truths in this matter which are as yet hidden. Other indications are given in the works of Aristotle, Pliny, Artephius, and others. A combination of gold, pearl, flower of sea-dew, spermaceti, aloes, bone of stag's heart, flesh of Tyrian snake and of Æthiopian dragon, properly prepared in due proportions, might promote longevity to an extent hitherto unimagined.

3. A third example may be found in Alchemy. The problem here is not merely to transmute the baser into the more precious metals, but to promote gold to its highest degree of perfection. In this perfected gold we should probably have a further aid to the prolongation of life.

THIRD PREROGATIVE OF EXPERIMENTAL SCIENCE

In this we leave altogether the domain of the sciences now recognized, and open out entirely new departments of research. At present the influences exerted on us by the stars can only be known through difficult astronomical calculations. Experimental science may enable us to estimate them directly. It may be possible for us to act on the character of the inhabitants of any region by altering their environment. Inventions of the greatest utility may be discovered, as perpetual fire, or explosive substances, or modes of counteracting dangerous poisons, and innumerable other properties of matter as yet unknown for want of experiment. The Magnet, of which use is already made, is but a type of other mutual attractions exerted by bodies at a distance. For instance, if a young sapling be longitudinally divided and the two divisions be brought near together, held each by the middle, the extremities will bend towards each other. In conclusion, I may point out the influence which the possessors of this science may exercise in the promotion of Christianity among the heathen, whether in subduing their pride, in disabusing them of false beliefs in magic, or in overcoming their material force.

C. DEDICATION OF *THE REVOLUTIONS OF THE HEAVENLY BODIES* BY NICOLAUS COPERNICUS (1543)

To Pope Paul III

I can easily conceive, most Holy Father, that as soon as some people learn that in this book which I have written concerning the revolutions of the heavenly bodies, I ascribe certain motions to the Earth, they will cry out at once that I and my theory should be rejected. For I am not so much in love with my conclusions as not to weigh what others will think about them, and although I know that the meditations of a philosopher are far removed from the judgment of the laity, because his endeavor is to seek out the truth in all things, so far as this is permitted by God to the human reason, I still believe that one must avoid theories altogether foreign to orthodoxy. Accordingly, when I considered in my own mind how absurd a performance it must seem to those who know that the judgment of many centuries has approved the view that the Earth remains fixed as centre in the midst of the heavens, if I should on the contrary, assert that the Earth moves; I was for a long time at a loss to know whether I should publish the commentaries which I have written in proof of its motion, or whether it were not better to follow the example of the Pythagoreans and of some others, who were accustomed to transmit the secrets of philosophy not in writing but orally, and only to their relatives and friends, as the letter from Lysis to Hipparchus bears witness. They did this, it seems to me, not as some think, because of a certain selfish reluctance to give their views to the world, but in order that the noblest truths, worked out by the careful study of great men, should not be despised by those who are vexed at the idea of taking great pains with any form of literature except such as would be profitable, or by those who, if they are driven to the study of philosophy for its own sake by the admonitions and the example of others, nevertheless, on account of their stupidity, hold a place among philosophers similar to that of drones among bees. Therefore, when I considered this carefully, the contempt which I had to fear because of the novelty and apparent absurdity of my view, nearly induced me to abandon utterly the work I had begun.

My friends, however, in spite of long delay and even resistance on my part, withheld me from this decision. First among these was Nicolaus Schonberg, Cardinal of Capua, distinguished in all branches of learning. Next to him comes my very dear friend, Tidemann Giese, Bishop of Culm, a most earnest student, as he is, of sacred and, indeed, of all good learning. The latter has often urged me, at times even spurring me on with reproaches, to publish and at last bring to light the book which had lain in my study not nine years merely, but already going on four times nine. Not a few other very eminent and scholarly men made the same request, urging that I should no longer through fear refuse to give out my work for the common benefit of students of Mathematics. They said I should find that the more absurd most men now thought this theory of mine concerning the motion of the Earth, the more admiration and gratitude it would command after they saw in the publication of my commentaries, the mist of absurdity cleared away by most transparent proofs. So, influenced by these advisers and this hope, I have at length allowed my friends to publish the work, as they had long besought me to do.

But perhaps your Holiness will not so much wonder that I have ventured to publish these studies of mine, after having taken such pains in elaborating them that I have not hesitated to commit to writing my views of the motion of the Earth, as you will be curious to hear how it occurred to me to venture, contrary to the accepted view of mathematicians, and well-nigh contrary to common sense, to form a conception of any terrestial motion whatsoever. Therefore I would not have it unknown to Your Holiness, that the only thing which induced me to look for another way of reckoning the movements of the heavenly bodies was that I knew that mathematicians by no means agree in their investigations thereof. For, in the first place, they are so much in doubt concerning the motion of the sun and the moon, that they cannot even demonstrate and prove by observation the constant length of a complete year; and in the second place, in determining the motions both of these and of the other five planets, they fail to employ consistently one set of first principles and hypotheses, but use methods of proof based only upon the apparent revolutions and motions. For some employ concentric circles only; others, eccentric circles and epicycles; and even by these means they do not completely attain the desired end. For, although those who have depended upon concentric circles have shown that certain diverse motions can be deduced from these, yet they have not succeeded thereby in laying down any sure principle, corresponding indisputably to the phenomena. These, on the other hand, who have devised systems of eccentric circles, although they seem in great part to have solved the apparent movements by calculations which by these eccentrics are made to fit, have nevertheless introduced many things which seem to contradict the first principles of the uniformity of motion. Nor have they been able to discover or calculate from these the main point, which is the shape of the world and the fixed symmetry of its parts; but their procedure has been as if someone were to collect hands, feet, a head, and other members from various places, all very fine in themselves, but not proportionate to one body, and no single one corresponding in its turn to the others, so that a monster rather than a man would be formed from them. Thus in their process of demonstration which they term a "method," they are found to have omitted something essential, or to have included something foreign and not pertaining to the matter in hand. This certainly would never have happened to them if they had followed fixed principles; for if the hypotheses they assumed were not false, all that resulted therefrom would be verified indubitably. Those things which I am saying now may be obscure, yet they will be made clearer in their proper place.

Therefore, having turned over in my mind for a long time this uncertainty of the traditional mathematical methods of calculating the motions of the celestial bodies, I began to grow disgusted that no more consistent scheme of the movements of the mechanism of the universe, set up for our benefit by that best and most law-abiding Architect of all things, was agreed upon by philosophers who otherwise investigate so carefully the most minute details of this world. Wherefore I undertook the task of rereading the books of all the philosophers I could get access to, to see whether anyone ever was of the opinion that the motions of the celestial bodies were other than those postulated by the men who taught mathematics in the schools. And I found first, indeed, in Cicero, that Hicetas perceived that the Earth moved; and afterward in Plutarch I found that some others were of this opinion, whose words I have seen fit to quote here, that they may be accessible to all: —

> Some maintain that the Earth is stationary, but Philolaus the Pythagorean says that it revolves in a circle about the fire of the ecliptic, like the sun and moon. Heraklides of Pontus and Ekphantus the Pythagorean make the Earth move, not changing its position, however, confined in its falling and rising around its own centre in the manner of a wheel.

Taking this as a starting-point, I began to consider the mobility of the Earth; and although the idea seemed absurd, yet because I knew that the liberty had been granted to others before me to postulate all sorts of little circles for explaining the phenomena of the stars, I thought I also might easily be permitted to try whether by postulating some motion of the Earth, more reliable conclusions could be reached regarding the revolution of the heavenly bodies, than those of my predecessors.

And so, after postulating movements, which, farther on in the book, I ascribe to the Earth, I have found by many and long observations that if the movements of the other planets are assumed for the circular motion of the Earth and are substituted for the revolution of each star, not only do their phenomena follow logically therefrom, but the relative positions and magnitudes both of the stars and all their orbits, and of the heavens themselves, become so closely related that in none of its parts can anything be changed without causing confusion in the other parts and in the whole universe. Therefore, in the course of the work I have followed this plan: I describe in the first book all the positions of the orbits together with the movements which I ascribe to the Earth, in order that this book might contain, as it were, the general scheme of the universe. Thereafter in the remaining books, I set forth the motions of the other stars and of all their orbits together with the movement of the Earth, in order that one may see from this to what extent the movements and appearances of the other stars and their orbits can be saved, if they are transferred to the movement of the Earth. Nor do I doubt that ingenious and learned mathematicians will sustain me, if they are willing to recognize and weigh, not superficially, but with that thoroughness which Philosophy demands above all things, those matters which have been adduced by me in this work to demonstrate these theories. In order, however, that both the learned and the unlearned equally may see that

I do not avoid anyone's judgment, I have preferred to dedicate these lucubrations of mine to Your Holiness rather than to any other, because, even in this remote corner of the world where I live, you are considered to be the most eminent man in dignity of rank and in love of all learning and even of mathematics, so that by your authority and judmgent you can easily suppress the bites of slanderers, albeit the proverb hath it that there is no remedy for the bite of a sycophant. If perchance there shall be idle talkers, who, though they are ignorant of all mathematical sciences, nevertheless assume the right to pass judgment on these things, and if they should dare to criticise and attack this theory of mine because of some passage of scripture which they have falsely distorted for their own purpose, I care not at all; I will even despise their judgment as foolish. For it is not unknown that Lactantius, otherwise a famous writer but a poor mathematician, speaks most childishly of the shape of the Earth when he makes fun of those who said that the Earth has the form of a sphere. It should not seem strange then to zealous students, if some such people shall ridicule us also. Mathematics are written for mathematicians, to whom, if my opinion does not deceive me, our labors will seem to contribute something to the ecclesiastical state whose chief office Your Holiness now occupies; for when not so very long ago, under Leo X, in the Lateran Council the question of revising the ecclesiastical calendar was discussed, it then remained unsettled, simply because the length of the years and the months, and the motions of the sun and moon were held to have been not yet sufficiently determined. Since that time, I have given my attention to observing these more accurately, urged on by a very distinguished man, Paul, Bishop of Fossombrone, who at that time had charge of the matter. But what I may have accomplished herein I leave to the judgment of Your Holiness in particular, and to that of all other learned mathematicians; and lest I seem to Your Holiness to promise more regarding the usefulness of the work than I can perform, I now pass to the work itself.

(*— From the Harvard Classics, Vol. 39, 55–60.*)

D. HARVEY'S DEDICATION OF HIS WORK ON THE MOTION OF THE HEART AND THE CIRCULATION OF THE BLOOD (1628)

TO HIS VERY DEAR FRIEND, DOCTOR ARGENT, THE EXCELLENT AND ACCOMPLISHED PRESIDENT OF THE ROYAL COLLEGE OF PHYSICIANS, AND TO OTHER LEARNED PHYSICIANS, HIS MOST ESTEEMED COLLEAGUES

I have already and repeatedly presented you, my learned friends, with my new views of the motion and function of the heart, in my anatomical lectures; but having now for more than nine years confirmed these views by multiplied demonstrations in your presence, illustrated them by arguments, and freed them from the objections of the most learned and skilful anatomists, I at length yield to the requests, I might say entreaties, of many, and here present them for a general consideration in this treatise.

Were not the work indeed presented through you, my learned friends, I should scarce hope that it could come out scatheless and complete; for you have in general been the faithful witnesses of almost all the instances from which I have either collected the truth or confuted error. You have seen my dissections, and at my demonstrations of all that I maintained to be objects of sense, you have been accustomed to stand by and bear me out with your testimony. And as this book alone declares the blood to course and revolve by a new route, very different from the ancient and beaten pathway trodden for so many ages, and illustrated by such a host of learned and distinguished men, I was greatly afraid lest I might be charged with presumption did I lay my work before the public at home, or send it beyond seas for impression, unless I had first proposed the subject to you, had confirmed its conclusions by ocular demonstrations in your presence, had replied to your doubts and objections, and secured the assent and support of our distinguished President. For I was most intimately persuaded, that if I could make good my proposition before you and our College, illustrious by its numerous body of learned individuals, I had less to fear from others. I even ventured to hope that I should have the comfort of finding all that you had granted me in your sheer love of truth, conceded by others who were philosophers like yourselves. True philosophers, who are only eager for truth and knowledge, never regard themselves as already so thoroughly informed, but that they welcome further information from whomsoever and from wheresoever it may come; nor are they so narrow minded as to imagine any of the arts or sciences transmitted to us by the ancients, in such a state of forwardness or completeness, that nothing is left for the ingenuity and industry of others. On the contrary, very many maintain that all we know is still infinitely less than all that still remains unknown; nor do philosophers pin their faith to others' precepts in such wise that they lose their liberty, and cease to give credence to the conclusions of their proper senses. Neither do they swear such fealty to their mistress Antiquity that they openly, and in sight of all, deny and desert their friend Truth. But even as they see that the credulous and vain are disposed at the first blush to accept and believe everything that is proposed to them, so do they observe that the dull and unintellectual are indisposed to see what lies before their eyes, and even deny the light of the noon-day sun. They teach us in our course of philosophy to sedulously avoid the fables of poets and the fancies of the vulgar, as the false conclusions of the sceptics. And then the studious and good and true, never suffer their minds to be warped by the passions of hatred and envy, which unfit men duly to weigh the arguments that are advanced in behalf of truth, or to appreciate the proposition that is even fairly demonstrated. Neither do they think it unworthy of them to change their opinion if truth and undoubted demonstration require them to do so. They do not esteem it discreditable to desert error, though sanctioned by the highest antiquity, for they know full well that to err, to be deceived, is human; that many things are discovered by accident and that many may be learned indifferently from any quarter, by an old man from a youth, by a person of understanding from one of inferior capacity.

My dear colleagues, I had no purpose to swell this treatise into a large volume by quoting the names and writings of anatomists, or to make a parade of the strength of my memory, the extent of my reading, and the amount of my pains; because I profess both to learn and to teach anatomy, not from books but from dissections; not from the positions of philosophers but from the fabric of nature; and then because I do not think it right or proper to strive to take from the ancients any honor that is their due, nor yet to dispute with the moderns, and enter into controversy with those who have excelled in anatomy and been my teachers. I would not charge with wilful falsehood anyone who was sincerely anxious for truth, nor lay it to anyone's door as a crime that he had fallen into error. I avow myself the partisan of truth alone; and I can indeed say that I have used all my endeavors, bestowed all my pains on an attempt to produce something that should be agreeable to the good, profitable to the learned, and useful to letters.

Farewell, most worthy Doctors,
And think kindly of your Anatomist
WILLIAM HARVEY.

E. GALILEO BEFORE THE INQUISITION (1633)

I. HIS CONDEMNATION

We, GASPARO del titolo di S. Croce, in Gierusalemme Borgia;
FRA FELICE CENTINO del titolo di S. Anastasia, detto d'Ascoli;
GUIDO del titolo di S. Maria del Popolo Bentivoglio;
FRA DESIDERIO SCAGLIA del titolo di S. Carlo, detto di Cremona;
FRA ANTONIO BARBERINO, detto di S. Onofrio;
LAUDIVIO ZACCHIA del titolo di S. Pietro in Vincolo, detto di S. Sisto;
BERLINGERO del titolo di S. Agostino, Gessi;
FABRICIO del titolo di S. Lorenzo in pane e perna, Verospi, chiamato Prete;
FRANCESCO di S. Lorenzo, in Damaso Barberino; e
MARTINO di S. Maria Nuova, Ginetti Diaconi;
by the Grace of God, Cardinals of the Holy Roman Church, Inquisitors-General throughout the whole Christian Republic, Special Deputies of the Holy Apostolical Chair against heretical depravity,

Whereas you, Galileo, son of the late Vincenzo Galilei of Florence, aged seventy years, were denounced in 1615 to this Holy Office, for holding as true the false doctrine taught by many, namely, that the sun is immovable in the centre of the world, and that the earth moves, and also with a diurnal motion; also for having pupils whom you instructed in the same opinions; also for maintaining a correspondence on the same with some German mathematicians; also for publishing certain letters on the solar spots, in which you developed the same doctrine as truth; also for answering the objections which were continually produced from the Holy Scriptures, by glozing the said Scriptures according to your own meaning; and whereas thereupon was produced the copy of a writing, in form of a letter, confessedly written by you to a person formerly your pupil, in which, following the hypotheses of Copernicus, you include several propositions contrary to the true sense and authority of the Holy Scripture:

Therefore this holy tribunal being desirous of providing against the disorder and mischief thence proceeding and increasing to the detriment of the holy faith, by the desire of His Holiness, and of the Most Eminent Lords Cardinals of this supreme and universal Inquisition, the two propositions of the stability of the sun, and the motion of the earth, were qualified by the Theological Qualifiers as follows:

The proposition that the sun is the centre of the world and immovable from its place is absurd, philosophically false, and formally heretical, because it is expressly contrary to the Holy Scripture.

The proposition that the earth is not the centre of the world, nor immovable, but that it moves, and also with a diurnal motion, is also absurd, philosophically false, and, theologically considered, at least erroneous in faith.

But whereas being pleased at that time to deal mildly with you, it was decreed in the Holy Congregation, held before His Holiness on the 25th day of February, 1616, that His Eminence, the Lord Cardinal Bellarmine, should enjoin you to give up altogether the said false doctrine; if you should refuse, that you should be ordered by the Commissary of the Holy Office to relinquish it, not to teach it to others, not to defend it, nor ever mention it, and in default of acquiescence that you should be imprisoned; and in execution of this decree, on the following day at the palace, in presence of His Eminence the said Lord Cardinal Bellarmine, after you had been mildly admonished by the said Lord Cardinal, you were commanded by the acting Commissary of the Holy Office, before a notary and witnesses to relinquish altogether the said false opinion, and in future neither to defend nor teach it in any manner, neither verbally nor in writing, and upon your promising obedience, you were dismissed.

And in order that so pernicious a doctrine might be altogether rooted out, nor insinuate itself further to the heavy detriment of the Catholic faith, a decree emanated from the Holy Congregation of the Index prohibiting the books which treat of this doctrine; and it was declared false and altogether contrary to the Holy and Divine Scripture.

And whereas a book has since appeared, published at Florence last year, the title of which showed that you were the author, which is: The Dialogue of Galileo Galilei, on the Two Principal Systems of the World, the Ptolemaic and Copernican; and whereas the Holy Congregation has heard that, in consequence of the printing of the said book, the false opinion of the earth's motion and stability of the sun is daily gaining ground; the said book has been taken into careful consideration, and in it has been detected a glaring violation of the said order, which had been intimated to you; inasmuch as in this book you have defended the said opinion, already in your presence condemned; although in the said book you labor with many circumlocutions to produce the belief that it is left by you undecided, and in express terms probable; which is equally a very grave error, since an opinion can in no way be probable which has been already declared and finally determined contrary to the Divine Scripture:

Therefore by our order you were cited to this Holy Office, where, on your examination upon oath, you acknowledged the said book as written and printed by you. You also confessed that you began to write the said book ten or twelve years ago, after the order aforesaid had been given; also, that you demanded licence to publish it, but without signifying to those who granted you this permission that you had been commanded not to hold, defend or teach the said doctrine in any manner.

You also confessed that the style of the said book was, in many places, so composed that the reader might think the arguments adduced on the false side to be so worded as more effectually to entangle the understanding than to be easily solved, alleging in excuse, that you have thus run into an error, foreign (as you say) to your intention, from writing in the form of a dialogue, and in consequence of the natural complacency which everyone feels in regard to his own subtilities, and in showing himself more skilful than the generality of mankind in contriving, even in favor of false propositions, ingenious and apparently probable arguments.

And, upon a convenient time having been given to you for making your defense, you produced a certificate in the handwriting of His Eminence the Lord Cardinal Bellarmine, procured as you said, by yourself, that you might defend yourself against the calumnies of your enemies, who reported that you had abjured your opinions, and had been punished by the Holy Office; in which certificate it is declared that you had not abjured, nor had been punished, but merely that the declaration made by His Holiness, and promulgated by the Holy Congregation of the Index, had been announced to you, which declares that the opinion of the motion of the earth, and the stability of the sun, is contrary to the Holy Scriptures, and, therefore, cannot be held or defended. Wherefore, since no mention is there made of two articles of the order, to wit, the order "not to teach," and "in any manner," you argued that we ought to believe that, in the lapse of fourteen or sixteen years, they had escaped your memory, and that this was also the reason why you were so silent as to the order, when you sought permission to publish your book, and that this is said by you not to excuse your error, but that it may be attributed to vainglorious ambition, rather than to malice. But this very certificate, produced on your behalf, has greatly aggravated your offense, since it is therein declared that the said opinion is contrary to the Holy Scripture; and yet you have dared to treat of it, to defend it, and to argue that it is probable; nor is there any extenuation in the licence artfully and cunningly extorted by you, since you did not intimate the command imposed upon you.

But whereas it appeared to us that you had not disclosed the whole truth in regard to your intentions, We thought it necessary to proceed to the rigorous examination of you, in which (without any prejudice to what you had confessed, and which is above detailed against you, with regard to your said intention) you answered like a good Catholic. Therefore, having seen and maturely considered the merits of your cause, with your said confessions and excuses, and everything else which ought to be seen and considered, We have come to the underwritten final sentence against you.

Invoking, therefore, the Most Holy name of Our Lord Jesus Christ, and of His Most Glorious Virgin Mother Mary, by this our final sentence, which, sitting in council and judgment for the tribunal of the Reverend Masters of Sacred Theology, and Doctors of Both Laws, Our Assessors, We put forth in this writing touching the matters and controversies before us, between the Magnificent Charles Sincerus, Doctor of Both Laws, Fiscal Proctor of this Holy Office, of the one part, and you, Galileo Galilei, an examined and confessed criminal from this present writing as above, of the other part. We pronounce, judge and declare, that you, the said Galileo, by reason of these things which have been detailed in the course of this writing, and which, as above, you have confessed, have rendered yourself vehemently suspected by this Holy Office of heresy; that is to say, that you believe and hold the false doctrine, and contrary to the Holy and Divine Scriptures, namely, that the sun is the centre of the world, and that it does not move from the east to west, that the earth does move and is not the centre of the world; also that an opinion can be held and supported as probable after it has been declared and finally decreed contrary to the Holy Scripture, and consequently that you have incurred all the censures and penalties enjoined and promulgated in the sacred canons, and other general and particular constitutions, against delinquents of this description. From which it is Our pleasure that you be absolved, provided that, first, with a sincere heart and unfeigned faith, in Our presence, you abjure, curse, and detest the said errors and heresies and every other error and heresy contrary to the Catholic and Apostolic Church of Rome, in the form now shown to you.

But, that your grievous and pernicious error and transgression may not go altogether unpunished, and that you may be made more cautious in the future, and may be a warning to others to abstain from delinquencies of this sort, We decree that the book of the Dialogues of Galileo Galilei be prohibited by a public edict.

We condemn you to the formal prison of this Holy Office for a period determinable at Our pleasure; and by way of salutary penance, We order you, during the next three years, to recite once a week the seven Penitential Psalms.

Reserving to ourselves the power of moderating, commuting, or taking off the whole or part of the said punishment or penance.

And so We say, pronounce, and by Our sentence declare, decree, and reserve, in this and every other better form and manner, which lawfully we may and can use.

So we, the undersigned Cardinals, pronounce.

FELICE, *Cardinal di Ascoli,*
GUIDO, *Cardinal Bentivoglio,*
DESIDERIO, *Cardinal di Cremona,*
ANTONIO, *Cardinal S. Onofrio,*
BERLINGERO, *Cardinal Gessi,*
FABRICIO, *Cardinal Verospi,*
MARTINO, *Cardinal Ginetti.*

II. HIS RECANTATION

I Galileo Galilei, son of the late Vincenzo Galilei of Florence, aged seventy years, being brought personally to judgment, and kneeling before you, Most Eminent and Most Reverend Lords Cardinals, General Inquisitors of the Universal Christian Republic against heretical depravity, having before my eyes the Holy Gospels, which I touch with my own hands, swear, that I have always believed, and now believe, and with the help of God will in future believe, every article which the Holy Catholic and Apostolic Church of Rome holds, teaches and preaches. But because I had been enjoined by this Holy Office altogether to abandon the false opinion which maintains that the sun is the centre and immovable, and forbidden to hold, defend, or teach the said false doctrine in any manner, and after it had been signified to me that the said doctrine is repugnant with the Holy Scripture, I have written and printed a book, in which I treat of the same doctrine now condemned, and adduce reasons with great force in support of the same, without giving any solution, and therefore, have been judged grievously suspected of heresy; that is to say, that I have held and believed that the sun is the centre of the world and immovable, and that the earth is not the centre and movable.

Wishing, therefore, to remove from the mind of Your Eminences, and of every Catholic Christian, this vehement suspicion rightfully entertained towards me, with a sincere heart and unfeigned faith, I abjure, curse and detest the said errors and heresies, and generally every other error and sect contrary to the said Holy Church; and I swear that I will nevermore in future say or assert anything verbally, or in writing, which may give rise to a similar suspicion of me; but if I shall know any heretic, or anyone suspected of heresy, that I will denounce him to this Holy Office, or to the Inquisitor and Ordinary of the place in which I may be.

I swear, moreover, and promise, that I will fulfill, and observe fully, all the penances which have been or shall be laid to me by this Holy Office. But if it shall happen that I violate any of my said promises, oaths and protestations (which God avert!), I subject myself to all the pains and punishments which have been decreed and promulgated by the sacred canons, and other general and particular constitutions, against delinquents of this description. So may God help me, and His Holy Gospels, which I touch with my own hands.

I, the above-named Galileo Galilei, have abjured, sworn, promised and bound myself, as above, and in witness thereof with my own hand have subscribed this present writing of my abjuration, which I have recited word for word. At Rome in the Convent of Minerva, 22nd June, 1633.

I, Galileo Galilei, have abjured as above with my own hand.

— From Routledge. *History of Science.* See also K. von Gebler. *Galileo Galilei and the Roman Curia. London, 1879.*

F. PREFACE TO THE *PHILOSOPHIÆ NATURALIS PRINCIPIA MATHEMATICA* (1686)

By Isaac Newton

Since the ancients (as we are told by Pappus) made great account of the science of mechanics in the investigation of natural things; and the moderns, laying aside substantial forms and occult qualities, have endeavored to subject the phenomena of nature to the laws of mathematics, I have in this treatise cultivated mathematics so far as it regards philosophy. The ancients considered mechanics in a twofold respect; as rational, which proceeds accurately by demonstration, and practical. To practical mechanics all the manual arts belong, from which mechanics took its name. But as artificers do not work with perfect accuracy, it comes to pass that mechanics is so distinguished from geometry, that what is perfectly accurate is called geometrical; what is less so is called mechanical. But the errors are not in the art but in the artificers. He that works with less accuracy is an imperfect mechanic: and if any could work with perfect accuracy, he would be the most perfect mechanic of all; for the description of right lines and circles, upon which geometry is founded, belongs to mechanics. Geometry does not teach us to draw these lines, but requires them to be drawn; for it requires that the learner should first be taught to describe these accurately, before he enters upon geometry; then it shows how by these operations problems may be solved. To describe right lines and circles are problems, but not geometrical problems. The solution of these problems is required from mechanics; and by geometry the use of them, when so solved, is shown; and it is the glory of geometry that from those few principles, fetched from without, it is able to produce so many things. Therefore geometry is founded in mechanical practice and is nothing but that part of universal mechanics which accurately proposes and demonstrates the art of measuring. But since the manual arts are chiefly conversant in the moving of bodies, it comes to pass that geometry is commonly referred to their magnitudes, and mechanics to their motion. In this sense rational mechanics will be the science of motions resulting from any forces whatsoever, and of the forces required to produce any motion, accurately proposed and demonstrated. This part of mechanics was cultivated by the ancients in the five powers which relate to manual arts, who considered gravity (it not being a manual power) no otherwise than as it moved weights by those powers. Our design, not respecting arts, but philosophy, and our subject, not manual, but natural, powers, we consider chiefly those things which relate to gravity, levity, elastic force, the resistance of fluids, and the like forces, whether attractive or impulsive; and therefore we offer this work as mathematical principles of philosophy; for all the difficulty of philosophy seems to consist in this — from the phenomena of motions to investigate the forces of nature, and then from these forces to demonstrate the other phenomena; and to this end the general propositions in the first and second book are directed. In the third book we give an example of this in the explication of the system of the World; for by the propositions mathematically demonstrated in the first book, we there derive from the celestial phenomena the forces of gravity with which bodies tend to the sun and the several planets. Then, from these forces, by other propositions which are also mathematical, we deduce the motions of the planets, the comets, the moon, and the sea. I wish we could derive the rest of the phenomena of nature by the same kind of reasoning from mechanical principles; for I am induced by many reasons to suspect that they may all depend upon certain forces by which the particles of bodies, by some causes hitherto unknown, are either mutually impelled towards each other, and cohere in regular figures, or are repelled and recede from each other; which forces being unknown, philosophers have hitherto attempted the search of nature in vain; but I hope the principles here laid down will afford some light either to that or some truer method of philosophy.

In the publication of this work, the most acute and universally learned Mr. Edmund Halley not only assisted me with his pains in correcting the press and taking care of the schemes, but it was to his solicitations that its becoming public is owing; for when he had obtained of me my demonstrations of the figures of the celestial orbits, he continually pressed me to communicate the same to the Royal

Society, who afterwards, by their kind encouragement and entreaties, engaged me to think of publishing them. But after I had begun to consider the inequalities of the lunar motions, and had entered upon some other things relating to the laws and measures of gravity, and other forces; and the figures that would be described by bodies attracted according to given laws; and the motion of several bodies moving among themselves; the motion of bodies in resisting mediums; the forces, densities, and motions of mediums; the orbits of the comets, and such like; I put off that publication until I had made a search into those matters, and could put out the whole together. What relates to the lunar motions (being imperfect) I have put all together in the corollaries of proposition 66, to avoid being obliged to propose and distinctly demonstrate the several things there contained in a method more prolix than the subject deserved, and interrupt the series of the several propositions. Some things, found out after the rest, I chose to insert in places less suitable, rather than change the number of the propositions and the citations. I heartily beg that what I have here done may be read with candor; and that the defects I have been guilty of upon this difficult subject may be not so much reprehended as kindly supplied, and investigated by new endeavors of my readers.

Isaac Newton.

Cambridge, Trinity College,
May 8, 1686.

(— *Translation by Andrew Motte. The Harvard Classics, Vol. 39, pp. 157–159.*)

G. AN INQUIRY INTO THE CAUSES AND EFFECTS OF THE *VARIOLÆ VACCINÆ*, A DISEASE DISCOVERED IN SOME OF THE WESTERN COUNTIES OF ENGLAND, PARTICULARLY GLOUCESTERSHIRE AND KNOWN BY THE NAME OF THE COW POX

By Edward Jenner, M.D., F.R.S., etc.

[*The first successful attempt — and this wholly empirical — to control smallpox in the human subject was the art of Inoculation with the virus of smallpox itself, a procedure derived from the East, and introduced about 1720 into Europe and America. In 1796 Jenner laid the foundation of experimental medicine, immunology and serology, by his work on Vaccination, i.e., inoculation with cow-pox, in a paper bearing the above title. The first edition was published in 1798 and the second, from which the following extracts are taken, in 1800.*]

The deviation of Man from the state in which he was originally placed by Nature seems to have proved to him a prolific source of Disease. From the love of splendour, from the indulgences of luxury, and from his fondness for amusement, he has familiarised himself with a great number of animals, which may not originally have been intended for his associates. The Wolf, disarmed of ferocity, is now pillowed in the lady's lap. The Cat, the little Tyger of our island, whose natural home is the forest, is equally domesticated and caressed. The Cow, the Hog, the Sheep, and the Horse, are all, for a variety of purposes, brought under his care and dominion.

There is a disease to which the Horse, from his state of domestication, is frequently subject. The Farriers have termed it the Grease. It is an inflammation and swelling of the heel, from which issues matter possessing properties of a very peculiar kind, which seems capable of generating a disease in the Human Body (after it has undergone the modification I shall presently speak of), which bears so strong a resemblance to the Small Pox, that I think it highly probable it may be the source of that disease.

In this Dairy Country a great number of cows are kept, and the office of milking is performed indiscriminately by men and maid servants. One of the former having been appointed to apply dressings to the heels of a horse affected with the Grease, and not paying due attention to cleanliness, incautiously bears his part in milking the cows, with some particles of the infectious matter adhering to his fingers. When this is the case, it commonly happens that a disease is communicated to the cows, and from the cows to the dairymaids, which spreads through the farm until most of the cattle and domestics feel its unpleasant consequences. This disease has obtained the name of Cow Pox. It appears on the nipples of the cows in the form of irregular pustules. At their first appearance they are commonly of a palish blue, or rather of a colour somewhat approaching to livid, and are surrounded by an inflammation. These pustules, unless a timely remedy be applied, frequently degenerate into phagedenic ulcers, which prove extremely troublesome. The animals become indisposed, and the secretion of milk is much lessened. Inflamed spots now begin to appear on different parts of the hands of the domestics employed in milking, and sometimes on the wrists, which quickly run on to suppuration, first assuming the appearance of the small vesications produced by a burn. Most commonly they appear about the joints of the fingers and at their extremities; but whatever parts are affected, if the situation will admit, these superficial suppurations put on a circular form, with their edges more elevated than their center, and of a colour distantly approaching to blue. Absorption takes place, and tumours appear in each. axilla. The system becomes affected, the pulse is quickened; shivering, succeeded by heat, general lassitude and pains about the loins and limbs, with vomiting, come on. The head is painful, and the patient is now and then affected with delirium. These symptoms, varying in their degrees of violence, generally continue from one day to three or four, leaving ulcerated sores about the hands, which, from the sensibility of the parts, are very troublesome, and commonly heal slowly, frequently becoming phagedenic, like those from whence they sprung. The lips, nostrils, eyelids, and other parts of the body, are somtimes affected with sores; but these evidently arise from their being heedlessly rubbed or scratched with the patient's infected fingers. No eruptions on the skin have followed the decline of the feverish symptoms in any instance that has come under my inspection, one only excepted, and in his case a very few appeared on the arms: they were very minute, of a vivid red colour, and soon died away without advancing to maturation; so that I cannot determine whether they had any connection with the preceding symptoms.

Thus the disease makes its progress from the Horse (as I conceive) to the nipple of the Cow, and from the Cow to the Human Subject.

Morbid matter of various kinds, when absorbed into the system, may produce effects in some degree similar; but what renders the Cow Pox virus so extremely singular, is, that the person who has been thus affected is forever after secure from the infection of the Small Pox; neither exposure to the variolous effluvia, nor the insertion of the matter into the skin, producing this distemper.

In support of so extraordinary a fact, I shall lay before my reader a great number of instances: but first it is necessary to observe, that pustulous sores frequently appear spontaneously on the nipples of the cows, and instances have occurred, though very rarely, of the hands of the servants employed in milking being affected with sores in consequence, and even of their feeling an indisposition from absorption. These pustules are of a much milder nature than those which arise from that contagion which constitutes the true Cow Pox. They are always free from the bluish or livid tint so conspicuous in the pustules in that disease. No erysipelas attends them, nor do they shew any phagedenic disposition as in the other case, but quickly terminate in a scab without creating any apparent disorder in the Cow. This complaint appears at various seasons of the year, but most commonly in the spring, when the Cows are first taken from their winter food and fed with grass. It is very apt to appear also when they are suckling their young. But this disease is not considered as similar in any respect to that of which I am treating, as it is incapable of producing any specific effects on the human constitution. However, it is of the greatest consequence to point it out here, lest the want of discrimination should occasion an idea of security from the infection of the Small Pox, which might prove delusive.

[*Hereupon follow detailed descriptions of numerous cases illustrating Jenner's ideas. Of these we quote only two: Case I, illustrating Jenner's observations of immunity to smallpox acquired naturally by accidental inoculation or vaccination with cowpox virus, and Case XVII, his first and therefore most famous example of experimental inoculation* (*or vaccination*) *upon the person of a boy named James Phipps. This, the earliest experiment of the kind ever made, occurred on May 14, 1796.*]

CASE I

Joseph Merret, now an Under Gardener of the Earl of Berkeley, lived as a servant with a Farmer near this place in the year 1770, and occasionally assisted in milking his master's cows. Several horses belonging to the farm began to have sore heels, which Merret frequently attended. The cows soon became affected with the Cow Pox, and soon after several sores appeared on his hands. Swellings and stiffness in each axilla followed, and he was so much indisposed for several days as to be incapable of pursuing his ordinary employment. Previously to the appearance of the distemper among the cows there was no fresh cow brought into the farm, nor any servant employed who was affected with the Cow Pox.

In April, 1795, a general inoculation taking place here, Merret was inoculated with his family; so that a period of twenty-five years had elapsed from his having the Cow Pox to this time. However, though the variolous matter was repeatedly inserted into his arm, I found it impracticable to infect him with it; an efflorescence only, taking on an erysipelatous look about the centre, appearing on the skin near the punctured parts. During the whole time that his family had the

Small Pox, one of whom had it very full, he remained in the house with them, but received no injury from exposure to the contagion.

It is necessary to observe, that the utmost care was taken to ascertain, with the most scrupulous precision, that no one whose case is here adduced had gone through the Small Pox previous to these attempts to produce that disease. . . .

Case XVII

The more accurately to observe the progress of the infection, I selected a healthy boy, about eight years old, for the purpose of inoculation for the Cow Pox. The matter was taken from a sore on the hand of a dairymaid (Sarah Nelmes), who was infected by her master's cows, and it was inserted, on the 14th of May, 1796, into the arm of the boy by means of two superficial incisions, barely penetrating the cutis, each about half an inch long.

On the seventh day he complained of uneasiness in the axilla, and on the ninth he became a little chilly, lost his appetite, and had a slight head-ache. During the whole of this day he was perceptibly indisposed, and spent the night with some degree of restlessness, but on the day following he was perfectly well.

The appearance of the incisions in their progress to a state of maturation were much the same as when produced in a similar manner by variolous matter. The only difference which I perceived was, in the state of the limpid fluid arising from the action of the virus, which assumed rather a darker hue, and in that of the efflorescence spreading round the incisions, which had more of an erysipelatous look than we commonly perceive when variolous matter has been made use of in the same manner; but the whole died away (leaving on the inoculated parts scabs and subsequent eschars) without giving me or my patient the least trouble.

In order to ascertain whether the boy, after feeling so slight an affection of the system from the Cow Pox virus, was secure from the contagion of the Small Pox, he was inoculated the 1st of July following with variolous matter, immediately taken from a pustule. Several slight punctures and incisions were made on both his arms, and the matter was carefully inserted, but no disease followed. The same appearances were observable on the arms as we commonly see when a patient has had variolous matter applied after having either the Cow

Pox or the Small Pox. Several months afterwards he was again inoculated with variolous matter, but no sensible effect was produced on the constitution. . . .

I shall now conclude from this Inquiry with some general observations on the subject, and on some others which are interwoven with it.

Although I presume it may be unnecessary to produce further testimony in support of my assertion "that the Cow Pox protects the human constitution from the infection of the Small Pox," yet it affords me considerable satisfaction to say, that Lord Somerville, the President of the Board of Agriculture, to whom this paper was shown by Sir Joseph Banks, has found upon inquiry that the statements were confirmed by the concurring testimony of Mr. Dollan, a surgeon, who resides in a dairy country remote from this, in which these observations were made. With respect to the opinion adduced "that the source of the infection is a peculiar morbid matter arising in the horse," although I have not been able to prove it from actual experiments conducted immediately under my own eye, yet the evidence I have adduced appears sufficient to establish it.

They who are not in the habit of conducting experiments may not be aware of the coincidence of circumstance necessary for their being managed so as to prove perfectly decisive; nor how often men engaged in professional pursuits are liable to interruptions which disappoint them almost at the instant of their being accomplished. However, I feel no room for hesitation respecting the common origin of the disease, being well convinced that it never appears among the cows (except it can be traced to a cow introduced among the general herd which has been previously infected, or to an infected servant) unless they have been milked by some one who, at the same time, has the care of a horse affected with diseased heels.

The spring of the year 1797, which I intended particularly to have devoted to the completion of this investigation, proved from its dryness, remarkably adverse to my wishes; for it frequently happens, while the farmers' horses are exposed to the cold rains which fall at that season that their heels become diseased, and no Cow Pox then appeared in the neighborhood.

The active quality of the virus from the horses' heels is greatly increased after it has acted on the nipples of the cow, as it rarely happens that the horse affects his dresser with sores, and as rarely that a milkmaid escapes the infection when she milks infected cows. It is most active at the commencement of the disease, even before it has acquired a pus-like appearance; indeed I am not confident whether this property in the matter does not entirely cease as soon as it is secreted in the form of pus. I am induced to think it does cease, and that it is the thin darkish-looking fluid only, oozing from the newly-formed cracks in the heels, similar to what sometimes appears from erysipelatous blisters, which give the disease. Nor am I certain that the nipples of the cows are at all times in a state to receive the infection. The appearance of the disease in the spring and the early part of the summer, when they are disposed to be affected with spontaneous eruptions so much more frequently than at other seasons, induces me to think, that the virus from the horse must be received upon them when they are in this state, in order to produce effects: experiments, however, must determine these points. But it is clear that when the Cow Pox virus is once generated, that the cows cannot resist the contagion, in whatever state their nipples may chance to be, if they are milked with an infected hand.

Whether the matter, either from the cow or the horse will affect the sound skin of the human body, I cannot positively determine; probably it will not, unless on those parts where the cuticle is extremely thin, as on the lips for example. I have known an instance of a poor girl who produced an ulceration on her lip by frequently holding her finger to her mouth to cool the raging of a Cow-Pox sore by blowing upon it. The hands of the farmers' servants here, from the nature of their employments, are constantly exposed to those injuries which occasion abrasions of the cuticle, to punctures from thornes and such like accidents; so that they are always in a state to feel the consequences of exposure to infectious matter.

It is singular to observe that the Cow Pox virus, although it renders the constitution unsusceptible of the variolous, should, nevertheless, leave it unchanged with respect to its own action. I have already produced an instance to point out this, and shall now corroborate it with another.

Elizabeth Wynne, who had the Cow Pox in the year 1759, was inoculated with variolous matter, without effect, in the year 1797, and again caught the Cow Pox in the year 1798. When I saw her, which was on the eighth day after she received the infection, I found her affected with general lassitude, shiverings, alternating with heat, coldness of the extremities, and a quick and irregular pulse. These symptoms were preceded by a pain in the axilla. On her hand was one large pustulous sore.

It is curious also to observe, that the virus, which with respect to its effects is undetermined and uncertain previously to it passing from the horse through the medium of the cow, should then not only become more active, but should invariably and completely possess those specific properties which induce in the human constitution symptoms similar to those of the variolous fever, and effect in it that peculiar change which forever renders it unsusceptible of the variolous contagion.

May it not then be reasonably conjectured, that the source of the Small Pox is morbid matter of a peculiar kind, generated by a disease in the horse, and that accidental circumstances may have again and again arisen, still working new changes upon it, until it has acquired the contagious and malignant form under which we now commonly see it making its devastations amongst us? And, from a consideration of the change which the infectious matter undergoes from producing a disease on the cow, may we not conceive that many contagious diseases, now prevalent amongst us, may owe their present appearance not to a simple, but to a compound origin? For example, is it difficult to imagine that the measles, the scarlet fever, and the ulcerous sore throat with a spotted skin, have all sprung from the same source, assuming some variety in their forms according to the nature of their new combinations? The same question will apply respecting the origin of many other contagious diseases, which bear a strong analogy to each other.

H. PRINCIPLES OF GEOLOGY: BEING AN ATTEMPT TO EXPLAIN THE FORMER CHANGES OF THE EARTH'S SURFACE BY REFERENCE TO CAUSES NOW IN OPERATION

By Charles Lyell, Esq., F.R.S.

[*The first edition of this epoch-making work appeared in 1830 and the second edition, from which the following excerpts are taken, in 1832. Few if any books of the nineteenth century have had greater influence upon human thought. The first four chapters constitute an invaluable review of previous geological opinion, from the earliest times. The following quotations are from the end of the fourth and the latter part of the fifth chapters.*]

We have now arrived at the era of living authors, and shall bring to a conclusion our sketch of the progress of opinion in Geology. . . . A new school at last arose who professed the strictest neutrality, and the utmost indifference to the systems of Werner and Hutton, and who were resolved diligently to devote their labours to observation. The reaction, provoked by the intemperance of the conflicting parties, now produced a tendency to extreme caution. Speculative views were discountenanced, and through fear of exposing themselves to the suspicion of a bias towards the dogmas of a party, some geologists became anxious to entertain no opinion whatever on the causes of phenomena, and were inclined to scepticism even where the conclusions deducible from observed facts scarcely admitted of reasonable doubt.

Geological Society of London. — But although the reluctance to theorize was carried somewhat to excess, no measure could be more salutary at such a moment than a suspension of all attempts to form what were termed "theories of the earth." A great body of new data were required, and the Geological Society of London, founded in 1807, conduced greatly to the attainment of this desirable end. To multiply and record observations, and patiently to await the result at some future period, was the object proposed by them, and it was their favourite maxim that the time was not yet come for a general system of geology, but that all must be content for many years to be exclusively engaged in furnishing materials for future generalizations. By acting up to these principles with consistency, they in a few years disarmed all prejudice, and rescued the science from the imputation of being a dangerous, or at best but a visionary pursuit.

Modern Progress of Geology

Study of Organic Remains. — Inquiries were at the same time prosecuted with great success by the French naturalists, who devoted their attention especially to the study of organic remains. They shewed that the specific characters of fossil shells and vertebrated animals might be determined with the utmost precision, and by their exertions a degree of accuracy was introduced into this department of science, of which it had never before been deemed susceptible. It was found that, by the careful discrimination of the fossil contents of strata, the contemporary origin of different groups could often be established, even where all identity of mineralogical character was wanting, and where no light could be derived from the order of superposition.

The minute investigation, moreover, of the relics of the animate creation of former ages, had a powerful effect in dispelling the illusion which had long prevailed concerning the absence of analogy between the ancient and modern state of our planet. A close comparison of the recent and fossil species, and the inferences drawn in regard to their habits, accustomed the geologist to contemplate the earth as having been at successive periods the dwelling-place of animals and plants of different races, some of which were discovered to have been terrestrial, and others aquatic — some fitted to live in seas, others in the waters of lakes and rivers. By the consideration of these topics, the mind was slowly and insensibly withdrawn from imaginary pictures of catastrophes and chaotic confusion, such as haunted the imagination of the early cosmogonists. Numerous proofs were discovered of the tranquil deposition of sedimentary matter and the slow development of organic life. If many still continued to maintain, that "the thread of induction was broken," yet in reasoning by the strict rules of induction from recent to fossil species, they virtually disclaimed the dogma which in theory they professed. The adoption of the same generic, and, in some cases, even the same specific, names for the exuviæ of fossil animals, and their living analogues, was an important step towards familiarising the mind with the idea of the identity and unity of the system in distant eras. It was an acknowledgment, as it were, that a considerable part of the ancient memorials of nature were written in a living language. The growing importance then of the natural history of organic remains, and its general application to geology, may be pointed out as the characteristic feature of the progress of the science during the present century. This branch of knowledge has already become an instrument of great power in the discovery of truths in geology, and is continuing daily to unfold new data for grand and enlarged views respecting the former changes of the earth.

When we compare the result of observations in the last thirty years with those of the three preceding centuries, we cannot but look forward with the most sanguine expectations to the degree of excellence to which geology may be carried, even by the labours of the present generation. Never, perhaps, did any science, with the exception of astronomy, unfold, in an equally brief period, so many novel and unexpected truths, and overturn so many preconceived opinions. The senses had for ages declared the earth to be at rest, until the astronomer taught that it was carried through space with inconceivable rapidity. In like manner was the surface of this planet regarded as having remained unaltered since its creation, until the geologist proved that it had been the theatre of reiterated change, and was still the object of slow but never-ending fluctuations. The discovery of other systems in the boundless regions of space was the triumph of astronomy — to trace the same system through various transformations — to behold it at successive eras adorned with different hills and valleys, lakes and seas, and peopled with new inhabitants, was the delightful meed of geological research. By the geometer were measured the regions of space, and the relative distances of the heavenly bodies — by the geologist myriads of ages were reckoned, not by arithmetical computation, but by a train of physical events — a succession of phenomena in the animate and inanimate worlds — signs which convey to our minds more definite ideas than figures can do, of the immensity of time.

Whether our investigation of the earth's history and structure will eventually be productive of as great practical benefits to mankind, as a knowledge of the distant heavens, must remain for the decision of posterity. It was not till astronomy had been enriched by the observations of many centuries, and had made its way against popular prejudices to the establishment of a sound theory, that its application to the useful arts was most conspicuous. The cultivation of geology began at a later period; and in every step which it has hitherto made towards sound ethical principles, it has had to contend against more violent prepossessions. The practical advantages already derived from it have not been inconsiderable: but our generalizations are yet imperfect, and they who follow may be expected to reap the most valuable fruits of our labour. Meanwhile the charm of first discovery is our own, and as we explore this magnificent field of inquiry, the sentiment of a great historian of our times may continually be present to our minds, that "he who calls what has vanished back again into being, enjoys a bliss like that of creating." . . .

ASSUMPTION OF THE DISCORDANCE OF THE ANCIENT AND EXISTING CAUSES OF CHANGE UNPHILOSOPHICAL

. . . For more than two centuries the shelly strata of the Sub-Apennine hills afforded matter of speculation to the early geologists of Italy, and few of them had any suspicion that similar deposits were then forming in the neighboring sea. They were as unconscious of the continued action of causes still producing similar effects, as the astronomers, in the case supposed by us, of the existence of certain heavenly bodies still giving and reflecting light, and performing their movements as in the olden time. Some imagined that the strata, so rich in organic remains, instead of being due to secondary agents, had been so created in the beginning of things by the fiat of the Almighty; and others ascribed the imbedded fossil bodies to some plastic power which resided in the earth in the early ages of the world. At length Donati explored the bed of the Adriatic, and found the closest resemblance between the new deposits there forming, and those which constituted hills above a thousand feet high in various parts of the peninsula. He ascertained that certain genera of living testacea were grouped together at the bottom of the sea in precisely the same manner as were their fossil analogues in the strata of the hills, and that some species were common to the recent and fossil world. Beds of shells, moreover, in the Adriatic, were becoming incrusted with calcareous rock; and others were recently enclosed in deposits of sand and clay, precisely as fossil shells were found in the hills. This splendid discovery of the identity of modern and ancient submarine operations was not made without the aid of artificial instruments, which, like the telescope, brought phenomena into view not otherwise within the sphere of human observation.

In like manner, in the Vicentin, a great series of volcanic and marine sedimentary rocks were examined in the early part of the last century; but no geologist suspected, before the time of Arduino, that these were partly composed of ancient submarine lavas. If, when these enquiries were first made, geologists had been told that the mode of formation of such rocks might be fully elucidated by the study of processes then going on in certain parts of the Mediterranean, they would have been as incredulous as geometers would have been before the time of Newton, if any one had informed them that, by making experiments on the motion of bodies on the earth, they might discover the laws which regulated the movements of distant planets.

The establishment, from time to time, of numerous points of identification, drew at length from geologists a reluctant admission, that there was more correspondence between the physical constitution of the globe, and more uniformity in the laws regulating the changes of its surface, from the most remote eras to the present, than they at first imagined. If, in this state of the science, they still despaired or reconciling every class of geological phenomena to the operations of ordinary causes, even by straining analogy to the utmost limits of credibility, we might have expected, that the balance of probability at least would now have been presumed to incline towards the identity of the causes. But, after repeated experience of the failure of attempts to speculate on different classes of geological phenomena, as belonging to a distinct order of things, each new sect persevered systematically in the principles adopted by their predecessors. They invariably began, as each new problem presented itself, whether relating to the animate or inanimate world, to assume in their theories, that the economy of nature was formerly governed by rules quite independent of those now established. Whether they endeavoured to account for the origin of certain igneous rocks, or to explain the forces which elevated hills or excavated valleys, or the causes which led to the extinction of certain races of animals, they first presupposed an original and dissimilar order of nature; and when at length they approximated, or entirely came round to an opposite opinion, it was always with the feeling, that they conceded what they were justified *à priori* in deeming improbable. In a word, the same men who, as natural philosophers, would have been greatly surprised to find any deviation from the usual course of Nature *in their own time*, were equally surprised, as geologists, not to find such deviations at every period of the past.

The Huttonians were conscious that no check could be given to the utmost license of conjecture in speculating on the causes of geological phenomena, unless we can assume invariable constancy in the order of Nature. But when they asserted this uniformity without any limitation as to time, they were considered, by the majority of their contemporaries, to have been carried too far, especially as they applied the same principle to the laws of the organic, as well as of the inanimate world.

We shall first advert briefly to many difficulties which formerly appeared insurmountable, but which, in the last forty years, have been partially or entirely removed by the progress of science; and shall afterwards consider the objections that still remain to the doctrine of absolute uniformity.

In the first place, it was necessary for the supporters of this doctrine to take for granted incalculable periods of time, in order to explain the formation of sedimentary strata by causes now in diurnal action. The time which they required theoretically, is now granted, as it were, or has become absolutely requisite, to account for another class of phenomena brought to light by more recent investigations. It must always have been evident to unbiassed minds, that successive strata, containing, in regular order of superposition, distinct beds of shells and corals, arranged in families as they grow at the bottom of the sea, could only have been formed by slow and insensible degrees in a great lapse of ages, yet, until organic remains were minutely examined and specifically determined, it was rarely possible to prove that the series of deposits met with in one country was not formed simultaneously with that found in another. But we are now able to determine, in numerous instances, the relative dates of sedimentary rocks in distant regions, and to show, by their organic remains, that they were not of contemporary origin, but formed in succession. We often find, that where an interruption in the consecutive formations in one district is indicated by a sudden transition from one assemblage of fossil species to another, the chasm is filled up, in some other district, by other important groups of strata.

The more attentively we study the European continent, the greater we find the extension of the whole series of geological formations. No sooner does the calendar appear to be completed, and the signs of a succession of physical events arranged in chronological order, than we are called upon to intercalate, as it were, some new periods of vast duration. A geologist, whose observations have been confined to England, is accustomed to consider the superior and newer groups of marine strata in our island as modern, and such they are, comparatively speaking; but when he has travelled through the Italian peninsula and in Sicily, and has seen strata of more recent origin forming mountains several thousand feet high, and has marked a long series both of volcanic and submarine operations, all newer than any of the regular strata which enter largely into the physical structure of Great Britain, he returns with more exalted conceptions of the antiquity of some of our modern deposits, than he before entertained of the oldest of the British series.

We cannot reflect on the concessions thus extorted from us, in regard to the duration of past time, without foreseeing that the period may arrive when part of the Huttonian theory will be combated on the ground of its departing too far from the assumption of uniformity in the order of nature. On a closer investigation of extinct volcanos, we find proofs that they broke out at successive eras, and that the eruptions of one group were often concluded long before others had commenced their activity. Some were burning when one class of organic beings were in existence, others came into action when different races of animals and plants existed, — it follows, therefore, that the convulsions caused by subterranean movements, which are merely another portion of the volcanic phenomena, occurred also in succession, and their efforts must be divided into separate sums, and assigned to separate periods of time; and this is not all: when we examine the volcanic products, whether they be lavas which flowed out under water or upon dry land, we find that intervals of time, often of great length, intervened between their formation, and that the effects of one eruption were not greater in amount than that which now results during ordinary volcanic convulsions. The accompanying or preceding earthquakes, therefore, may be considered to have been also successive, and to have been in like manner interrupted by intervals of time, and not to have exceeded in violence those now experienced in the ordinary course of nature.

Already, therefore, may we regard the doctrine of the sudden elevation of whole continents by paroxysmal eruptions as invalidated; and there was the greatest inconsistency in the adoption of such a tenet by the Huttonians, who were anxious to reconcile former changes to the present economy of the world. It was contrary to analogy to suppose that Nature had been at any former epoch parsimonious of time and prodigal of violence — to imagine that one district was not at rest while another was convulsed — that the disturbing forces were not kept under subjection, so as never to carry simultaneous havoc and desolation over the whole earth, or even over one great region. If it could have been shown, that a certain combination of circumstances would at some future period produce a crisis in the subterranean action, we should certainly have had no right to oppose our experience for the last three thousand years as an argument against the probability of such occurrences in past ages; but it is not pretended that such a combination can be foreseen.

In speculating on catastrophes by water, we may certainly anticipate great floods in future, and we may therefore presume that they have happened again and again in past times. The existence of enormous seas of fresh water such as the North American lakes, the largest of which is elevated more than six hundred feet above the level of the ocean, and is in parts twelve hundred feet deep, is alone sufficient to assure us, that the time will come, however distant, when a deluge will lay waste a considerable part of the American continent. No hypothetical agency is required to cause the sudden escape of the confined waters. Such changes of level, and opening of fissures, as have accompanied earthquakes since the commencement of the present century, or such excavation of ravines as the receding cataract of Niagara is now effecting, might breach the barriers. Notwithstanding, therefore, that we have not witnessed within the last three thousand years the devastation by deluge of a large continent, yet, as we may predict the future occurrence of such catastrophes, we are authorized to regard them as part of the present order of Nature, and they may be introduced into geological speculations respecting the past, provided we do not imagine them to have been more frequent or general than we expect them to be in time to come.

The great contrast in the aspect of the older and newer rocks, in their texture, structure, and in the derangement of the strata, appeared formerly one of the strongest grounds for presuming that the causes to which they owed their origin were perfectly dissimilar from those now in operation. But this incongruity may now be regarded as the natural result of subsequent modifications, since the difference of the relative age is demonstrated to have been so immense, that, however slow and insensible the change, it must have become important in the course of so many ages. In addition to the volcanic heat, to which the Vulcanists formerly attributed too much influence, we must allow for the effect of mechanical pressure, of chemical affinity, of percolation by mineral waters, of permeation by elastic fluids, and the action, perhaps, of many other forces less understood, such as electricity and magnetism. In regard to the signs of upraising and sinking, of fracture and contortion in rocks, it is evident that newer strata cannot be shaken by earthquakes, unless the subjacent rocks are also affected; so that the contrast in the relative degree of disturbance in the more ancient and the newer strata, is one of many proofs that the convulsions have happened in different eras, and the fact confirms the uniformity of the action of subterranean forces, instead of their greater violence in the primeval ages.

The science of Geology is enormously indebted to Lyell — more so, as I believe, than to any other man who ever lived. — Darwin. *Autobiography.*

Pour juger de ce qui est arrivé, et même de ce qui arrivera, nous n'avons qu'à examiner ce qui arrive. — Buffon. *Théorie de la Terre.*

I. SOME INVENTIONS OF THE EIGHTEENTH AND NINETEENTH CENTURIES. APPLIED SCIENCE AND ENGINEERING

He who seeks for immediate practical use in the pursuit of science, may be reasonably sure that he will seek in vain. Complete knowledge and complete understanding of the action of the forces of nature and of the mind, is the only thing that science can aim at. The individual investigator must find his reward in the joy of new discoveries . . . in the consciousness of having contributed to the growing capital of knowledge. . . . Who could have imagined, when Galvani observed the twitching of the frog muscles as he brought various metals in contact with them, that eighty years later Europe would be overspun with wires which transmit messages from Madrid to St. Petersburg with the rapidity of lightning, by means of the same principle whose first manifestations this anatomist then observed. — *Helmholtz.*

The place of inventions in the history of science is hard to define. Conditioned as they doubtless are by a favorable environment — at least for survival — they do not always obviously arise as a direct or logical consequence of preceding discoveries, or even of known principles, but seem sometimes to spring almost *de novo* from the brain of the inventor. And yet such an origin is probably more apparent than real. The steam-engine could hardly have come from Watt without Newcomen and Black as his predecessors, the telegraph from Morse or the telephone from Bell except after Franklin, Oersted and Faraday. Probably the truth is that if we only knew all the facts, instead of only some of them, we should find every invention the natural descendant, near or remote, of science already existing. And as inheritance often seems to skip a generation or two and children sometimes show no discoverable resemblance to their immediate forbears, so inventions may come without disclosing any resemblance to parent inventions or ideas, while yet really intimately related to knowledge that has gone before.

Nor is it easy to estimate the reciprocal debt of science to inventions and the arts. That this debt is large there can be no doubt. To illustrate this fact it is hardly necessary to do more than mention examples, such as the service of the compass to the sciences of geography, navigation and surveying; of the telescope and the chronometer to astronomy; of the microscope to biology; of the air pump to natural philosophy; or of the abacus or the Arabic numerals to arithmetic.

Among the more notable of the inventions of the nineteenth century were the locomotive, the steamboat, the friction match, the sewing-machine, the steel pen, the telegraph, the telephone and the phonograph; labor-saving machinery; explosives; and the internal combustion engine, with its numerous offspring (motor vehicles, airplanes, motor boats, etc.).

POWER: ITS SOURCES AND SIGNIFICANCE. — The recent progress of science and of civilization has been accompanied by a remarkable extension of man's control over his environment, which has come largely with his ability to develop, transmit, and utilize chemical, gravitational and electrical energy or *power*. The ancients and the men of the Middle Ages used chiefly the power of man and other animals and of winds (windmills) and to some extent water (*i.e.* gravitation), as in water-wheels, but knew little of heat power or chemical power and nothing of electrical power, or of power transmission of any kind, — except in moving herds, treadmills, or marching armies.

> In past times the chief store of national power was manual labor: to-day it is the machine that does the work. — *K. Pearson.*

The first step in the modern direction was apparently toward chemical power, in the invention of gunpowder.

GUNPOWDER, NITROGLYCERINE, DYNAMITE. — Gunpowder is believed to have been known to the Chinese long belong it appeared in Europe. An explosive mixture of charcoal, sulphur, and nitre was apparently also known to the Arabians, but the first important appearance of gunpowder in Europe was about the fourteenth century, and since the sixteenth it has played an all-important part in war and in peace. Its effects upon society and civilization have been profound, and with society and civilization the progress of science is always closely bound up.

The manufacture of gunpowder marks the beginning of the manufacture of power, if we may describe the controlled accumulation, storage and liberation of energy by that convenient term. In 1845 gun-cotton was invented by Schönbein, and in 1847 nitroglycerine by Sobrero, and both explosives were found to be far more copious and powerful sources of energy than gunpowder. It was Alfred Nobel, however, a Swedish engineer, who after mixing nitroglycerine with gunpowder first made practical use of this for blasting. It was also Nobel who in 1867 made nitroglycerine less dangerous by diluting it with inert substances such as silicious earth, — mixtures to which he gave the name dynamite.[1]

The manufacture of power from gravitational sources, such as water-power and wind power, goes back to the earliest times — sails, wind-mills and water-wheels being of very ancient origin. Power from fuel begins with Newcomen, Watt and the steam-engine. Electrical power is at present chiefly derived indirectly from gravitational (hydraulic) or from chemical (fuel) sources.

THE STEAM-ENGINE. — The last half of the eighteenth century was not merely an era of great revolutions: it was also an age of great inventions and among these, first in importance as well as first to arise, was the steam-engine.

Various and more or less successful attempts to utilize heat or steam as a source of power had been made before Watt's time, such, for example, as those of Hero in Alexandria (120 B.C.) the Marquis of Worcester (1663) and Newcomen (1705). Of these only Newcomen's need be dwelt upon here. In Newcomen's engine a vertical cylinder with piston was used, the piston-rod, also vertical, being fixed above to one end of a walking-beam of which the other end carried a parallel rod. Thus the rise and fall of the piston caused a corresponding fall and rise of a parallel rod, which could be attached to anything, *e.g.* to a pump. The cylinder was connected with a steam boiler by a pipe fitted with a stopcock, and was filled with steam below the piston by opening the stopcock. The steam pressing upon the boiler raised the piston and depressed the parallel (pump) rod. The stopcock was then closed, a "vent" in the cylinder was opened, cold water was introduced from another pipe to condense the steam, whereupon a vacuum formed, and the atmospheric pressure depressed the piston and lifted the pump rod. By having the various stopcocks carefully worked by hand a certain regularity of operation could be obtained, but before long improvements were made and the stopcocks were caused to work automatically. But since the cold (condensing) water chilled the cylinder, much heat was necessarily wasted.

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[1] Nobel died in 1896, bequeathing his fortune, estimated at $9,000,000, to the founding of a fund which supports the international "prizes" — usually $40,000 each — which bear his name and are annually awarded to those who have most contributed to "the good of humanity." Five prizes have been usually given: viz. one in physics, one in chemistry, one in medicine or physiology, one in literature and one for the promotion of peace.

Watt began by inventing (in 1765) a separate condenser, for cooling the steam without cooling the cylinder, — thus saving a vast amount of heat. He next abandoned altogether the use of atmospheric pressure for depressing the piston, employing steam above as well as below the piston, to lower as well as to lift it: and with these improvements, to which he added many others, he soon had in his possession a serviceable and automatic steam-engine, rudimentary in many respects, but not essentially unlike that of to-day.

THE SPINNING JENNY, THE WATER-FRAME AND THE MULE.—In 1770 James Hargreaves patented the spinning jenny, a frame with a number of spindles side by side, by which many threads could be spun at once instead of only one, as in the old, one-thread, distaff or the spinning wheel. In 1771 Arkwright operated successfully in a mill a patent spinning machine which, because actuated by water power, was known as the "water-frame." In 1779 Crompton combined the principles involved in Hargreaves' and Arkwright's machines into one, which, because of this hybrid origin, became known as the spinning "mule." This proved so successful that by 1811 more than four and a half million spindles worked as "mules" were in operation in England.

A similar machine for weaving was soon urgently needed, and in 1785 the "power loom" of Cartwright appeared, although it required much improvement and was not widely used before 1813.

THE COTTON GIN (ENGINE). — With the inventions just described facilities arose for the manufacture of cotton as well as woollen, but the supply of raw cotton was limited, chiefly because of the difficulty of separating the staple (fibres) from the seeds upon which they are borne. Cotton had for centuries been grown and manufactured in

India, the fibres being separated from the seeds by a rude hand machine known as a *churka,* used by the Chinese and Hindus. By this it was impossible to clean cotton rapidly. The invention therefore in 1793 by Eli Whitney of Connecticut of the saw cotton-gin which enormously facilitated this separation was one of the most important inventions ever made. This consisted in a series of saws revolving between the interstices of an iron bed upon which the cotton was so placed as to be drawn through while the seeds were left behind. The value of the saw gin was instantly recognized and the output of cotton in America was rapidly and immensely increased by its use.

STEAM TRANSPORTATION. — Boats and ships propelled by man power or by the wind have been used from time immemorial, and parallel rails for wheeled conveyors moved by animal power or by gravity preceded the steam locomotive. The steamboat and the steam vehicle appeared at (or in the case of the latter even before) the opening of the nineteenth century.

The first practically successful steamboat was a tug, the *Charlotte Dundas,* built and operated in Soctland for the towing of canal boats by Symmington in 1802. The first commercially successful steamboat was Fulton's *Clermont,* on the Hudson, in 1807. The first steam-engine to run on roads appears to have been Cugnot's in France in 1769. The first to run on rails was Trevithick's, in 1804, built to fit the rails of a horse railway. This engine also discharged its exhaust steam into the funnel to aid the draught of the furnace, — a device of fundamental importance to the further development of the locomotive. The first practically successful locomotive was Stephenson's *Rocket* (1829).

The compound (double or triple expansion) engine, which dates from 1781 (Hornblower), 1804 (Woolf), and 1845 (McNaughton), embodies what is perhaps the greatest single improvement in the steam-engine in the nineteenth century. The turbine has begun to replace the reciprocating engine only very recently (1900).

THE ACHROMATIC COMPOUND MICROSCOPE. — The compound microscope, after its introduction about the middle of the seventeenth century, and its use by Malphigi, Kircher, Leeuwenhoek, Grew, and others, was of only limited value because of the spherical, and especially the chromatic, aberration of its lenses. This remained true until long after Huygens had perfected the eye-piece of the telescope, and Hall and Dolland had succeeded in correcting chromatic aberration in telescope objectives by the combination of crown and flint glass, in the eighteenth century.

Amici, of Modena, in 1812, Fraunhofer of Munich in 1816, Tully of London in 1824, J. J. Lister in 1830 and others gradually perfected the achromatic microscope objective, so that about 1835 really excellent instruments became accessible to microscopical investigators. The numerous discoveries in cellular biology and in pathology which soon followed testify to the extent and importance of these improvements.

ILLUMINATING GAS, — made by the destructive distillation of coal, was invented and introduced in 1792 by William Murdock, who in 1802 had so far perfected the process that even the exterior of his factory in Birmingham was illuminated with gas in celebration of the peace of Amiens.

FRICTION MATCHES,— were preceded early in the nineteenth century by splinters of wood coated with sulphur and tipped with a mixture of chlorate of potash and sugar. These when touched with sulphuric acid ignited. It was not, however, until 1827 that practical friction matches were made and sold. These were known, after their inventor, as "Congreves" and consisted of wooden splints coated with sulphur and tipped with a mixture of sulphide of antimony, chlorate of potash, and gum. When subjected to severe friction, specially arranged for, these took fire. The phosphorus friction match was introduced commercially in 1833.

THE SEWING-MACHINE. — Very few labor-saving inventions surpass in efficiency sewing-machines. These also were invented in the nineteenth century and had a gradual development, in which various inventors participated. The first which need be mentioned was that of a French tailor, named Thimonier, patented in 1830. It is said that although made of wood and clumsy, eighty of these machines were in use in Paris in 1841, when an ignorant mob wrecked the establishment in which they were located and nearly murdered the inventor. The most important ideas embodied in modern machines are, however, of strictly American origin, the work of Walter Hunt of New York, and of Elias Howe of Spencer, Massachusetts being of principal importance (1846). Other Americans, especially Singer, Grover, Wilson and Gibbs, afterwards contributed to the present excellence and variety of the sewing-machine.

PHOTOGRAPHY. — Scheele, the Swedish chemist, appears to have been the first to study the effect of sunlight on silver chloride. Others, including Rumford and Davy, observed the chemical properties of light, but it was Wedgwood who, in 1802, made the first photograph by throwing shadows upon white paper moistened with nitrate of silver. Wedgwood was unable, however, to fix his prints.

Daguerreotypes, taken on silver plated copper, date from 1839, and were made by covering the copper with a thin film of silver iodide, — a compound sensitive to light. The image was developed by mercury vapor and fixed by sodium hyposulphite. The discovery of the fixing power of hyposulphite was in itself alone of immense importance. With the name of Daguerre, who began experimenting in 1826, that of a fellow countryman and partner, Niepce, is intimately associated.

The subsequent development of photography is due to a host of workers. The collodion film which underlies all modern work was first introduced in 1850. It is said to be a practically perfect medium because totally unaffected by silver nitrate.

ANÆSTHESIA. THE OPHTHALMOSCOPE. — Anæsthesia, or insensibility to pain, during dental surgical operations was introduced, if not discovered, by Wells, a dentist of Hartford, Connecticut, who himself took nitrous oxide gas for anæsthesia in 1844. The first public demonstration of surgical anæsthesia under ether was made by a dentist, Morton, and a surgeon, Jackson, at the Massachusetts General Hospital in Boston in 1846. Anæsthesia by chloroform was introduced by Simpson of Edinburgh, in 1847.

The ophthalmoscope, an instrument for examination of the interior of the eye, of inestimable value to medicine, was invented by Helmholtz in 1851. It is said that when von Graefe, an eminent ophthalmologist, first saw with it the interior of the eye he cried out, "Helmholtz has unfolded to us a new world."

INDIA-RUBBER, — the coagulated and dried juice of the rubber tree, first reported by Herrera, "who in the second voyage of Columbus observed that the inhabitants of Hayti played a game with balls made 'of the gum of a tree' and that the balls although large were lighter, and bounced better, than the windballs of Castile," was at the end of the eighteenth century still a curiosity, employed by Priestley, among others, as an eraser or "rubber."

Rubber is a hydrocarbon soft when pure but readily hardened by "vulcanization," *i.e.* treatment with sulphur or certain sulphur compounds (chloride, carbon bisulphide), a process introduced by Goodyear in 1839.

ELECTRICAL APPARATUS; TELEGRAPH, TELEPHONE, ELECTRIC LIGHTING, ELECTRIC MACHINERY. — The first important application of electricity to the service of man was the telegraph. This is too well known to require more than the briefest description. An electric circuit in a wire "made" or "broken" at one point is likewise made or broken at all other points. Hence, it is only necessary to employ a preconcerted system of make-and-break signals to dispatch messages. This plan was first employed in 1836 by S. F. B. Morse, a native of Charlestown, Massachusetts, and the first telegraph line between two cities was installed between Baltimore and Washington in 1844. The first transatlantic cable was laid in 1858.

The telephone, invented by Alexander Graham Bell, is even more familiar. This, also, depends on the making and breaking of an electric circuit, not (as is usual in the telegraph) by a key manipulated by the finger, but by sound waves of the human voice impinging upon a delicate membrane (the transmitter) and reproduced at a distance by corresponding vibrations of another delicate membrane (the receiver).

Wireless telegraphy and wireless telephony differ from ordinary telegraphy and telephony merely in the use of signal waves set up in the ether instead of signal waves (*i.e.* making and breaking) set up in the current carried by a wire. Both arts are inventions of very recent date.

The electric light, which had long been known as a laboratory experiment, became of practical utility about 1880, with the invention of the incandescent lamp, first the carbon arc and then the carbon filament, the former by Brush, the latter by Edison.

The phonograph was invented by Edison in 1876, and was the culmination of attempts extending over many years to record and reproduce sound waves. In these attempts Young, König, Fleeming Jenkin and many others participated.

FOOD PRESERVING BY CANNING AND REFRIGERATION. — In 1810 Appert of France succeeded in preserving foods in closed vessels by heating and sealing while hot. In 1816 a small amount of food preserved in this way found its way into the British Navy, where its value was recognized to some extent as a preventive of scurvy. It was not, however, until after the American Civil War that the industry began to assume anything like the vast extent and importance it has since reached.

Refrigeration in various forms has been used for food preserving probably from the earliest times, but the present enormous industry of cold storage has all grown up since the middle of the nineteenth century with the invention and development of refrigerators (domestic and commercial) and especially of machines for producing and distributing compressed air or other vapors or brine ammonia and other liquids at very low temperatures. These have been perfected rather rapidly since 1860, but did not become common before 1880. The first cargo of fresh meat successfully exported from America to Europe was shipped in March, 1879, and from New Zealand to Europe in February, 1880, arriving after a passage of 98 days in excellent condition.

THE INTERNAL-COMBUSTION ENGINE. — For a century or thereabouts the steam-engine stood without a rival as a thermodynamic machine and prime mover. Innumerable attempts had been made meantime to construct other kinds of engines to convert heat more directly into power for mechanical work; but it was not until 1876 that the internal-combustion engine as improved by Otto became a practical success.

In the steam-engine, the furnace in which the heat is generated is external to the cylinder in which that heat does its work, the steam being merely an intermediary. It is therefore an external-combustion engine. Obviously, if the fuel burned is made to liberate its heat in the cylinder instead of the furnace, the steam can be dispensed with. This is what actually happens in the internal-combustion engine. The present enormous extent of the use of such engines for motors of all kinds, testifies to the importance of this invention.

ANILINE, — was first obtained from indigo in 1826 by Unverdorben and named by him *crystalline.* In 1834 Runge prepared a similar substance from coal tar, and in 1841 Fritsche obtained from indigo an oil which he called *aniline,* — a word derived from the Sanskrit *Nila,* the indigo plant. The commercial importance of aniline in the dye-stuffs industry dates from the discovery of mauve by Perkin in 1858. This was the first of the notable series of aniline dyes now so well known, and the forerunner of the immense color industry of to-day.

THE MANUFACTURE OF STEEL; BESSEMER. — The making of steel by the decarbonization of cast-iron, a process which initiated what has been called the "age of steel," was introduced by Bessemer (1813–1898) in 1856. Bessemer's attention was drawn to the subject by his recognition of the necessity of improving gun-metal. Bessemer's process was at first only partially successful, but since others have shown how to improve it (by the addition of spiegeleisen, etc.) it has reached enormous proportions.

AGRICULTURAL APPARATUS AND INVENTIONS. — Beginning about 1850 an era of improved agricultural apparatus began, of which one result has been the opening of vast tracts of farm lands which might otherwise have remained unproductive. Steel plows, better harrows, mowing-machines, horse-power rakes, haymaking machinery, and especially harvesters of ingenious design for cereal crops (first introduced by McCormick in 1834), threshing-machines and spraying-machines are to-day common, where these were almost unknown before 1875. Machinery has also been applied to dairying, first to the making of butter and cheese, and more recently even to the milking of cows. Progress has also been made in the preservation of milk and of eggs by condensing, drying, freezing, etc. by new and economical processes invented and applied since that time.

APPLIED SCIENCE. ENGINEERING. — Very much as discoveries and inventions blend together and as both spring from a common source, manifested as curiosity, inquiry, experimentation and correlation (*i.e.* from science), so applied science, including engineering, comes from a common ancestry, *i.e.* from correlated knowledge,—which is science. Both terms are loosely used and both cover to-day a multitude of diversified human activities.

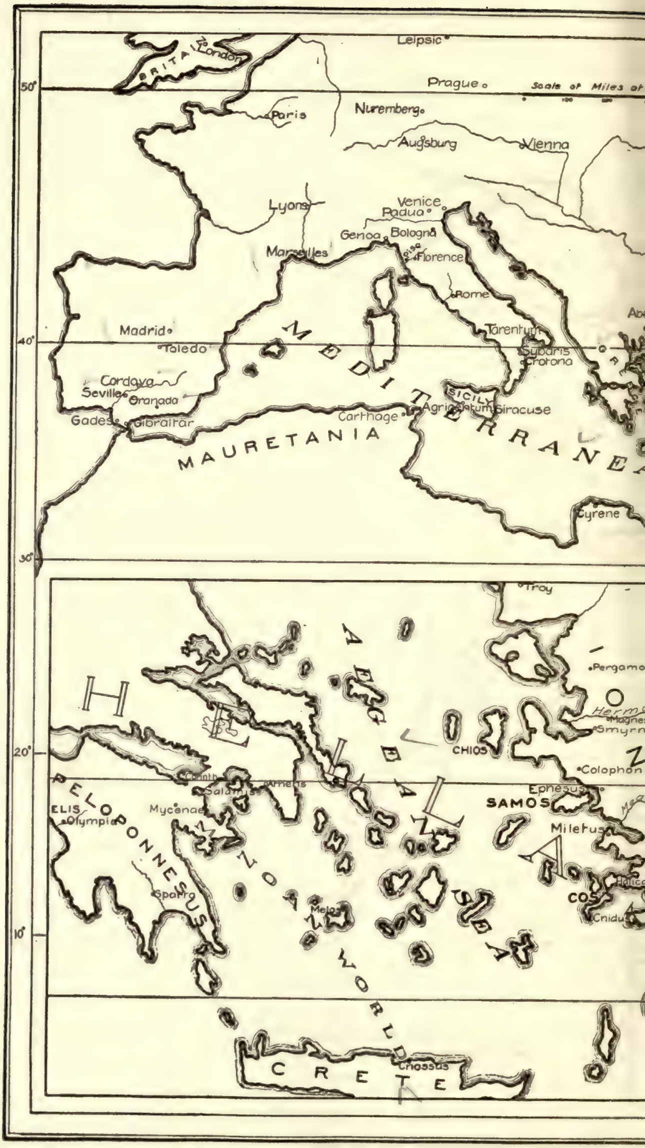
With the progress of science, arts and invention, engineering and other forms of applied science have developed so that these frequently have their own schools, either with or apart from universities and colleges; the school for miners at Freiberg, in Saxony, begun in 1765, being now only one of hundreds of technological and scientific schools for the training of engineers and others. Up to 1850 most engineers in America were trained in military schools and were primarily military engineers. But from that time forward the civil, as opposed to the military, engineer began to appear, and from the parent stem of civil engineering we now have mechanical, mining, electrical, sanitary, chemical, marine and other branches of engineering, often highly specialized. The term "engineer" is now very widely employed, with more or less appropriateness, to occupations remote from those of the military or civil engineer, as for example, the "illuminating engineer," the "efficiency engineer," the "public health engineer," etc. We may soon expect to have added to these many others, such as the agricultural engineer, the forest engineer and even the fishery engineer.

An historical sketch of applied science and engineering would obviously include the work of Archimedes, Vitruvius, Frontinus, and Leonardo, and proceed with the applications made of the discoveries and inventions of the Renaissance and modern times. Some of this ground is covered in the present volume, and more of it in the series of books by Smiles entitled Lives of the Engineers.

— There is scarcely a department of science or art which is the same, or at all the same, as it was fifty years ago. A new world of inventions — of railways and of telegraphs — has grown up around us which we cannot help seeing; a new world of ideas is in the air and affects us, though we do not see it.

— Bagehot. *Physics and Politics* (*1868*).

— Only since continental ideas and influences have gained ground in this country (*Great Britain*) *has the word science gradually taken the place of that which used to be termed natural philosophy or simply philosophy. One reason why science forms such a prominent feature in the culture of this age is the fact that only within the last hundred years has scientific research approached the more intricate phenomena and the more hidden forces and conditions which make up and govern our everyday life. The great inventions of the sixteenth, seventeenth and eighteenth centuries were made without special scientific knowledge, and frequently by persons who possessed skill rather than learning. They greatly influenced science and promoted knowledge, but they were brought about more by accident or by the practical requirements of the age than by the power of an unusual insight acquired by study. But in the course of the last hundred years the scientific investigation of chemical and electric phenomena has taught us to disentangle the intricate web of the elementary forces of nature, to lay bare the many interwoven threads, to break up the equilibrium of actual existence, and to bring within our power and under our control forces of undreamed-of magnitude. The great inventions of former ages were made in countries where practical life, industry and commerce were most advanced; but the great inventions of the last fifty years in chemistry and electricity and the science of heat have been made in the scientific laboratory: the former were stimulated by practical wants; the latter themselves produced new practical requirements, and created new spheres of labor, industry, and commerce. Science and knowledge have in the course of this century overtaken the march of practical life in many directions.* — Merz.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Bologna
Marseilles
Pisa
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Rome
Madrid
Toledo
40°
MEDITERRANEA
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Sybaris
Crotona
Cordova
Seville
Granada
Gades
Gibraltar
SICILY
Agrigentum
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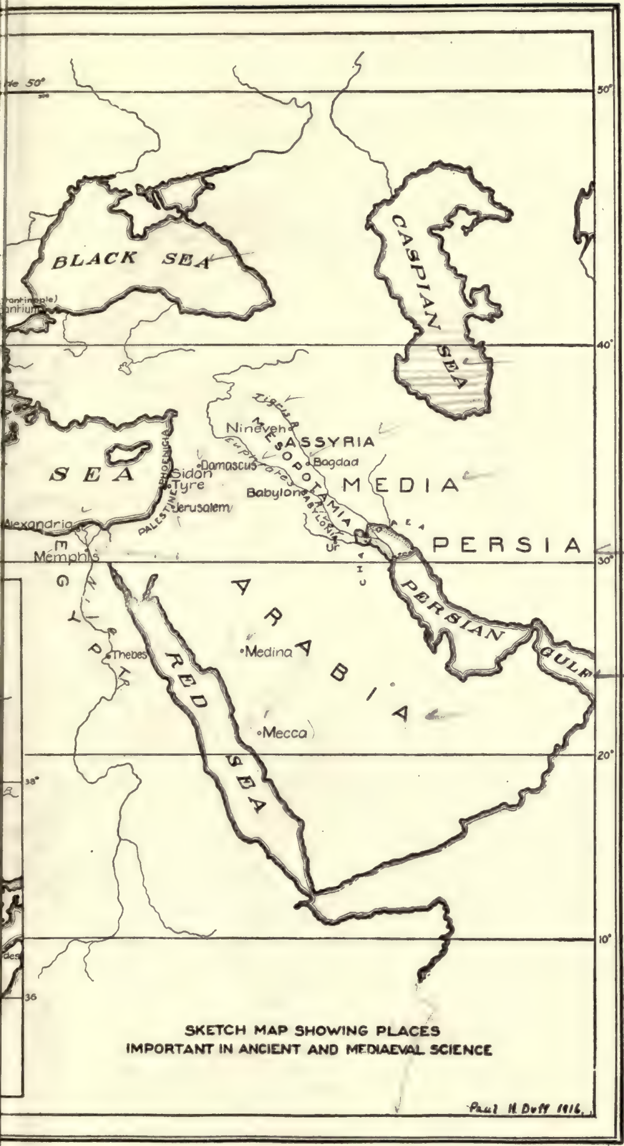
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Tigris
Nineveh
MESOPOTAMIA
ASSYRIA
Euphrates
Bagdad
Babylon
BABYLONIA
Ur
CHALDEA
ELAM
MEDIA
PERSIA
ARABIA
Medina
Mecca
RED SEA
PERSIAN GULF
38°
36
Paul H Duff 1916.

SKETCH MAP SHOWING PLACES
IMPORTANT IN ANCIENT AND MEDIAEVAL SCIENCE

SOME IMPORTANT NAMES, DATES AND EVENTS IN THE HISTORY OF SCIENCE AND CIVILIZATION

(For certain earlier events, see Chapters I and II.)

c. = *circa*, about.

	SCIENCE		GENERAL HISTORY, LITERATURE, ART, ETC.	
Before the Sixth Century B.C.	c. 2000–1700.	Ahmes Papyrus.	c. 1100.	*Gades (Cadiz) founded by the Phœnicians.*
			c. 1000.	Homer. David. Solomon.
			c. 850.	*Carthage founded.*
			c. 800.	Hesiod.
			c. 753.	*Rome founded (legendary).*
			c. 700.	*Nineveh flourishes under Sennacherib.*
	c. 640–546.	Thales.	c. 660.	*Byzantium founded.*
	c. 611–545.	Anaximander.	610.	Sappho and other Greek poets.
				Necho II undertakes to connect River Nile and Red Sea by Canal. His sailors circumnavigate Africa.
			c. 606.	*Nineveh destroyed.*
Sixth Century B.C.	c. 588–524.	Anaximenes.	c. 600.	*Marseilles founded.*
	c. 582–500.	Pythagoras.	c. 560.	Crœsus and Solon.
	c. 576–480.	Xenophanes.	c. 550–478.	Confucius.
	c. 540–475.	Heraclitus.	c. 538.	*Babylon taken by Cyrus.*
	c. 539.	Parmenides.	525–456.	Æschylus.
Fifth Century B.C.	c. 500.	Alcmæon.	c. 500.	*Carthaginians explore west coast of Africa.*
	c. 500–428.	Anaxagoras.		
	c. 470.	Hippocrates of Chios (Mathematician).	490–429.	Pericles.
			490.	*Marathon, Battle of.*
	469–399.	Socrates.	c. 484–425.	Herodotus.
	c. 465.	Empedocles.	480.	*Thermopylæ, Battle of.*
	c. 460.	Leucippus.	480.	*Salamis, Battle of.*
	c. 460–370.	Democritus.		
	c. 460.	Hippocrates of Cos (Physician).	480–406.	Euripides. Phidias.
			450–385.	Aristophanes.
	c. 428–347.	Archytas.		
	427–347.	Plato.	450–400.	Thucydides.
	c. 420.	Hippias.	c. 434–359.	Xenophon.
	c. 408–?	Eudoxus.	c. 430.	*The plague at Athens.*

	Science		General History, Literature, Art, etc.	
Fourth Century B.C.	c. 400.	Meton (Calendar).		
	c. 400.	*Motion of Earth* (Philolaus).	384–322.	Demosthenes.
			c. 370.	Diogenes. Scopas. Praxiteles.
	384–322.	Aristotle.		
	375–325.	Menæchmus.	356–323.	Alexander the Great.
	c. 375.	Heraclides of Pontus.	338.	*Chæronea, Battle of.*
	372–287.	Theophrastus.		
	c. 350–260.	Zeno (Stoic).	332.	*Alexandria founded.*
	342–270.	Epicurus.		
	c. 330–275.	Euclid.		
	c. 325.	Eudemus.	323–30.	The Ptolemies, I–VI.
			c. 300.	*Museum and Library of Alexandria.*
Third Century B.C.			300.	Epicurus.
			283.	*The Pharos built at Alexandria.*
	c. 300.	Herophilus. Erasistratus.		
			280.	*The Colossus of Rhodes.*
	287–212.	Archimedes.		
	c. 276–194.	Erastosthenes.		
	270–	Aristarchus.	c. 285.	Theocritus.
	c. 260–200.	Apollonius.		
	238.	*Decree of Canopus (Leap Year).*	269.	*Silver money first coined in Rome.*
			c. 210.	*The Great Chinese Wall begun.*
Second Century B.C.				*Paper made in China.*
			c. 170.	Polybius.
			c. 166.	Terence.
			161.	*Philosophers and Rhetoricians banished from Rome.*
	c. 135.	Ctesibius.		
	c. 146–126.	Hipparchus.		
			146.	*Carthage destroyed* (*rebuilt in 123*).

	Science		General History, Literature, Art, etc.	
First Century B.C.	98–55.	Lucretius.	106–43.	Cicero.
	c. 70–	Geminus.	102–44.	Cæsar.
	c. 63 B.C.–24 A.D.	Strabo.	59 B.C.–17 A.D.	Livy.
		Varro.	54 B.C.–39 A.D.	Seneca.
	47.	Julian Calendar.	47.	*Cæsar takes Alexandria.*
	14.	Vitruvius. *De Architectura.*	39.	*Pollio founds First Public Library.*
			27.	*End of Roman Republic.*
				Golden Age of Roman Literature. (Horace, Virgil, Livy, etc.)
First Century A.D.		Nicomachus.		
	23–79.	Pliny.		
	c. 40–103.	Frontinus.		
	c. 75.	Hero.		
Second Century	c. 130.	Galen.		
	c. 140.	Ptolemy. (*Almagest.*)		
	c. 140.	Theon of Smyrna.		
Third Century A.D.	c. 250.	Diophantus.	272–337.	Constantine (*First Christian Emperor*).
	c. 300.	Pappus.		
Fourth Century	c. 370.	Theon of Alexandria.	354–430.	St. Augustine.

	Science		General History, Literature, Art, etc.	
A.D. Fifth Century	410–485.	Proclus.		
	476.	Brahmagupta.	476.	*Fall of Rome.*
		Martianus Capella (*Liberal Arts*).		
Sixth Century	c. 480–524.	Boethius.		
	c. 530.	Arya-bhata.	529.	*Edict of Justinian. Schools of Athens closed.*
			569–632.	Mohammed.
Seventh Century			622.	*The Hegira.*
			641.	*Fall of Alexandria.*
Eighth			711.	*Moorish Conquest of Spain.*
	781–790.	*Schools of Alcuin.*	732.	*Moorish Invasion of Western Europe checked by Charles Martel.*
			c. 742–814.	Charlemagne.
Ninth Century	c. 830.	*Algebra of Alkarismi.*		
Tenth	940–1003.	Gerbert (Pope Sylvester II).	962.	*Holy Roman Empire.*
				Abelard.
	980–1037.	Avicenna.		
Eleventh Century	c. 1000.	Bhaskara.	1066.	*Battle of Hastings.*
	c. 1038.	Alhazen.	1096–1270.	*Crusades.*

	Science		General History, Literature, Art, etc.	
Twelfth Century		*Arabic numerals.*		
	1113–1150.	*Translations of Greek Classics from Arabic.*		
	1126–1198.	Averroes.		
		Jordanus Nemorarius.		
	1175.	Pisano (Fibonacci).		
Thirteenth	1206–1280.	Albertus Magnus.		
	1210.	*Aristotle's Physics proscribed in Paris.*	1215.	*Magna Charta.*
			c. 1254–1324.	Marco Polo.
			1265–1321.	Dante Alighieri.
	1214–1294.	Roger Bacon.		
	c. 1219.	*University of Bologna.*	c. 1300.	*Spectacles invented.*
	1249.	*University College, Oxford.*		
	1284.	*Peterhouse College, Cambridge.*		
		Mongolian Observatory at Meraga.		
Fourteenth			1304–1374.	Petrarch.
	1364.	University of Vienna.	1337–1453.	*The Hundred Years' War.*
			c. 1340–1400.	Chaucer.
			1340–1450.	*The Black Death.*
			1379–1446.	Brunelleschi.
Fifteenth Century	1401–1464.	Nicolas of Cusa.		
	1423–1461.	Peurbach.		
	1436–1476.	Regiomontanus.	1444–1511.	Bramante.
		Tartar Observatory (Samarcand).	c. 1450.	*Invention of Printing.*
			1453.	*Fall of Constantinople to the Turks.*
	1452–1519.	Leonardo da Vinci.		
	1473–1543.	Copernicus.	1471–1528.	Dürer.
	1486–1567.	Stifel.	1492.	*Discovery of America.*
	1490–1555.	Agricola.	1497.	*Vasco da Gama rounds Cape of Good Hope.*
	1493–1541.	Paracelsus.		
Sixteenth	1501–1576.	Cardan.		
	1503.	*Margarita philosophica.*		

	Science		General History, Literature, Art, etc.	
Sixteenth Century *(Continued)*	1510–1558.	Recorde.	1513.	*Balboa reaches Pacific Ocean.*
	c. 1506–1559.	Tartaglia.		
	1510–1589.	Palissy.	1517.	*Protestant Reformation.*
	1512–1594.	Mercator.		
	1514–1564.	Vesalius.	1519–1522.	*First Circumnavigation of the Globe by Magellan.*
	1514–1576.	Rheticus.		
	1516–1565.	Gesner.		
	1522–1565.	Ferrari.	1524–1580.	Camoens.
	1540–1603.	Vieta.	1530.	*Spinning wheel.*
	1543.	*De Revolutionibus of Copernicus.*	1547–1616.	Cervantes.
			c. 1552–1599.	Spenser.
	1543–1615.	Baptista della Porta.	1564–1616.	Shakespeare.
	1544–1603.	Gilbert.	1573–1637.	Ben Jonson.
	1546–1601.	Tycho Brahe.		
	1548–1600.	Bruno.		
	1548–1620.	Stevinus.	1588.	*Defeat of the Spanish Armada.*
	1550–1617.	Napier.		
	1560–1621.	Harriott.	1598.	*Edict of Nantes.*
	1561–1626.	Francis Bacon.		
	1564–1642.	Galileo.	1600–1681.	Calderon.
	1571–1630.	Kepler.		
	1575–1660.	Oughtred.		
	1577–1644.	Van Helmont.		
	1578–1657.	W. Harvey.		
	1582.	*Gregorian Calendar.*		
	1591–1626.	Snellius.		
	1593–1662.	Desargues.		
	1596–1650.	Descartes.		
	1598–1647.	Cavalieri.		
Seventeenth Century	1601–1665.	Fermat.		
	1602–1686.	Von Guericke.	1605.	*Don Quixote.*
	1608–1647.	Torricelli.	1607.	*First Permanent English Colony in America.*
	1616–1703.	Wallis.		
	1623–1662.	Pascal.		
	1624–1689.	Sydenham.	1608–1774.	Milton.
	1627–1691.	Boyle.	1618–1648.	*Thirty Years' War.*
	1629–1695.	Huygens.	1622–1673.	Molière.
	1630–1677.	Barrow.	1631–1700.	Dryden.
	1635–1703.	Hooke.	1636.	*Harvard College founded.*
	1635–1672.	Willughby.		
	1642–1727.	Newton.	1638–1715.	Louis XIV.
	1646–1716.	Leibnitz.	1639–1699.	Racine.

	Science		General History, Literature, Art, etc.	
Seventeenth Century (*Continued*)	1637.	*Discours sur la Méthode.* (*Analytic Geometry.*)	1649–1660.	*English Commonwealth.*
	1644–1710.	Roemer.		
	1656–1742.	Halley.	1660–1731.	De Foe.
	1660–1734.	Stahl.		
	1668–1738.	Boerhaave.	1672–1725.	Peter the Great.
	1677–1761.	Hales.		
	1687.	*Principia of Newton.*	1683.	*Siege of Vienna by Turks.*
	1698–1746.	Maclaurin.	1688–1744.	Pope.
	1699–1739.	Dufay.	1694–1778.	Voltaire.
	1699–1777.	Jussieu.		
Eighteenth Century	1700–1782.	Bernouilli, D.		
	1705.	*Newcomen's Engine.*		
	1706–1790.	Franklin.		
	1707–1778.	Linnæus.		
	1707–1783.	Euler.		
	1707–1788.	Buffon.		
	1707–1777.	Haller.	1709–1784.	Samuel Johnson.
	1717–1783.	d'Alembert.	1711–1776.	Hume.
	1726–1797.	Hutton.		
	1728–1793.	Hunter.	1728–1774.	Goldsmith.
	1731–1810.	Cavendish.	1732–1790.	Washington.
	1733–1804.	Priestley.		
	1736–1813.	Lagrange.		
	1736–1819.	Watt.	1737–1794.	Gibbon.
	1738–1822.	Herschel, F. W.		
	1742–1786.	Scheele.		
	1743–1794.	Lavoisier.		
	1744–1829.	Lamarck.		
	1746–1818.	Monge.		
	1749–1827.	Laplace.	1749–1832.	Goethe.
	1750–1817.	Werner.		
	1752–1833.	Legendre.	1759–1796.	Burns.
	1753–1814.	Rumford.	1759–1805.	Schiller.
	1764.	*Watt's Steam Engine.*		
	1766–1844.	Dalton.		
	1767.	*Spinning Jenny.*		
	1769–1832.	Cuvier.		
	1769–1859.	Humboldt.	1769–1821.	Napoleon I.

	Science		General History, Literature, Art, etc.	
Eighteenth Century *(Continued)*	1769.	*Spinning Frame.*	1770–1850.	Wordsworth.
	1773–1829.	Young.	1771–1832.	Scott.
	1774.	*Discovery of Oxygen.*	1772–1834.	Coleridge.
			1773–1859.	Metternich.
	1775–1836.	Ampère.	1775–1781.	*American Revolution.*
	1776–1839.	Treviranus.		
	1777–1855.	Gauss.		
	1778–1829.	Davy.		
	1778–1841.	De Candolle.		
	1779–1848.	Berzelius.		
	1781–1848.	Stephenson.		
	1781.	*Discovery of Uranus.*		
	1783.	*Air Balloon.*		
	1784–1846.	Bessel. *Parallax of stars.*	1789–1794.	*French Revolution.*
	1791–1867.	Faraday.		
	1791–1872.	Morse.		
	1792.	*Cotton Gin.*	1792–1822.	Shelley.
	1792–1876.	von Baer.		
	1793–1856.	Lobatchewski.		
	1794.	*École Polytechnique.*	1795–1821.	Keats.
			1795–1881.	Carlyle.
	1796.	*Vaccination.*		
	1796–1832.	Carnot.		
	1799–1853.	St. Hilaire.		
Nineteenth Century	1801–1858.	Müller.		
	1803–1873.	Liebig.	1802–1885.	Victor Hugo.
	1807–1873.	Agassiz, L.	1802–1894.	Kossuth.
	1809–1882.	Darwin.	1803–1882.	Emerson.
	1810–1888.	Gray, Asa.	1804–1865.	Cobden.
	1811–1877.	Leverrier.	1805.	*Battle of Trafalgar.*
	1811–1899.	Bunsen.	1805–1872.	Mazzini.
	1813–1878.	Bernard.	1807–1882.	Longfellow.
	1813–1898.	Bessemer.	1807–1882.	Garibaldi.
	1817–1911.	Hooker.	1807–1892.	Whittier.
	1818–1889.	Joule.	1809–1865.	Lincoln.
	1819–1892.	Adams.	1809–1892.	Tennyson.
	1820–1903.	Spencer.	1810–1861.	Cavour.
	1820–1910.	Florence Nightingale.	1811–1863.	Thackeray.
			1812–1870.	Dickens.
	1821–1894.	Helmholtz.	1815.	*Battle of Waterloo.*
	1822–1882.	Mendel.	1815–1898.	Bismarck.

Nineteenth Century *(Continued)*

Science	
1822–1888.	Clausius.
1822–1895.	Pasteur.
1822–1908.	Gibbs.
1823–1913.	Wallace.
1824–1887.	Kirchhoff.
1824–1907.	Kelvin.
1825–1894.	Huxley.
1827–1914.	Lister.
1828.	*Synthesis of Urea.*
1828.	*Stephenson's "Rocket."*
1829–1896.	Kekulé.
1830.	*Lyell's Principles of Geology.*
1831–1879.	Maxwell.
1834–1906.	Langley.
1834–1907.	Mendeléeff.
1834–1914.	Weismann.
1835–1909.	Newcomb.
1836.	*The Telegraph.*
1838–1907.	Perkin.
1839.	*The Daguerreotype.*
1843–1910.	Koch.
1846.	*Anæsthesia.*
	Discovery of Neptune.
1847–	Graham Bell.
1847.	*Die Erhaltung der Kraft.*
1847–	Edison.
1848–	De Vries.
1852–	Van't Hoff.
1857–1894.	Hertz.
1858.	*Atlantic Cable.*
1859–1860.	*Spectroscope.*
1859–	Arrhenius.
1859.	*Origin of Species.*
1868.	*Antiseptic Surgery.*
1876.	*Germ Theory.*
	Telephone.
1877.	*Phonograph.*
1896.	*First Successful Flight* (Langley).

General History, Literature, Art, etc.	
1834.	*Poor Law Reform in England.*
1837.	*Accession of Queen Victoria.*
1846–1848.	*War between United States and Mexico.*
1846.	*Repeal of the Corn Laws.*
1848.	*Abdication of Louis Philippe of France.*
1852.	*Accession of Napoleon III.*
1853–1856.	*Crimean War.*
1854.	*First (Perry) Treaty between United States and Japan.*
1859–	William II of Germany.
1859.	*Peace of Villafranca.*
1861.	*First Italian Parliament.*
1861–1865.	*Civil War in the United States.*
1866.	*War between Prussia and Austria.*
1867.	*End of the Shogunate of Japan.*
1870–1871.	*War between Prussia and France.*
1890.	*Promulgation of Constitution of Japan.*
1894.	*War between China and Japan.*
1898.	*War between Spain and the United States.*
1899.	*Boer War in South Africa.*
1900.	*Boxer Uprising in China.*

A SHORT LIST OF REFERENCE BOOKS

Particular attention may be called to the important and valuable publications of the John Crerar Library, Chicago, prepared by Aksel G. S. Josephson: viz. *A List of Books on the History of Science* (*1911*); *Supplement to the Same* (*1916*); and *A List of Books on the History of Industry and Industrial Arts* (*1915*).

A. General

Abelson, Paul.	The Seven Liberal Arts. A study in medieval culture.
Arago, F. E.	Biographies of Distinguished Scientific Men.
Avebury.	(See Lubbock.)
Bacon, Francis.	The Advancement of Learning.
Bacon, Roger.	(See Little.)
Bury, J. B.	A History of Freedom of Thought.
Dannemann, F.	*Die Naturwissenschaften in ihrer Entwickelung und in ihrem Zusammenhange.* 4 vols. (1910–1913.)
	Aus der Werkstatt grosser Forscher.
Draper, J. W.	History of the Conflict between Religion and Science.
	History of the Intellectual Development of Europe.
Griffiths, A. B.	Biographies of Scientific Men.
Henderson, L. J.	The Order of Nature.
Holland, R. S.	Historic Inventions.
Hume, M.	The Spanish People.
Huxley, T. H.	Advance of Science in the Last Half Century.
Isis.	*Revue consacrée à l'histoire de la science.* (March 1913–June 1914.) Edited by G. Sarton.

Ker, W. P.	The Dark Ages.
Lee, Sidney.	Great Englishmen of the Sixteenth Century.
✓ Libby, W.	Introduction to the History of Science.
Little, A. G.	Roger Bacon, Commemoration Essays.
Lubbock, J.	Fifty Years of Science.
Merz, J. T.	A History of European Thought in the Nineteenth Century.
Nineteenth Century, The.	A review of progress during the past one hundred years in the chief departments of human activity.
Ostwald, W. F.	*Grosse Männer.*
Ostwald, W. F., Editor.	*Klassiker der Exacten Wissenschaften* (contains many volumes of historical interest).
Pearson, K.	The Grammar of Science.
Phillips, L. March.	In the Desert.
Picard, Émile.	*La science moderne et son état actuel.*
Putnam, G. H.	Books and their Makers during the Middle Ages.
Rashdall, H.	Universities of Europe in the Middle Ages.
Rolt-Wheeler, F. W.	The Science-History of the Universe.
Routledge, R.	A Popular History of Science.
Sarton, G.	(*See p. 459.*)
Schaff, P.	The Renaissance.
Smiles, Samuel.	Lives of the Engineers.
Symonds, J. A.	Renaissance in Italy. The Revival of Learning.
Thomson, J. A.	Progress of Science in the Century.
Vernon-Harcourt, L. F.	Achievements in Engineering during the Last Half Century.
Wallace, A. R.	The Wonderful Century.
Walsh, J. J.	Catholic Churchmen in Science.
	The Popes and Science.
	The Thirteenth, Greatest of Centuries.
Whewell, W.	History of the Inductive Sciences, from the Earliest to the Present Times. (1837.)
	The Philosophy of the Inductive Sciences, founded upon their history. (1847.)

Whewell, W.	History of Scientific Ideas. Being the first part of the philosophy of the inductive sciences. (1858.)
White, A. D.	A History of the Warfare of Science with Theology in Christendom.
✓ Williams, H. S.	A History of Science. Nineteenth Century Science.

B. Ancient Science

Allman, G. J.	Greek Geometry from Thales to Euclid.
Apollonius of Perga.	Treatise on Conic Sections. (c. 220 B.C.)
Aristotle.	On the Parts of Animals; on Generation, etc.
Baikie, James.	Sea Kings of Crete.
Breasted, J. H.	Ancient Times: a history of the early world.
Butcher, S. H.	Some Aspects of the Greek Genius.
Davidson, Thomas.	Aristotle and Ancient Educational Ideals.
Euclid.	Elements. (c. 300 B.C.)
Fairbanks, A.	First Philosophers of Greece.
Freeman, K. E.	Schools of Hellas.
Frontinus.	The Waterworks of Rome. (c. 100 A.D.)
Galen.	On the Natural Faculties. (c. 180 A.D.)
Gibbon, Edward.	Decline and Fall of the Roman Empire.
Gomperz, Theodor.	Greek Thinkers: a history of ancient philosophy.
Gow, James.	A Short History of Greek Mathematics.
Grote, George.	Plato and the Other Companions of Socrates.
Hawes, C. H. and H.	Crete the Forerunner of Greece.
Heath, Thomas L.	Apollonius of Perga. Archimedes. Aristarchus of Samos. The Thirteen Books of Euclid's Elements. Diophantus of Alexandria.
Jastrow, M.	The Civilization of Babylonia and Assyria.
Lewes, G. H.	Aristotle; a Chapter in the History of Science.

Lubbock, John.	Prehistoric Times.
Lucretius.	On the Nature of Things. (c. 90 B.C.)
Mahaffy, J. P.	What have the Greeks done for Modern Civilization? Alexander's Empire.
Osborn, H. F.	Men of the Old Stone Age.
Pliny.	Natural History. (c. 70 A.D.)
Ptolemy.	Almagest. (c. 150 A.D.)
Seneca.	Physical Science in the Time of Nero. (c. 30 A.D.)
Strabo.	Geography. (c. 10 A.D.)
Tannery, Paul.	*La Géométrie Grecque.*
Walden, J. W. H.	The Universities of Ancient Greece.

C. Mathematical Science

(Mathematics, Astronomy, Mechanics)

Arthur, James.	Time and its Measurement.
Ball, R. S.	Great Astronomers.
Ball, W. W. R.	A Primer of the History of Mathematics. A Short Account of the History of Mathematics.
Berry, Arthur.	A Short History of Astronomy.
Bibliotheca Mathematica.	*Zeitschrift für Geschichte der mathematischen Wissenschaften.*
Brewster, David.	Martyrs of Science. Memoirs of Sir Isaac Newton.
Bryant, W. W.	A History of Astronomy.
Cajori, Florian.	A History of Elementary Mathematics. A History of Mathematics. A History of the Logarithmic Slide-rule and Allied Instruments. The Teaching and History of Mathematics in the United States.
Cantor, Moritz.	*Vorlesungen über Geschichte der Mathematik.* Vols. I–IV. (1892–1898.)
Chasles, Michel.	*Aperçu historique sur l'origine et le dévéloppement des méthodes en géométrie.*

Clerke, A. M.	A Popular History of Astronomy during the Nineteenth Century.
	The Herschels and Modern Astronomy.
	Modern Cosmogonies.
de Morgan, Augustus.	A Budget of Paradoxes.
Descartes, René.	A Discourse on Method, etc. (1637.)
Dreyer, J. L. E.	Tycho Brahe.
	History of Planetary Systems from Thales to Kepler.
Duhem, Pierre.	*Les origines de la statique.*
Enriques, F.	Problems of Science.
Fink, Karl.	A Brief History of Mathematics.
Forbes, George.	History of Astronomy.
Galileo, G.	Dialogues concerning Two New Sciences, (1638) translated by H. Crew and A. de Salvio.
Grant, Robert.	History of Physical Astronomy, from the earliest ages to the middle of the nineteenth century.
Haldane, E. S.	Descartes, His Life and Times.
Hale, G. E.	The Study of Stellar Evolution.
Hobson, E. W.	"Squaring the Circle." A History of the Problem.
Lockyer, J. N.	The Dawn of Astronomy. A Study of the Temple Worship and Mythology of the Ancient Egyptians.
Lodge, Oliver.	Pioneers of Science.
Mach, Ernst.	The Science of Mechanics. A Critical and Historical Exposition of its Principles.
Miller, G. A.	Historical Introduction to Mathematical Literature.
Morley, Henry.	Jerome Cardan.
Napier, John.	Tercentenary Memorial Volume.
Newton, Isaac.	*Philosophiæ naturalis principia mathematica.* (1687.)
Pierpont, James.	The History of Mathematics in the Nineteenth Century.
Poincaré, H.	Science and Hypothesis.

Poincaré, H.	The Value of Science.
Reeves, E.	Maps and Map Making. Three lectures delivered under the auspices of the Royal Geographical Society.
Smith, D. E.	*Rara Arithmetica.* A catalogue of the arithmetics written before the year MDCI with a description of those in the library of George Arthur Plimpton of New York.
	History of Modern Mathematics.
Todhunter, I.	History of the Mathematical Theories of Attraction and the Figure of the Earth from the Time of Newton to that of Laplace.
	A History of the Theory of Elasticity and of the Strength of Materials, from Galileo to Present Time.
	A History of the Mathematical Theory of Probability from the Time of Pascal to that of Laplace.

D. Physical and Chemical Science

Armitage, F. P.	A History of Chemistry.
Arrhenius, S. A.	Theories of Chemistry, being lectures delivered at the University of California.
Barus, Carl.	The Progress of Physics in the Nineteenth Century.
Bauer, Hugo.	A History of Chemistry.
Berry, A. J.	The Atmosphere.
Boyle, Robert.	Sceptical Chymist. (1660.)
Cajori, Florian.	A History of Physics in its Elementary Branches, including the Evolution of Physical Laboratories.
Clarke, F. W.	The Progress and Development of Chemistry during the Nineteenth Century.
Fahie, J. J.	Galileo, His Life and Work.

Gilbert, W.	On the Loadstone and Magnetic Bodies, and on the Great Magnet, the Earth.
Glazebrook, R. T.	James Clerke Maxwell and Modern Physics.
Gray, A.	Lord Kelvin.
Hales, S.	Hæmostatics, etc. (1733).
Hilditch, T.	A Concise History of Chemistry.
Jones, Bence.	Life and Letters of Faraday.
Jones, H. C.	A New Era in Chemistry; some of the more important developments in General Chemistry during the last quarter of a century.
Kelvin.	(See Thomson.)
Lodge, Oliver.	Signalling through Space without Wires. Being a Description of the Work of Hertz and His Successors.
Mach, Ernst.	History and Root of the Principle of the Conservation of Energy.
Mendenhall, T. C.	A Century of Electricity.
Meyer, Ernst.	A History of Chemistry from Earliest Times to the Present Day.
Muir, M. M. Pattison.	A History of Chemical Theories and Laws.
	The Story of Alchemy and the Beginnings of Chemistry.
O'Reilly, M. F. (Potamian).	Makers of Electricity.
Priestley, Joseph.	Experiments and Observations on Different Kinds of Air; On the Generation of Air from Water, etc.
Ramsay, William.	Essays Biographical and Chemical.
Ramsay, William.	The Gases of the Atmosphere. The history of their discovery.
Redgrove, H. S.	Alchemy: Ancient and Modern.
Roberts, E.	Famous Chemists.
Schorlemmer, Carl.	The Rise and Development of Organic Chemistry.
Tait, P. G.	Lectures on Some Recent Advances in Physical Science.
Thomson, William.	Popular Lectures and Addresses.

Thorpe, T. E.	Essays in Historical Chemistry. History of Chemistry.
Tyndall, John.	Faraday as a Discoverer.
Venable, F. P.	The Development of the Periodic Law.
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Whetham, W. C. D.	The Recent Development of Physical Science.
Whittaker, E. T.	A History of the Theories of Æther and Electricity from the Age of Descartes to the Close of the Nineteenth Century.

E. Natural Science

(See also p. 398)

Agassiz, E. C.	Louis Agassiz, His Life and Correspondence.
Allbutt, T. C.	The Historical Relations of Medicine and Surgery to the End of the Sixteenth Century.
Buck, A. H.	The Growth of Medicine to the Nineteenth Century.
Buckley, A. B.	A Short History of Natural Science and of the Progress of Discovery from the Time of the Greeks to the Present Day.
Clodd, Edward.	Pioneers of Evolution from Thales to Huxley.
Darwin, Francis.	Life of Charles Darwin.
Duncan, P. M.	Heroes of Science. Botanists, Zoölogists and Geologists.
Foster, Michael.	Lectures on the History of Physiology during the Seventeenth and Eighteenth Centuries.
Garrison, F. H.	Introduction to the History of Medicine.
Geikie, Archibald.	The Founders of Geology.
Goode, G. Brown.	The Beginnings of Natural History in America.
Green, E. L.	Landmarks of Botanical History.
Green, J. R.	A History of Botany.

Haddon, A. C.	History of Anthropology.
Harvey, William.	An Anatomical Dissertation upon the Movement of the Heart and the Blood in Animals. (1628).
Huxley, Leonard.	Life and Letters of Thomas Henry Huxley.
Huxley, T. H.	Man's Place in Nature. Essays.
	Advance of Science in Last Half Century.
Judd, J. W.	The Coming of Evolution.
Lankester, E. R.	History and Scope of Zoölogy.
Locy, W. A.	Biology and Its Makers.
Lull, R. S.	Organic Evolution.
Osborn, H. F.	From the Greeks to Darwin. An outline of the development of the evolution idea.
	The Origin and Evolution of Life.
Osler, William.	The Growth of Truth as illustrated in the discovery of the circulation of the blood.
Paget, Stephen.	Pasteur and After Pasteur.
Park, Roswell.	An Epitome of the History of Medicine.
Payne, J. F.	Thomas Sydenham.
Power, D'Arcy.	William Harvey.
Romanes, G. J.	Darwin and After Darwin.
Sachs, J	History of Botany.
Thomson, J. A.	The Science of Life. An outline of the history of biology and its recent advances.
Vallery-Radot, R.	Life of Pasteur.
Woodward, H. B.	History of Geology.
Zittel, K. A. von.	History of Geology and Palæontology to the End of the Nineteenth Century.

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